The Razor’s Edge: Dynamical Models of Thin Accretion Disks around Black Holes

Scott C. Noble (RIT)
Julian Krolik (JHU) & John Hawley (UVa)

TAPIR Seminar -- Caltech -- May 28, 2010
“The sharp edge of a razor is difficult to pass over; thus the wise say the path to Salvation is hard.” -- M. Somerset Maugham
“The sharp edge of a disk is difficult to resolve; thus the wise say the path to Solution is hard.”
"The sharp edge of a disk is difficult to resolve; thus the wise say the path to Solution is hard."
The Exciting World of Black Hole Accretion!

AGN!!

XRBs!!

Feedback!!

Monday, May 31, 2010
Probing the Spacetime of BHs

- Variability: e.g. QPOs, short time scale fluctuations

- Polarization
  (e.g. Schnittman & Krolik 2009)

- Spectral Fitting of Thermal Emission

\[ L = AR_{\text{in}}^2 T_{\text{max}}^4 \]
\[ R_{\text{in}}^2 = f(a, M) \]

McClintock et al. 2006, Shafee et al. 2006

- Relativistic Iron Lines

- Directly Resolving the BH Silhouette
  - e.g. Sgr A* with sub-mm/mm VLBI

Thermal Spectral Fitting for BH Spin

\[ L = A R_{\text{in}}^2 T_{\text{max}}^4 \]

\[ R_{\text{in}} = R_{\text{in}} (M, a) \simeq R_{\text{ISCO}} \]

Done, Gierlinski & Kubota (2007)

\[ T_{\text{max}} \]

Table 1

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Observation Date</th>
<th>Satellite</th>
<th>Detector</th>
<th>( a_* ) (D05)</th>
<th>( a_* ) (ST95)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRO J1655–40</td>
<td>1995 Aug 15</td>
<td>ASCA</td>
<td>GIS2</td>
<td>(~0.85)</td>
<td>(~0.8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>GIS3</td>
<td>(~0.80)</td>
<td>(~0.75)</td>
</tr>
<tr>
<td></td>
<td>1997 Feb 25–28</td>
<td>ASCA</td>
<td>GIS2</td>
<td>(~0.75^a)</td>
<td>(~0.70)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>GIS3</td>
<td>(~0.75^a)</td>
<td>(~0.7)</td>
</tr>
<tr>
<td></td>
<td>1997 Feb 26</td>
<td>RXTE</td>
<td>PCA</td>
<td>(~0.75^a)</td>
<td>(~0.65)</td>
</tr>
<tr>
<td></td>
<td>1997 (several)</td>
<td>RXTE</td>
<td>PCA</td>
<td>0.65–0.75^a</td>
<td>0.55–0.65</td>
</tr>
<tr>
<td>4U 1543–47</td>
<td>2002 (several)</td>
<td>RXTE</td>
<td>PCA</td>
<td>0.75–0.85^a</td>
<td>0.55–0.65</td>
</tr>
</tbody>
</table>

\(^a\) Values adopted in this Letter.

Shafee et al. (2006)

McClintock et al. (2006)

\[ \text{Power Law} \]

<table>
<thead>
<tr>
<th>Object</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRS 1915+105(^a)</td>
<td>0.998</td>
<td>0.001</td>
</tr>
<tr>
<td>GRS 1915+105(^b)</td>
<td>0.998</td>
<td>0.001</td>
</tr>
</tbody>
</table>
**Disk “Dichotomy”**

**Thin Disks:**
- Dissipation Rate < Cooling Rate
- “Cold”, Optically Thick
- Thermal or Multi-temperature black body

**Thick Disks:**
- Dissipation Rate > Cooling Rate
- “Hot”, optically thin, outflows
- 2 Temperature flow, advected heat

---

Narayan & Sunyaev (1973)
Novikov & Thorne (1973)
Page & Thorne (1974)

Narayan & Yi (1994-5) (ADAF)
Blandford & Begelman (1999) (ADIOS)
Quataert & Gruzinov (2000) (CDAF)

---

Monday, May 31, 2010
Steady-state Thin Disk Models

Novikov & Thorne (1973)
- Stationary gravity
- Perfect radiator
- Work done by stress locally dissipated & radiated
- Zero stress at ISCO as boundary condition
- Luminosity as total liberation of binding energy up until plunge into ISCO

\[ L = \eta \dot{M} c^2 \]
\[ \eta = 1 - \epsilon_{\text{ISCO}} \]

Shakura & Sunyaev (1973)

\[ T^r_\phi = -\alpha P \quad P = \rho c_s^2 \]
\[ \tau^r_\phi = -\alpha c_s^2 \]

