1016-351-70
Probability

Problem Set 8

Assigned 2010 May 4
Due 2010 May 11

Show your work on all problems!

1 Devore Chapter 5, Problem 38

2 Devore Chapter 5, Problem 46

3 Devore Chapter 5, Problem 50

4 Devore Chapter 5, Problem 72

5 Devore Chapter 5, Problem 89 (Extra Credit)

[This problem provides the theoretical justification for our original definition of a chi-squared random variable as the sum of the squares of independent standard normal rvs.–JTW]

6 Computational Exercise (Extra Credit)

A random variable X obeying a χ^2 distribution with ν degrees of freedom has a pdf

$$f(x;\nu) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)}\, x^{(\nu/2)-1}\, e^{-x/2} & x > 0 \\ 0 & x < 0 \end{cases} \tag{6.1}$$

as well as a mean $\mu = \nu$ and variance $\sigma^2 = 2\nu$. Since it is the sum of ν iid rvs (each of which is the square of a standard normal random variable), the central limit theorem says that it should be approximated, in the limit that ν is large, by a normal distribution

$$f(x;\nu) \approx f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \tag{6.2}$$

a. For $0 < x < 20$, plot the exact chi-squared pdf and the normal approximation for $\nu = 5$.
b. For $0 < x < 200$, plot the exact chi-squared pdf and the normal approximation for $\nu = 50$.

Warning: If you use matplotlib via

```
ipython -pylab
```

the `gamma` imported into your namespace produces gamma-distributed random variables; if you want the gamma function to calculate $\Gamma(\nu/2)$ you'll need

```
from scipy.special import gamma
```