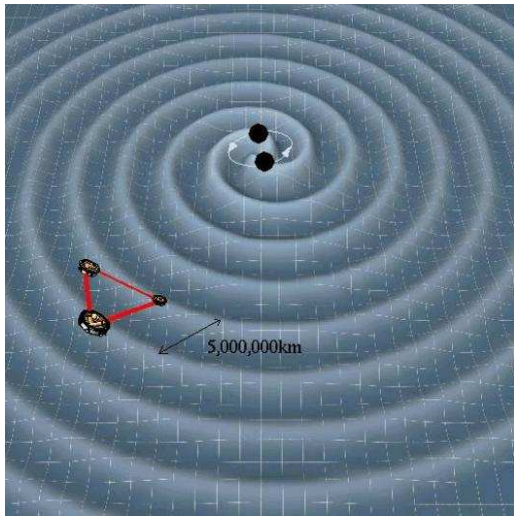


The nonlinear gravitational-wave memory in binary black hole mergers

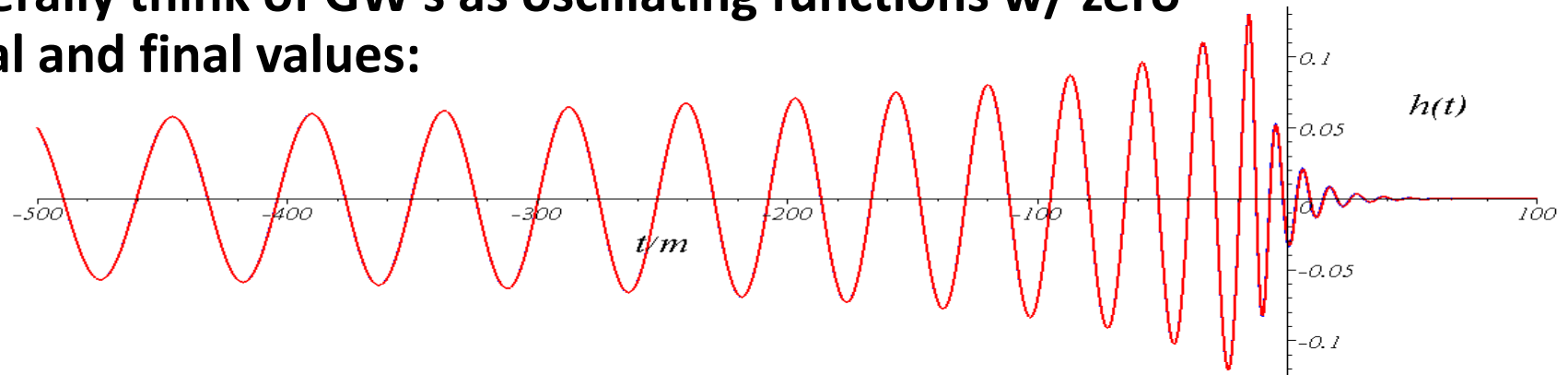


Marc Favata

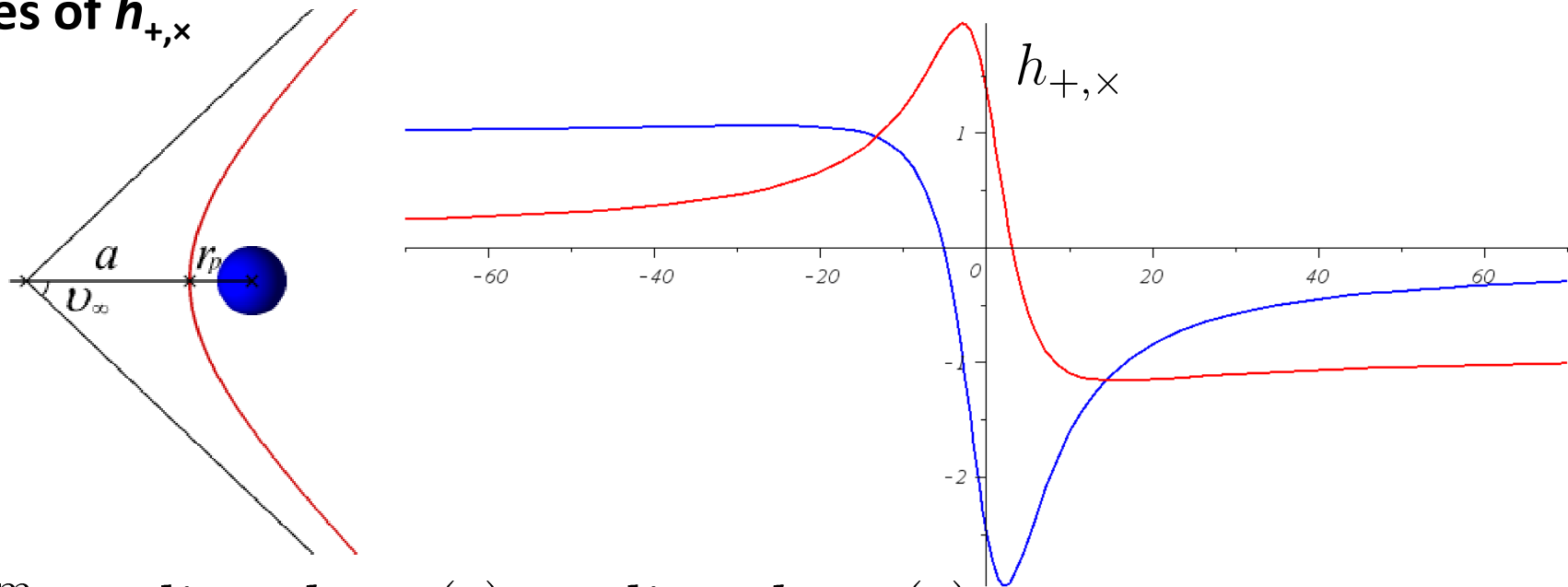
Kavli Institute for Theoretical Physics
University of California, Santa Barbara