No stress at sonic point:
\[ \rightarrow R_{\text{in}} = R_s \simeq R_{\text{ISCO}} \]

Muchotzeb & Paczynski (1982)
Abramowicz et al. (1988)
2 It is conceivable that the disk material might contain extremely strong magnetic fields, and that these fields might transport a torque from the infalling material at \( r < r_{\text{ms}} \) to the disk at \( r \geq r_{\text{ms}} \). In this case the boundary condition at \( r_{\text{ms}} \) would be modified, and the solution for \( f \) would be changed. It seems to us unlikely that the changes would be substantial, except very near \( r_{\text{ms}} \) (i.e., at \( r - r_{\text{ms}} \lesssim 0.1r_{\text{ms}} \)). But when constructing explicit disk models, one should examine this possibility carefully.

In these three cases it seems almost certain that the ultimate, limiting value of \( a_* \) will not exceed our value of 0.998—and, hence, that the efficiency for converting rest mass into escaping radiation will not exceed 30 percent.

Other ways in which our assumptions may fail are these:

i) Magnetic fields attached to the disk may reach into the horizon, producing a torque on the hole (Ya. B. Zel’dovich and V. F. Schwartzman, private communication).

ii) The disk will recapture some of the photons it emits, thereby preventing them from going down the hole.

iii) The time-averaged, radial disk structure will be changed by photon recapture and resultant heating, and by magnetic torques that couple the innermost parts of the disk to the hole and couple them to matter that has fallen out of the disk and is plunging down the hole. The result will be deviations of the emitted photon flux \( F(r) \) from the law derived in Paper I, and deviations of the specific energy and angular momentum of the infalling matter from \( E_{\text{ms}}^+ \) and \( L_{\text{ms}}^+ \).

Gammie (1999)

- Magnetized inflow model matched to thin disk
- Efficiency tied to mag. flux BC

Agol & Krolik (2000)

- Magnetic torques at ISCO can affect radiative efficiency
Magneto-rotational Instability (MRI)

- Velikhov (1959)
- Chandrasekhar (1960)
- Balbus & Hawley (1991)

- Growth on orbital time scale.
- MRI develops from weak initial field --- relevant for any (partially) ionized gas.
- Magnetic coupling over different radii is not well described by local viscosity.
- Can explain high accretion rates where hydrodynamic viscosity cannot.
- Fastest instability known that feeds off free energy of differential rotation.
Disk Morphology

Hawley, De Villiers, Krolik, Hirose 2003+
Canonical Magnetic Field Distribution

Hirose et al. (2004)
Krolik, Hawley & Hirose (2005)

- Non-conservative
- 3D GRMHD
- H/R ~ 0.12
- Boyer-Lindquist Coordinates
SCN, Krolik & Hawley (2009)

**HARM3D:**
- Based on Gammie’s Harm (2D) and HAM (non-rel) codes
- 3D Ideal GRMHD
- Kerr-Schild coordinates
- Modern high-res. shock-capturing methods
- Flux (energy) conserving
- Contrained Transport scheme
- Optically-thin cooling function
- Maintains constant H/R
- Cooling on orbital timescale

\[
\nabla_\mu T^{\mu \nu} = -\mathcal{L} u_\nu
\]

\[
\mathcal{L} = \Omega_K u \Delta^q
\]

\[
T_\circ = \frac{\pi}{2} \left( \frac{H}{r} r \Omega_K \right)^2
\]

\[
N_r \times N_\theta \times N_\phi = 192 \times 192 \times 64
\]

\[
r \in \left[< r_{\text{hor}}, 120M \right] \quad \theta \in \pi [\delta, 1 - \delta] \quad \phi \in [0, \pi/2]
\]

\[
a = 0.9M
\]
SCN, Krolik & Hawley (2009)

- **HARM3D:**
  - Based on Gammie’s Harm (2D) and HAM (non-rel) codes
  - 3D Ideal GRMHD
  - Kerr-Schild coordinates
  - Modern high-res. shock-capturing methods
  - Flux (energy) conserving
  - Constrained Transport scheme
  - Optically-thin cooling function
  - Maintains constant H/R
  - Cooling on orbital timescale

\[
\nabla_\mu T^\mu_\nu = -\mathcal{L}u_\nu \\
\mathcal{L} = \Omega_K u \Delta^q \\
T_\phi = \frac{\pi}{2} \left( \frac{H}{r} \Omega_K \right)^2
\]

- \( N_r \times N_\theta \times N_\phi = 192 \times 192 \times 64 \)

\( r \in [r_{\text{hor}}, 120M] \quad \theta \in \pi [\delta, 1 - \delta] \quad \phi \in [0, \pi/2] \)

