

Physics A301: Classical Mechanics II

Problem Set 1

Assigned 2005 January 13
Due 2005 January 20

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems.

1 Position-Dependent Mass Distribution

Consider a spherically symmetric distribution of mass with density

$$\rho(\vec{r}') = \frac{Ma^2}{2\pi r'(r'^2 + a^2)^2} \quad 0 \leq r' < \infty \quad (1.1)$$

- Show by explicit calculation that the parameter M appearing in (1.1) is indeed the total mass of the distribution.
- Calculate (by direct evaluation of the integral over source points) the gravitational potential $\varphi(\vec{r})$ due to this mass distribution. (Hint: do the integral over the source point \vec{r}' in spherical coordinates oriented so that θ' is the angle between \vec{r} and \vec{r}' . You'll also need to break up the integral over r' into a piece where $r' < r$ and a piece where $r' > r$.)
- From your result to part b), calculate the gravitational field $\vec{g}(\vec{r})$ of this mass distribution, taking the gradient ($\vec{g} = -\vec{\nabla}\varphi$) in spherical coordinates.

2 Flat Earth Society

Consider an infinite sheet of mass of thickness L and uniform density ρ . Use symmetry properties of the mass distribution, along with Gauss's law for gravitation (which says that the flux of the gravitational field through any closed surface is equal to $-4\pi G$ times the mass enclosed within the surface) to deduce the gravitational field as follows:

- By considering the flux through a surface lying entirely above or below the sheet, show that the gravitational field is constant outside the sheet. Be sure to explain clearly and carefully your reasoning in each step.
- By considering the flux through a surface whose top is above and whose bottom is below the sheet, find the magnitude of the gravitational field outside the sheet.
- If the Earth were an infinite sheet of density 5.5 g/cm^3 , how thick would it need to be to produce the observed gravitational acceleration of 9.8 m/s^2 ?

3 Dig a Hole to China

Assume the Earth is a uniform-density sphere of mass M and radius R .

- a) What is the density ρ inside the Earth in terms of M and R ? For the rest of the problem, express your final answers in terms of M and R , without referring to ρ .
- b) Defining spherical coordinates with their center at the center of the Earth, find the gravitational field $\vec{g}(\vec{r})$ at any point, inside or outside the Earth. (To make this as easy as possible, you should use the physical result concerning the gravitational influence of a spherical shell of mass from section 6.2 of Symon, also derived in class January 18th.¹)
- c) By solving the differential equation $\vec{g} = -\vec{\nabla}\varphi$, find the gravitational potential $\varphi(\vec{r})$ both inside and outside the Earth.
- d) Consider a tunnel of negligible width (so we can ignore the mass removed to drill it) through the Earth along the z axis.
 - i) Show that the equation of motion for a particle dropped into the tunnel is that of a simple harmonic oscillator and write the oscillation frequency in terms of M and R (and G).
 - ii) Supposing the particle is dropped from rest at the surface of the Earth (so that $\vec{r}(0) = R\hat{z}$ and $\vec{v}(0) = \vec{0}$), what is its speed as it passes the center of the Earth?
 - iii) Look up the actual mass and radius of the Earth and express the results to parts i) and ii) in reasonable physical units for those values of M and R . Make sure your results are expressed to an appropriate number of significant figures for the values you quote.

¹Succinctly stated, this is that the gravitational field due to a spherical shell vanishes if you're inside the shell and is the same as that of a point mass at the center if you're outside the shell.