What is “memory”?

- Generally think of GW's as oscillating functions w/ zero initial and final values:



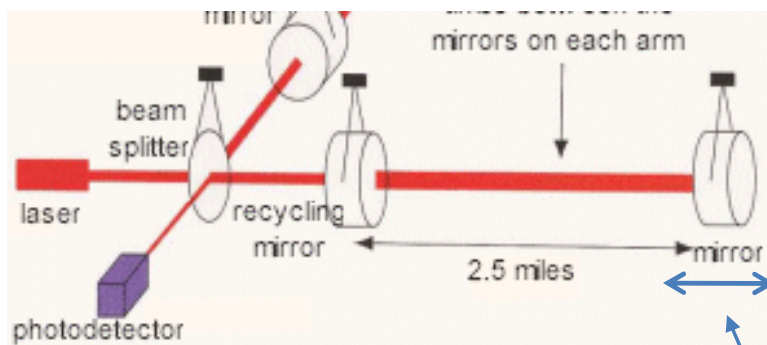
- But some sources exhibit differences in the initial & final values of $h_{+, \times}$



$$\Delta h_{+, \times}^{\text{mem}} = \lim_{t \rightarrow +\infty} h_{+, \times}(t) - \lim_{t \rightarrow -\infty} h_{+, \times}(t)$$

What is the GW memory?

- An “ideal” (freely-falling) GW detector would experience a permanent displacement after the GW has passed---leaving a “memory” of the signal.



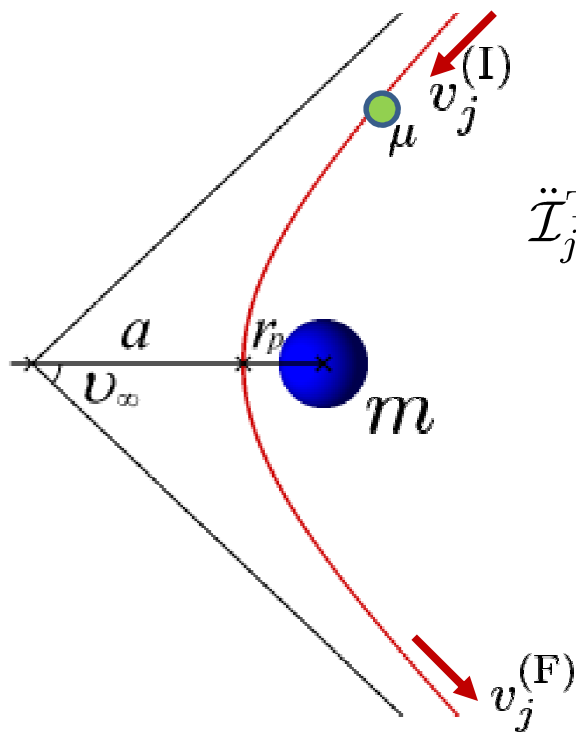
$$\delta x(t) = \frac{L}{2} h_+(t) \Rightarrow \delta x^{\text{mem}} = \frac{L}{2} h_+^{\text{mem}}$$

- The late-time constant displacement is not directly measureable, but its buildup is.
- While the memory’s buildup is in principle measureable in both LIGO and LISA, in LIGO the mirror displacement would not be truly permanent, **but it would be in LISA.**

Origin of the memory?

