

STAT 405-01: Mathematical Statistics I

Problem Set 5

Assigned 2013 October 3

Due 2013 October 10

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Hogg 2.6.1

2 Hogg 2.6.2

3 Hogg 2.6.5

4 Hogg 2.6.8

5 Hogg 2.6.9

6 Multivariate Transformation

Let X_1, X_2, X_3 be iid with common pdf $f(x) = (2\pi)^{-1/2} \exp(-x^2/2)$, $-\infty < x < \infty$. Define $Y_1 = \sqrt{X_1^2 + X_2^2 + X_3^2}$, $Y_2 = \text{atan2}(\sqrt{X_1^2 + X_2^2}, X_3)$, $Y_3 = \text{atan2}(X_2, X_1)$ so that $X_1 = Y_1 \sin Y_2 \cos Y_3$, $X_2 = Y_1 \sin Y_2 \sin Y_3$, and $X_3 = Y_1 \cos Y_2$. (Recall that

$$\text{atan2}(\eta, \xi) = \begin{cases} \tan^{-1}(\eta/\xi) - \pi & \xi < 0 \text{ and } \eta < 0 \\ -\pi/2 & \xi = 0 \text{ and } \eta < 0 \\ \tan^{-1}(\eta/\xi) & \xi > 0 \\ \pi/2 & \xi = 0 \text{ and } \eta > 0 \\ \tan^{-1}(\eta/\xi) + \pi & \xi < 0 \text{ and } \eta \geq 0 \end{cases}$$

so that $\text{atan2}(\eta, \xi) \in (-\pi, \pi]$ and in particular, if $\xi \geq 0$ then $\text{atan2}(\eta, \xi) \in [-\pi/2, \pi/2]$.) Find the transformed pdf $f_{Y_1, Y_2, Y_3}(y_1, y_2, y_3)$.