\( a = 0.9M \)
Comparison to NT

- Retained Heat --> Stress Deficit
- Continuity through the ISCO

- Fits approx. to Agol & Krolik (2000)
  \[ \Delta \eta = 0.01 \quad \Delta \eta / \eta = 7\% \]
- ~5% flux deficit at all radii
  - Due to retained thermal and magnetic energy densities.
GR Radiative Transfer

\[ \frac{d}{d\lambda} \left( \frac{I_\nu}{\nu^3} \right) = \frac{j_\nu}{\nu^2} \]

\[ j_\nu = \mathcal{L}/4\pi \]

*GR geodesic integration*
*Doppler shift*
*Gravitational redshift*
*Relativistic beaming*
*Interpolates simulation data in space & time*

Allows us to explore dependence on time and disk orientation on the sky.
\[ L = \eta \dot{M} c^2 \]
\[ \eta_{NT} = 0.143 \]
\[ \Delta \eta/\eta = 6\% \]
\[ \Delta T_{\text{max}}/T_{\text{max}} = 7\% \]
\[ \Delta R_{\text{in}}/R_{\text{in}} = 80\% \]
\[ T \rightarrow 0 : \Delta \eta/\eta = 20\% \]

Suggests previous spectral fits may overestimate spin.

NT model may underestimate luminosity in some disks.
ThinHR: $H/R = 0.06$  
912x160x64  
$\alpha = 0 M$

$\rho$
ThinHR: $H/R = 0.06$ 912x160x64 $a = 0 M$

$\rho$

$\frac{t}{M} = 0$. 

$\frac{y}{M}$

$\frac{x}{M}$

$-5$ $0$ $5$ $10$ $15$ $20$ $25$ $30$ $35$

$-10$ $-8$ $-6$ $-4$ $-2$
<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>ThinLR</th>
<th>MediumLR</th>
<th>ThinHR</th>
<th>MediumHR</th>
<th>ThickHR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH Spin</td>
<td>0.9M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Resolution $N_r \times N_\theta \times N_\phi$</td>
<td>192x192x64</td>
<td>192x192x64</td>
<td>192x192x64</td>
<td>912x160x64</td>
<td>512x160x64</td>
<td>348x160x64</td>
</tr>
<tr>
<td>Target H/R</td>
<td>0.1</td>
<td>0.06</td>
<td>0.08</td>
<td>0.06</td>
<td>0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>Actual H/R</td>
<td>0.07-0.12</td>
<td>0.085</td>
<td>0.091</td>
<td>0.061</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>Init. Inner Edge</td>
<td>15M</td>
<td>15M</td>
<td>15M</td>
<td>20M</td>
<td>20M</td>
<td>20M</td>
</tr>
<tr>
<td>Init. Radius of $P_{\text{max}}$</td>
<td>25M</td>
<td>25M</td>
<td>25M</td>
<td>35M</td>
<td>35M</td>
<td>35M</td>
</tr>
<tr>
<td>Start at Target H/R?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N_{\text{cells per H/R}}$</td>
<td>15-30</td>
<td>60</td>
<td>35</td>
<td>81</td>
<td>103</td>
<td>74</td>
</tr>
</tbody>
</table>

**Motivation:**
- Explore H/R dependence;
- Resolve height with >60 cells (Davis++ 2009);
- Attempt at isotropic dissipation with nearly cubical cells;
Inflow Equilibrium

Defined to be when:
1) Accreted specific angular momentum ($j_{\text{net}}$) is steady;
2) Mass flux shows no trends in time over radius;

Remember these are turbulent MHD flows---they need not reach any kind of steady-state!
• No trend seen in Maxwell Stress
• Minor “sqrt” trend seen in spec. ang. mom.
• Due to additional Reynolds stress for thicker disks
Preliminary Results!!!
$i = 5^\circ$

$i = 41^\circ$

$i = 77^\circ$

$i = 89^\circ$
Time-averaged ThinHR

NT

50 M

Monday, May 31, 2010
\[ \Delta \eta / \eta \]

\(-1\%\)

\[ \Delta \eta / \eta \]

\(-3\%\)

\[ \Delta \eta / \eta \]

\(-12\%\)

\[ \Delta \eta / \eta \]

\(-1\%\)
MediumHR

\[
\frac{\Delta \eta}{\eta} = +3\%
\]

\[
\frac{\Delta \eta}{\eta} = +5\%
\]

\[
\frac{\Delta \eta}{\eta} = +6\%
\]

\[
\frac{\Delta \eta}{\eta} = +4\%
\]
Efficiency Trend with Scaleheight

\[ R_{NT} = 11.4 \]
\[ R_{ThinHR} = 10.3 \]
\[ \Delta T_{\text{max}} / T_{\text{max}} = 8\% \]
\[ \Delta R_{\text{in}} / R_{\text{in}} = 11\% \]
\[ \Delta \eta / \eta = 10\% \]