Linear memory: (Zel'dovich & Polnarev '74; Braginsky & Grishchuk'78; Braginsky & Thorne '87)

- non-oscillatory change in the time-derivatives of the quadrupole and higher multipole moments
 - **Example:** unbound (hyperbolic) orbits (Turner '77)



$$h_{jk}^{\text{TT}} \approx \frac{2}{R} \ddot{\mathcal{I}}_{jk}^{\text{TT}} \quad \mathcal{I}_{jk}^{\text{TT}} = \mu [x_j x_k]^{\text{TT}}$$

$$\ddot{\mathcal{I}}_{jk}^{\text{TT}} = \mu [x_j \ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j \dot{x}_k]^{\text{TT}}$$

$\ddot{x}_j = -\frac{M}{r^3} x_j$

$$= 2\mu \left[\dot{x}_j \dot{x}_k - \frac{M}{r^3} x_j x_k \right]^{\text{TT}}$$



$$\Delta h_{jk}^{\text{TT}} = \frac{4\mu}{R} \Delta [\dot{x}_j \dot{x}_k]^{\text{TT}}$$

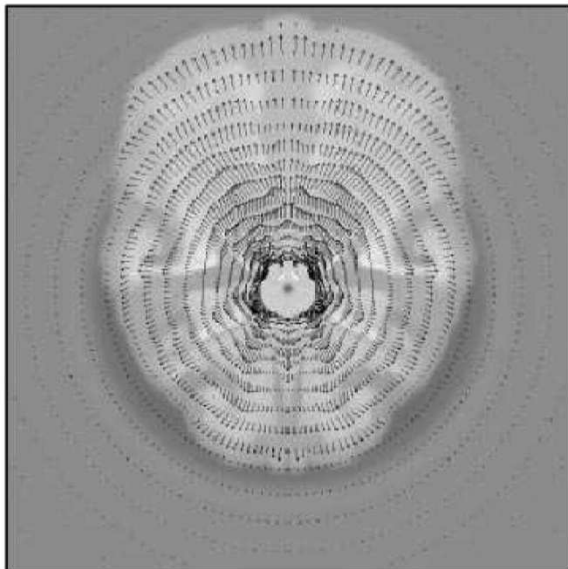
Origin of the memory?

Linear memory: (Zel'dovich & Polnarev '74; Braginsky & Grishchuk'78; Braginsky & Thorne '87)

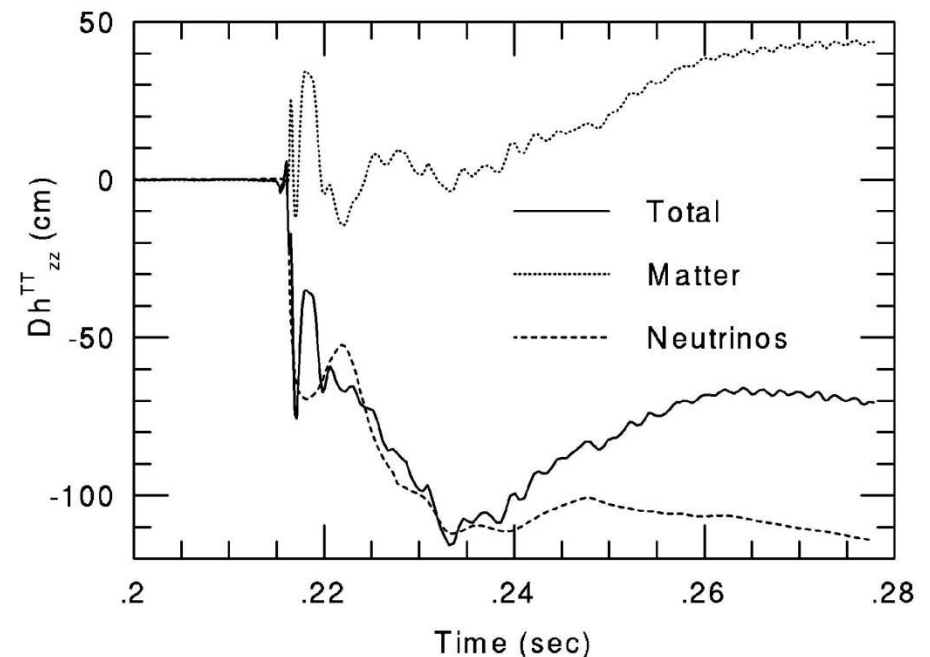
- non-oscillatory change in the time-derivatives of the quadrupole and higher multipole moments

– Examples:

- unbound (hyperbolic) orbits (Turner '77)
- Binary that becomes unbound (eg., due to mass loss)
- Anisotropic neutrino emission (Epstein '78)
- Asymmetric supernova explosions (see Ott '08 for a review)
- GRB jets (Sago et al., '04)



[Burrows & Hayes '96]



Origin of the memory?

