Simulating and Imaging Accretion Disks
(e.g. Sgr A*, i.e. the Galactic Center)
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FLAMR 2 Workshop
CITA
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Outline:

- HARM Primer
- When Good Codes go Bad
  - Charles Gammie, Jon McKinney, L. Del Zanna
- Outflow Evolutions
  - Charles Gammie
- Sgr A* Introduction
- Radiative Transfer Calculations of Simulations
  - Charles Gammie, Po Kin Leung, Laura Book
HARM (Gammie, McKinney, Toth 2003)

- Axi., fixed background (Kerr-Schild), MHD
- **Conservative** (not Zeus-like!)
- HLL, LF KT fluxes, need e. values (good enough)
- 1st-order limiters (minmod, MC)
- Flux-CT scheme imposes divergence constraint
- Parallelization via Domain Decomposition
- “5D” and “Del Zanna” Inversion routines
- \[ r = e^{X_1} \]

\[ \theta = X_1 + \sin(2\pi X_2) \]

Covariant Infrastructure
Failure Modes

- Most failures along axis and horizon, especially both: $P_{mag} >> P_{gas}, \ v \sim 1$
  - Leads to “stiff” PDE's where $\text{Trunc}(B) \sim p, \rho$

- Large gradients/velocities lead to large changes in $dU/dt \rightarrow \text{bad for } P(U)$
  - $\rightarrow \text{Need better inversion routines... ?}$
Inversion Methods

\[ Q_\mu = -n_\nu T^\nu_\mu \]

\[ Q_\mu n^\mu = -\frac{B^2}{2} (1 + v^2) + \frac{(Q.B)^2}{2W^2} - W + p \]

\[ \tilde{Q}_\mu = (g_{\mu\nu} + u_\mu u_\nu) Q^\nu \]

\[ \tilde{Q}^2 = v^2 (B^2 + W)^2 - \frac{(Q.B)^2 (B^2 + 2W)}{W^2} \]

- 5D: Num. Inv. U(P)
- 2D: solve both for \( v^2 \) and \( W \)
- 1DW: eliminate \( v^2 \) by \( Q^2 \), solve \( Q.n \)
- 1Dvsq: solve quartic \( Q.n \) for \( W \), NR \( Q^2 \) for \( v^2 \)
  - Only relevant for ideal-gas EOS
- Poly.: \( Q^2 \rightarrow Q.n \rightarrow 8\text{th order poly. (EOS)} \)
Phase Space Survey

\[ \log_{10} \rho \in [-7, 1] , \quad \log_{10} u \in [-10, 0] , \quad \log_{10} \gamma \in [0.002, 2.9] , \quad \log_{10} B^2 \in [-8, 1] , \quad \cos \Phi \in [-1, 1] . \]
Phase Space Survey

<table>
<thead>
<tr>
<th>Method</th>
<th>NR steps per sol.</th>
<th>Sol. per sec.</th>
<th>Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>8.45</td>
<td>$1.66 \times 10^5$</td>
<td>$8.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>$1D_w$</td>
<td>7.45</td>
<td>$1.68 \times 10^5$</td>
<td>$8.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$1D_{v^2}$</td>
<td>7.08</td>
<td>$1.06 \times 10^5$</td>
<td>$3.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>$1D_{v^2}$</td>
<td>7.05</td>
<td>$1.24 \times 10^5$</td>
<td>$2.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>5D</td>
<td>19.3</td>
<td>$1.89 \times 10^4$</td>
<td>$4.2 \times 10^{-1}$</td>
</tr>
<tr>
<td>Poly</td>
<td>—</td>
<td>$9.21 \times 10^3$</td>
<td>$4.1 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Accretion Disk Evolution

\[ \frac{\dot{M}}{M_0}, \frac{\dot{E}}{\dot{M}_0}, \frac{\dot{L}}{\dot{M}_0} \]

\[ t \frac{c^3}{2GM} \]

\[ 10^{-20}, 10^{-15}, 10^{-10}, 10^{-5}, 10^0 \]

\[ E_{NR} \]