Possibly, more light can be generated from retained heat and magnetic field.
Bonus Material:

Variability
Coronal X-ray Variability

X-ray Binaries

AGN Markowitz et al 2003

X-ray variability:
- is always dominated by corona;
- is dependent on spectral state;

\[ P \sim \nu^\alpha \]

\[ -3 < \alpha < -1 \]
Variability Models

\[ P \sim \nu^{\alpha} \]

Lyubarskii et al 1997
- Total variability is a superposition of independent variability from larger radii modulating interior annuli on inflow (viscous) times scales

Churazov et al 2001
- Outer radius of corona may be cause of (temporal) spectral slope

\[ \tau_a = \left[ \alpha \left( \frac{H}{r} \right)^2 \Omega_K \right]^{-1} \]

Armitage & Reynolds 2003
Machida & Matsumoto 2004
Schnittman et al 2006
Reynolds & Miller 2009
- Accretion rate modulation modeled as variability of \( \alpha \) (disk parameter)
- Predicts phase coherence at frequencies longer than inverse of inflow timescale
- Used accretion rate or stress as dissipation proxies
- PLD breaks at local orbital frequency per annulus
- Composite PLD \( \rightarrow \alpha \simeq -2 \)
SCN & Krolik 2009

- Use "thin disk" cooling rate in corona as emissivity
- Thomson Opacity model (e- scattering)
- Integrate to photosphere \( (\tau = 1) \)
- Include finite light speed effect
- Parameterized by accretion rate and inclination

\( \dot{m} = 0.003 \)

\( i = 41^\circ \)

\( \dot{m} = 0.003 \)

\( i = 53^\circ \)
\[ \dot{m} = 0.01 \]

\[ i = 29^\circ \]
Dissipation approximately follows accretion rate
Not all accretion rate modes are dissipated
Variability at infinity follows local dissipation var.

\[
\log \frac{P_{\text{diss}}(\nu, r)}{P_{M}(\nu, r)} \quad \text{and} \quad \log \frac{P_I(\nu, r)}{P_{\text{diss}}(\nu, r)}
\]
Mostly incoherent between adjacent radii and frequencies;

Possible coherence at
\[ \nu < \frac{1}{T_{\text{inflow}}(r)} \]

Need longer runs to verify;

Degenerate Result;

No inclination angle effect;

Consistent w/ observed power-law exponents

See no QPOs, though we lie between LFQPO and HFQPO range

Monday, May 31, 2010
Degeneracy Explanation

$\alpha_a > -2$

$\alpha_c < -2$

$\alpha_b > -2$

$\alpha_d < -2$

$i \approx 0^\circ$

$\alpha_i \approx -2$
Out-standing Issues in black hole accretion

Warped Disks  Fragile et al. 2007-2009

Initial Field Topology  Beckwith et al. 2008

Poloidal  Quadrupolar  Toroidal
Jet  Jet  “No” Jet

McKinney & Blandford 2009

Gammie et al (unpub.)

Full 2π Evolutions  m=1 mode dominance

Monday, May 31, 2010
Summary & Conclusion:

- Moving towards fully self-consistent accretion models;
- Building the analytical tools to evaluate disks’ statistical steady-state;
- Magnetic fields can change the “thin disk” picture within the ISCO;
- MRI turbulence can explain the high frequency X-ray variability in AGN and low/hard state of galactic black holes;
  - Emissivity is not trivially dependent on accretion rate;

Future Work:

- Fill in H/R vs. spin parameter space;
- Further magnetic field topology studies;
- What are “natural” initial disk conditions?
- Does variability depend on disk thickness?
- How does Unary Black Hole accretion physics carry over to Binary Black Holes?

Monday, May 31, 2010
Extra Slides
Track MRI Resolution for all time!

Suggestions from local shearing box simulations:

Sano et al. 2004

\[ \lambda_{\text{MRI}} \equiv \frac{1}{\sqrt{4\pi \rho \Omega(R)}} b_\mu \hat{e}_\theta^\mu \]

\[ \frac{\lambda_{\text{MRI}}}{\Delta z} > 6 \]

Davis, Stone, & Pessah 2009

\[ \frac{H}{\Delta z} > 60 \]

Monday, May 31, 2010
Spin Over-estimation

\[ a/M = 0.16 \]

\[ a/M = 0.92 \]