Nonlinear memory: (Christodoulou '91 ; see also Blanchet & Damour '92)

- Contribution to the distant GW field sourced by the emission of GWs
- Recall previous form of the Einstein's equations:

$$\square h_{\alpha\beta} = 16\pi \det(g_{\mu\nu}) T_{\alpha\beta} + \mathcal{F}[h, h]$$

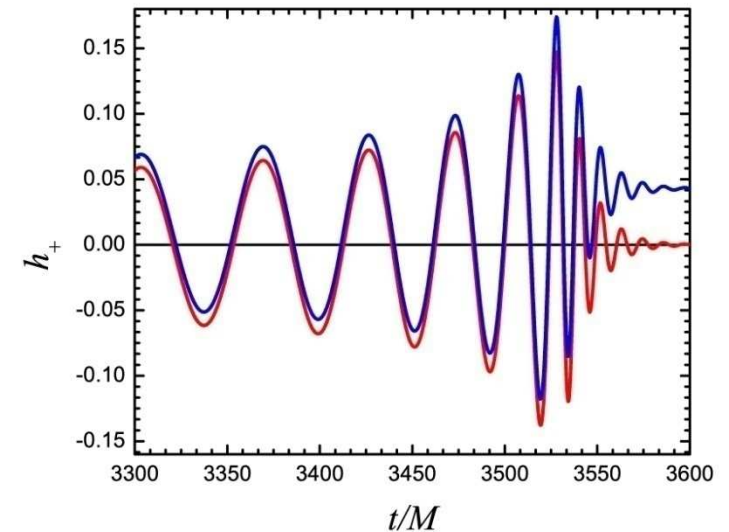
Grav'l wave stress-energy tensor...

$$T_{\alpha\beta}^{\text{gw}} \propto \frac{dE^{\text{gw}}}{dt d\Omega} \sim O(h^2)$$

$$\ddot{I}_{jk} \rightarrow \ddot{I}_{jk} + U_{jk}^{\text{gw}} \quad \dots\text{contributes to the changing multipole moments...}$$

$$h_{jk}^{\text{TT}} \approx \frac{2}{R} \ddot{I}_{jk}^{\text{TT}} \quad \dots\text{which determines the GW field...}$$

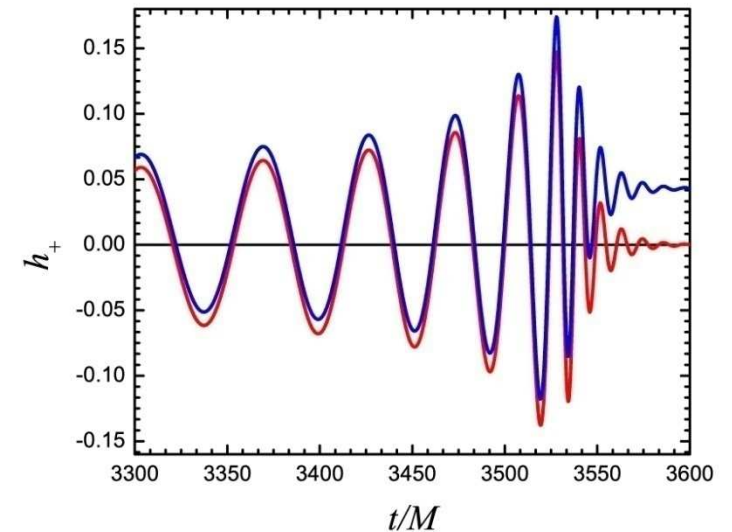
$$\Delta h^{(\text{mem})} \sim \frac{\Delta E^{\text{gw}}}{R} \quad \dots\text{which has a slowly-growing, non-oscillatory piece related to the radiated GW energy.}$$



Origin of the memory?

Nonlinear memory: (Christodoulou '91 ; see also Blanchet & Damour '92)

- Contribution to the distant GW field sourced by the emission of GWs



In analogy to the linear memory, the nonlinear memory can be interpreted as arising from changes in the mass quadrupole moment due to the **individual radiated gravitons** (Thorne '92) (just as radiated neutrinos cause linear memory in supernovae).