2D, 1D_w, 1D_{v^2}, 5D
Accretion Disk Evolution

Table 6. Accretion Disk Efficiency Comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>NR steps per sol.</th>
<th>Zone-cycles/node/sec.</th>
<th>Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>4.19</td>
<td>24535</td>
<td>9.57 x 10^{-5}</td>
</tr>
<tr>
<td>1D_{W}</td>
<td>4.18</td>
<td>23860</td>
<td>9.33 x 10^{-5}</td>
</tr>
<tr>
<td>1D_{v2}</td>
<td>5.22</td>
<td>20585</td>
<td>9.46 x 10^{-5}</td>
</tr>
<tr>
<td>5D</td>
<td>4.52</td>
<td>14741</td>
<td>9.22 x 10^{-5}</td>
</tr>
</tbody>
</table>
Outflow Evolutions

\[ r = e^{X_1} \]

\[ \theta = \pi X_2 + h(X_1) \sin(2 \pi X_2) \]

\[ h(X_1) = h_0 \arctan \left[ s(X_1^0 - X_1) \right] \]
Too much contribution from floor, McKinney (2006) says to use PPM, 3rd-order Runge-Kutta for time-integration and:

\[ \rho_{\text{floor}} \sim u_{\text{floor}} r^{-2.7} \]
Sagitarius A*

NASA/UMass/D.Wang et al. (Chandra)

120x48 arcmin or 900x400 light-year
It's a Black Hole, ok?

It's a Black Hole, ok?

- Very few possible compact sources
- Who's seen a scalar boson anyway?
- Spectra fits well with jet & accretion models
- Some spectra features seem to indicate variability < 10 Rs
- Dark star clusters are short lived
IR Observations

IR Observations

**Figure 1**: Graphs showing the observed flux density over time and the associated orbital period. The data points are indicated with error bars. The graphs are labeled with dates and times of observation. The bottom graph plots the orbital period against the ratio of semi-major axis to the semi-major axis of the stable orbit. The period is noted as 16.8 ± 0.2 minutes. The ratio of semi-major axis to the semi-major axis of the stable orbit is denoted as \( a'/0.52 \), with uncertainties given as ±0.1, ±0.07, ±0.14. The graph is labeled with 3) VLT.
X-Ray Observations

1.23 arcmin

8.4 arcmin

1.4 arcsec

NASA/CXC/MIT/ F.K. Baganoff et al.
X-Ray and Bondi

Modeling it as $kT \sim 1.3$ keV hot, optically-thin emission:

$$n_e = 30 \, cm^{-3}$$

$$c_s^2 = \gamma kT / \mu m = 550 \, km/s = v_{wind}$$

$$R_B = 2G M_{SgrA} / c_s^2 = 0.1 \, pc = 2.7 \, arcsec.$$  

$$\rightarrow \quad R_B = 2 \, R_{X-rays}$$
Radio (VLBA)

- Position best determined in radio (best res. on the planet)
- Visible Size $\sim 1/\nu$
- Optically thin at $\sim \text{submm } \lambda$
  $\longrightarrow$ horizon images
Composite Spectrum

\[ \log[E_\nu(\text{keV})] \]

\[ \log[\nu L_\nu(\text{erg s}^{-1})] \]

\[ \log[\nu(\text{Hz})] \]
The Luminosity Problem

\[ L_{\text{SgrA}} = 10^{36} \text{erg/s} \]

\[ L_{\text{Edd}} = 4\pi c G M \frac{\mu_e}{\sigma_T} = 1.51 \times 10^{38} \left( \frac{M}{M_{\odot}} \right) \text{erg/s} \]

\[ L_{\text{Edd}} \left( \frac{M_{\text{Sgr}}}{M_{\odot}} \right) = 5.44 \times 10^{44} \text{erg/s} \]

\[ \rightarrow L_{\text{Sgr}} = 10^{-8} L_{\text{Edd}} \]

\[ \dot{M}_{\text{X-rays}} = 4\pi R_B^2 \rho c_s = 4 \times 10^{-5} M_{\odot} / \text{yr} \]

\[ \rightarrow L = \eta c^2 \dot{M}_{\text{X-rays}} = \eta \ 2.1 \times 10^{43} \text{erg/s} \]
The Luminosity Problem

\[ \rightarrow L_{SgrA} = 5 \times 10^{-6} L_{\text{thin}} \]

