Physics A300: Classical Mechanics I

Problem Set 2

Assigned 2004 September 9
Due 2004 September 16

Show your work on all problems! Note that the answers to the some problems may be in the appendix of Symon, but they should only be used to check your work, since the answers alone are an insufficient solution to the problem.

1 Exponentially Decaying Sinusoidal Driving Force

In this problem you will consider the effect on a particle starting at rest at the origin of the force

$$F = F_0\, e^{-\gamma t} \sin \omega t \tag{1.1}$$

a) **SHOW** that

$$\int e^{-\gamma t} \sin \omega t\, dt = \frac{-\gamma e^{-\gamma t} \sin \omega t - \omega e^{-\gamma t} \cos \omega t}{\gamma^2 + \omega^2} \tag{1.2a}$$

$$\int e^{-\gamma t} \cos \omega t\, dt = \frac{-\gamma e^{-\gamma t} \cos \omega t + \omega e^{-\gamma t} \sin \omega t}{\gamma^2 + \omega^2} \tag{1.2b}$$

Yes, you could look this up in the endpapers of your calculus book or find it using Maple or Mathematica, but it's useful to be self-reliant. I can think of several different ways to obtain the expression above: (1) Integrate by parts to get two equations relating the two integrals to each other and then solve those equations algebraically, (2) Take the derivatives $\frac{d}{dt}(e^{-\gamma t} \sin \omega t)$ and $\frac{d}{dt}(e^{-\gamma t} \cos \omega t)$, both of which are linear combinations of $e^{-\gamma t} \sin \omega t$ and $e^{-\gamma t} \cos \omega t$ and then find algebraically the functions whose derivatives are just $e^{-\gamma t} \sin \omega t$ and $e^{-\gamma t} \cos \omega t$, or (3) Use the Euler relation $e^{i\theta} = \cos\theta + i \sin\theta$ to rewrite both integrals as combinations of exponentials, and then to recombine the complex exponentials in the result of the integrals into sines and cosines. Choose whichever method you like to evaluate the integrals, but **DON'T** just start from the expressions (1.2) and verify that they have the right derivatives. (And, obviously, don't just look them up or have a computer do them for you.)

b) Write the initial conditions on $x(0)$ and $\dot{x}(0)$ implied by the setup of this problem.

c) Find the velocity $\dot{x}(t)$ of the particle at an arbitrary time $t > 0$.

d) Find the position $x(t)$ of the particle at an arbitrary time $t > 0$.

2 Symon Chapter Two Problem 11

3 Motion in a Potential

Throughout this problem, you should limit your attention to $x > 0$.

Consider the potential energy

$$V(x) = -\frac{a}{x} + \frac{b}{x^2}$$

where a and b are positive constants.

a) Construct the following combinations of a and b (of the form $a^n b^m$ for particular values of m and n): i) V_c, with units of energy and ii) x_c, with units of length.

b) Write V/V_c as a function of x/x_c and use a computer plotting program to plot V/V_c versus x/x_c. Be sure to include the commands used as well as the plot itself.

c) Find the equilibrium point(s) associated with this potential and state whether they're stable or unstable. [Show this from the mathematical form of the potential, and verify that this is consistent with your graph from part b).]

d) If a particle of mass m has an initial position $x_0 = 3x_c$ and initial velocity v_0, what is the energy of its trajectory?

e) What is the smallest velocity v_{esc} such that if a particle starts off at $x_0 = 3x_c$ and $v_0 > v_{\text{esc}}$, it will never turn around and move back towards the origin? (You should be able to determine this from the potential energy without calculating any forces.)

f) For a particle with initial $x_0 = 3x_c$ and $v_0 = -v_{\text{esc}}$, at what value of x will the velocity vanish? What will the force be at that point?