Why is this interesting?:

- The Christodoulou memory is a **unique, nonlinear effect** of general relativity
- The memory is non-oscillatory and only affects the “+” polarization (for quasi-circular orbits with the standard choices for e^+_{ij}, e^x_{ij})
- Although it is a 2.5PN correction to the mass multipole moments, it affects the waveform amplitude at **leading (Newtonian) order**.

$$h_+ = -2\frac{\mu}{R} \left(\frac{M}{r}\right) \left[(1 + \cos^2 \Theta) \cos[2\varphi(t) - 2\Phi] + \frac{1}{96} \sin^2 \Theta (17 + \cos^2 \Theta) \right] + O\left(\frac{M}{r}\right)^{1/2}$$

- The memory is **hereditary**: it depends on the entire past-history of the source

calculation of the memory: motivation

- Little is known about the memory
 - For quasi-circular orbits, inspiral memory has only been computed to leading PN order [Wiseman & Will '92; Kennefick '94; Arun et al. '04; Blanchet et al. '08]
 - Zero knowledge about how the memory grows and saturates to its final value post-inspiral
- Since the memory enters the waveform at leading order, its not crazy to think we could detect it.
- **Two calculations:**
 - Compute all PN corrections to the memory up to 3PN order
 - Use “effective-one-body” formalism to compute evolution of memory during the inspiral, merger, and ringdown.

Post-Newtonian calculation of the memory:

- oscillatory PN waveform amplitude known to 3PN [Blanchet et.al '08]
- But memory terms computed only to leading order.

Post-Newtonian calculation of the memory:

Result: expressions for waveform modes h_{lm} and “+” polarization:

$$h_{+, \times} = \frac{2\eta M x}{R} H_{+, \times} + O\left(\frac{1}{R^2}\right), \text{ where } H_{+, \times} = \sum_{n=0}^{\infty} x^{n/2} H_{+, \times}^{(n/2)}.$$

$$H_+^{(0, \text{mem})} = \alpha \frac{1}{96} s_{\Theta}^2 (17 + c_{\Theta}^2),$$

[see MF, 0812.0069,
Phys. Rev. D (in press)]

$$H_+^{(0.5, \text{mem})} = 0,$$

$$H_+^{(1, \text{mem})} = \alpha s_{\Theta}^2 \left[-\frac{354\,241}{2\,064\,384} - \frac{62\,059}{1\,032\,192} c_{\Theta}^2 - \frac{4195}{688\,128} c_{\Theta}^4 + \left(\frac{15\,607}{73\,728} + \frac{9373}{36\,864} c_{\Theta}^2 + \frac{215}{8192} c_{\Theta}^4 \right) \eta \right],$$

$$H_+^{(1.5, \text{mem})} = 0,$$

$$H_+^{(2, \text{mem})} = \alpha s_{\Theta}^2 \left[-\frac{3\,968\,456\,539}{9\,364\,045\,824} + \frac{570\,408\,173}{4\,682\,022\,912} c_{\Theta}^2 + \frac{122\,166\,887}{3\,121\,348\,608} c_{\Theta}^4 + \frac{75\,601}{15\,925\,248} c_{\Theta}^6 + \left(-\frac{7\,169\,749}{18\,579\,456} \right. \right. \\ \left. \left. - \frac{13\,220\,477}{18\,579\,456} c_{\Theta}^2 - \frac{1\,345\,405}{6\,193\,152} c_{\Theta}^4 - \frac{25\,115}{884\,736} c_{\Theta}^6 \right) \eta + \left(\frac{10\,097}{147\,456} + \frac{5179}{36\,864} c_{\Theta}^2 + \frac{44\,765}{147\,456} c_{\Theta}^4 + \frac{3395}{73\,728} c_{\Theta}^6 \right) \eta^2 \right],$$

$$H_+^{(2.5, \text{mem})} = -\alpha \frac{5\pi}{21\,504} (1 - 4\eta) s_{\Theta}^2 (509 + 472c_{\Theta}^2 + 39c_{\Theta}^4),$$