Radio Linear Polarization constraints:

\[ \dot{M} = 10^{-3} \dot{M}_{X-\text{rays}} \rightarrow \eta = 10^{-3} \eta_{\text{thin}} \]

\[ \rightarrow \dot{M} = 4 \times 10^{-8} M_{\text{sun} / \text{year}} \]
Realistic Bondi Spectrum

- Including Synch., Bremsstrahlung (+IC)
- At low $\rho$, e's & ions decouple since $t_{\text{Coulomb}} > t_{\text{infall}}$
- Shapiro, Lightman, & Eardley (1976)
RIAF's (Radiatively Inefficient Accretion Flows)

• ADAF's (Advection Dominated Accretion Flows):
  • Narayan & Yi (1994-5), Yuan et al. (2003-4)
    • “At least I didn't name them Type II accretion flows!”
      Narayan, KITP SgrA* Conf. 2005

\[ Q_{\text{diss}} > Q_{\text{rad.}} \]
\[ \rho \propto r^{-3/2} \]

• 2-T flows, ala Shapiro et al., advection stabilizes
• Thick disks, ~spherical
• Convectively unstable
CDAF's & ADIOS's

\[ \rho \propto r^{-3/2+s} \]

\[ \dot{M}_{\text{in}} = \dot{M}_{\text{out}} \left( \frac{R_{\text{in}}}{R_{\text{out}}} \right)^s \]

- **ADIOS** (Advection Dominated Inflow/Outflow Sol's)
  - Blandford & Begelman (1999)
  - Much of the energy is blown away in a wind

- **CDAF** (Convection-Dominated Accretion Flows)
  - Quataert & Gruzinov (2000)
  - Ang. Mom. Transported inward
  - Energy Transported outward
  - Weakest accretion of the RIAF's
RIAF Simulations

- 3D, MHD, Paczynski & Wiita Pot., Viscous, Resistive
- Toroidal B fed in from outer boundary
- Similar $\rho \sim 1/R$ close to analytic CDAF
- Goldston, Quataert, Igumenshchev (2005)
- 3D RIAF Sim. as before
- $T_e = a T_{\text{tot}}$
- $n_e \sim$ Maxwellian + PLT
- Opt. thin at 450 GHz
- $t_{\text{orbit}}$ timescale for opt. thin emission
- $> t_{\text{orbit}}$ timescale for opt. thick emission
• RIAF's have problem with var. of brem. since $R_{\text{brem}} \sim 10^5 R_s$
• Instead, add PL $n_e$ gives hard IC/SSC photons
• Solves Radio under-lum.
• Modern RIAF's have many parameters, need better constraints: simult.
  wide-freq. survey, submm VLBI

• Jets lack a mechanism, no launching mechanism
• Reliant on a disk model of some type
• Can it predict X-ray flare state?
• SgrA* may have been more active in the past...?
Relativistic Radiative Transfer

\[ \frac{\partial I_\nu}{\partial s} = j_\nu - \alpha_\nu I_\nu \]

\[ I = I_\nu / \nu^3 \]

\[ j = j_\nu / \nu^2 \]

\[ \alpha = \alpha_\nu \nu \]

\[ \frac{\partial I}{\partial \lambda} = j - \alpha I \]

• Calc. Kerr-Schild geod.

• Use geodesics as rays along which to integrate I
  - Interpolate HARM data for density, temperature, B^i, v^i

• Synchrotron and Bremsstrahlung
  - Thermal equilibrium (i.e. No power-law electrons)

• Currently uses “frozen-in” approx.
Camera Resolution
Camera Resolution
Inclination
Inclination
Spin
Spin