$$H_+^{(3, \text{mem})} = \alpha s_{\Theta}^2 \left\{ -\frac{69\,549\,016\,726\,181}{46\,146\,017\,820\,672} + \frac{6\,094\,001\,938\,489}{23\,073\,008\,910\,336} c_{\Theta}^2 - \frac{1\,416\,964\,616\,993}{15\,382\,005\,940\,224} c_{\Theta}^4 - \frac{2\,455\,732\,667}{78\,479\,622\,144} c_{\Theta}^6 \right. \\ \left. - \frac{9\,979\,199}{2\,491\,416\,576} c_{\Theta}^8 + \left[\frac{1\,355\,497\,856\,557}{149\,824\,733\,184} - \frac{3485\pi^2}{9216} + \left(-\frac{3\,769\,402\,979}{4\,682\,022\,912} - \frac{205\pi^2}{9216} \right) c_{\Theta}^2 + \frac{31\,566\,573\,919}{49\,941\,577\,728} c_{\Theta}^4 \right. \right. \\ \left. \left. + \frac{788\,261\,497}{3\,567\,255\,552} c_{\Theta}^6 + \frac{302\,431}{9\,437\,184} c_{\Theta}^8 \right] \eta + \left(\frac{5\,319\,395}{28\,311\,552} - \frac{24\,019\,355}{99\,090\,432} c_{\Theta}^2 - \frac{4\,438\,085}{3\,145\,728} c_{\Theta}^4 - \frac{3\,393\,935}{7\,077\,888} c_{\Theta}^6 - \frac{7835}{98\,304} c_{\Theta}^8 \right) \eta^2 \right. \\ \left. + \left(\frac{1\,433\,545}{63\,700\,992} + \frac{752\,315}{15\,925\,248} c_{\Theta}^2 + \frac{129\,185}{2\,359\,296} c_{\Theta}^4 + \frac{389\,095}{1\,179\,648} c_{\Theta}^6 + \frac{9065}{131\,072} c_{\Theta}^8 \right) \eta^3 \right\},$$

Post-Newtonian calculation of the memory:

Result: expressions for waveform modes h_{lm} and “+” polarization:

$$h_{+, \times} = \frac{2\eta M x}{R} H_{+, \times} + O\left(\frac{1}{R^2}\right), \text{ where } H_{+, \times} = \sum_{n=0}^{\infty} x^{n/2} H_{+, \times}^{(n/2)}.$$

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$$H_+^{(1.5, \text{mem})} = 0,$$

$$H_+^{(2, \text{mem})} = \alpha s_{\Theta}^2 \left[-\frac{3968456539}{936465824} - \frac{59408175}{4682022912} c_{\Theta}^2 + \frac{1221687}{312348608} c_{\Theta}^4 + \frac{75601}{15925248} c_{\Theta}^6 + \left(-\frac{7169749}{18579456} - \frac{13220477}{18579456} c_{\Theta}^2 - \frac{115401}{193152} c_{\Theta}^4 - \frac{25115}{873696} c_{\Theta}^6 \right) \eta + \left(\frac{10097}{147456} + \frac{5179}{36864} c_{\Theta}^2 + \frac{44765}{147456} c_{\Theta}^4 + \frac{3395}{73728} c_{\Theta}^6 \right) \eta^2 \right],$$

$$H_+^{(2.5, \text{mem})} = -\alpha \frac{5\pi}{21504} (1 - 4\eta) s_{\Theta}^2 (509 + 472c_{\Theta}^2 + 39c_{\Theta}^4),$$

$$H_+^{(3, \text{mem})} = \alpha s_{\Theta}^2 \left\{ -\frac{69549016726181}{46146017820672} + \frac{6094001938489}{23073008910336} c_{\Theta}^2 - \frac{1416964616993}{15382005940224} c_{\Theta}^4 - \frac{2455732667}{78479622144} c_{\Theta}^6 - \frac{9979199}{2491416576} c_{\Theta}^8 + \left[\frac{1355497856557}{149824733184} - \frac{3485\pi^2}{9216} + \left(-\frac{3769402979}{4682022912} - \frac{205\pi^2}{9216} \right) c_{\Theta}^2 + \frac{31566573919}{49941577728} c_{\Theta}^4 + \frac{788261497}{3567255552} c_{\Theta}^6 + \frac{302431}{9437184} c_{\Theta}^8 \right] \eta + \left(\frac{5319395}{28311552} - \frac{24019355}{99090432} c_{\Theta}^2 - \frac{4438085}{3145728} c_{\Theta}^4 - \frac{3393935}{7077888} c_{\Theta}^6 - \frac{7835}{98304} c_{\Theta}^8 \right) \eta^2 + \left(\frac{1433545}{63700992} + \frac{752315}{15925248} c_{\Theta}^2 + \frac{129185}{2359296} c_{\Theta}^4 + \frac{389095}{1179648} c_{\Theta}^6 + \frac{9065}{131072} c_{\Theta}^8 \right) \eta^3 \right\},$$

**THIS COMPLETES THE WAVEFORM
AMPLITUDE TO 3PN ORDER.**

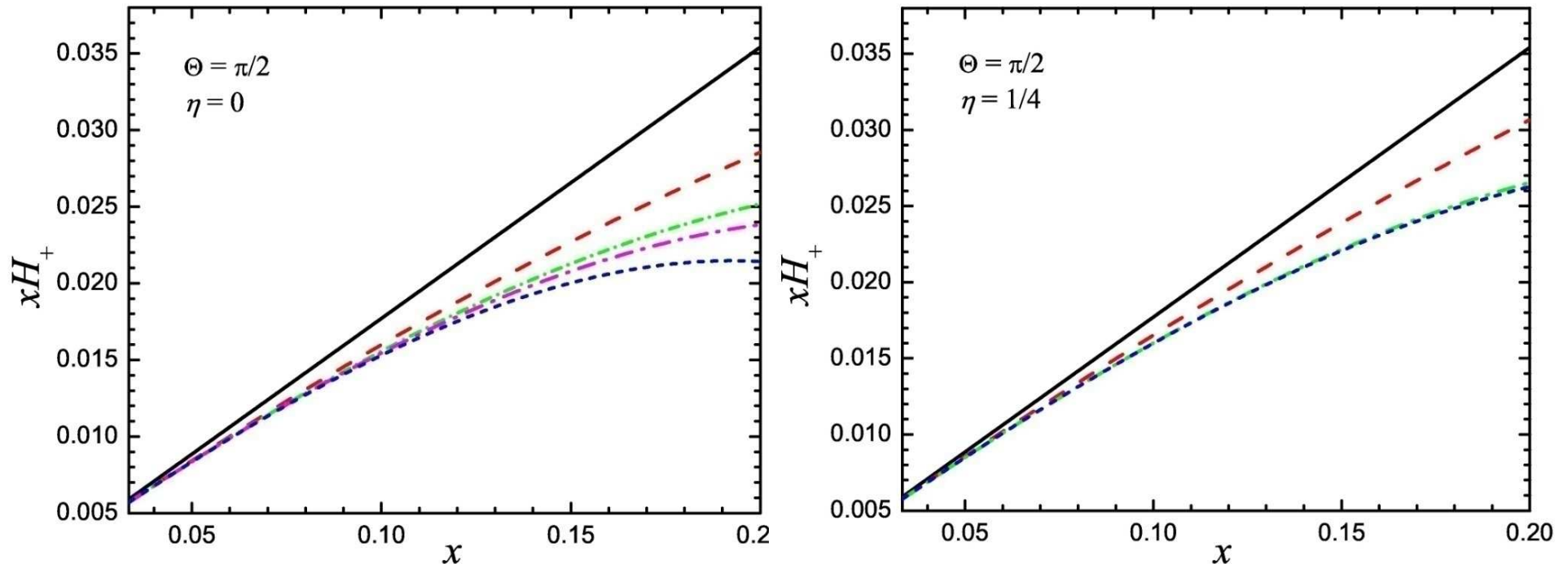
Post-Newtonian calculation of the memory:

Result: expressions for waveform modes h_{lm} and “+” polarization:

$$h_+^{\text{mem}} = \frac{2\eta M}{R} x H_+$$
$$x = (M\omega)^{2/3} \approx \frac{M}{r} [1 + O(c^{-2})]$$

0PN **1PN** **2PN** **2.5PN** **3PN**

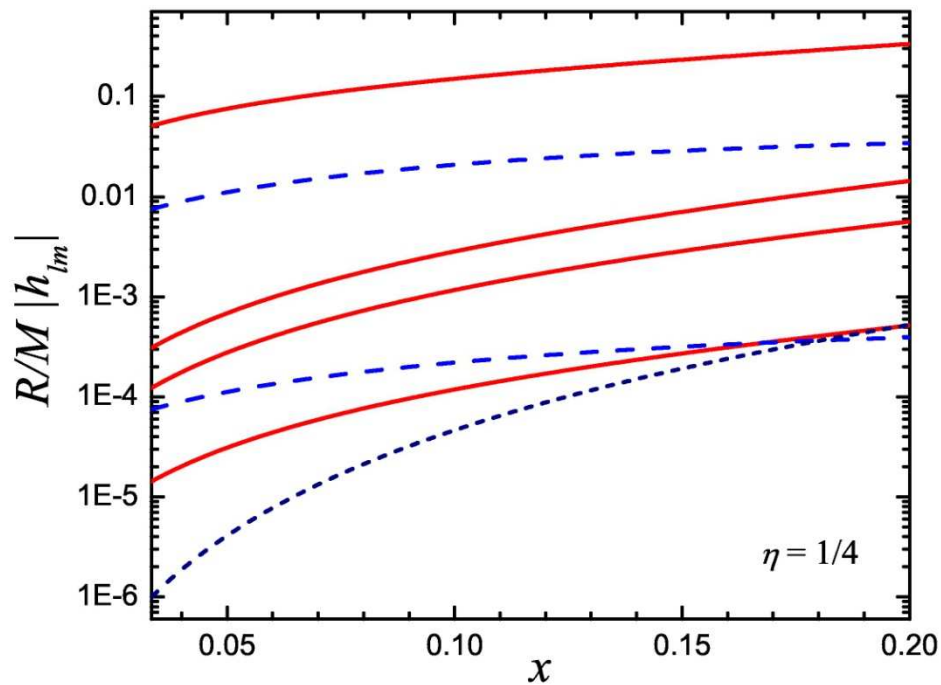
[MF, 0812.0069, Phys. Rev. D (in press)]



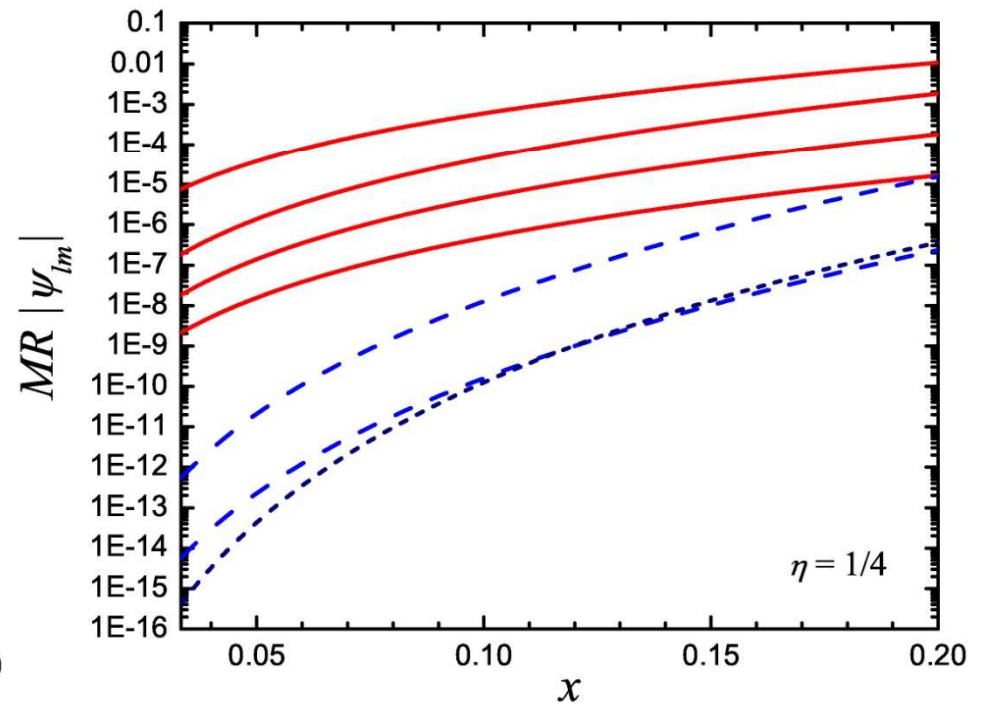
Memory in numerical relativity simulations:

- Extracting the memory from NR simulations faces several challenges:
 - Physical memory only present in $m=0$ modes (for planar, quasi-circular orbits), which are numerically suppressed

(2,2), (4,4), (3,2), (4,2) modes are much larger than the memory modes **(2,0), (4,0)**, etc..



Orbital separation decreasing \rightarrow



Orbital separation decreasing \rightarrow

[see MF, 0812.0069, Phys. Rev. D (in press)]

Memory in numerical relativity simulations:

- Other problems with NR computations of the memory:
 - Need to choose two integration constants to go from curvature to metric perturbation $\psi_{lm} = \dot{\dot{h}}_{lm}$
 - Choosing these incorrectly leads to “artificial” memory (Berti et al. '07)
 - Memory sensitive to past-history of the source (depends on initial separation)
 - Consider leading-order (2,0) memory mode, with a finite separation r_0

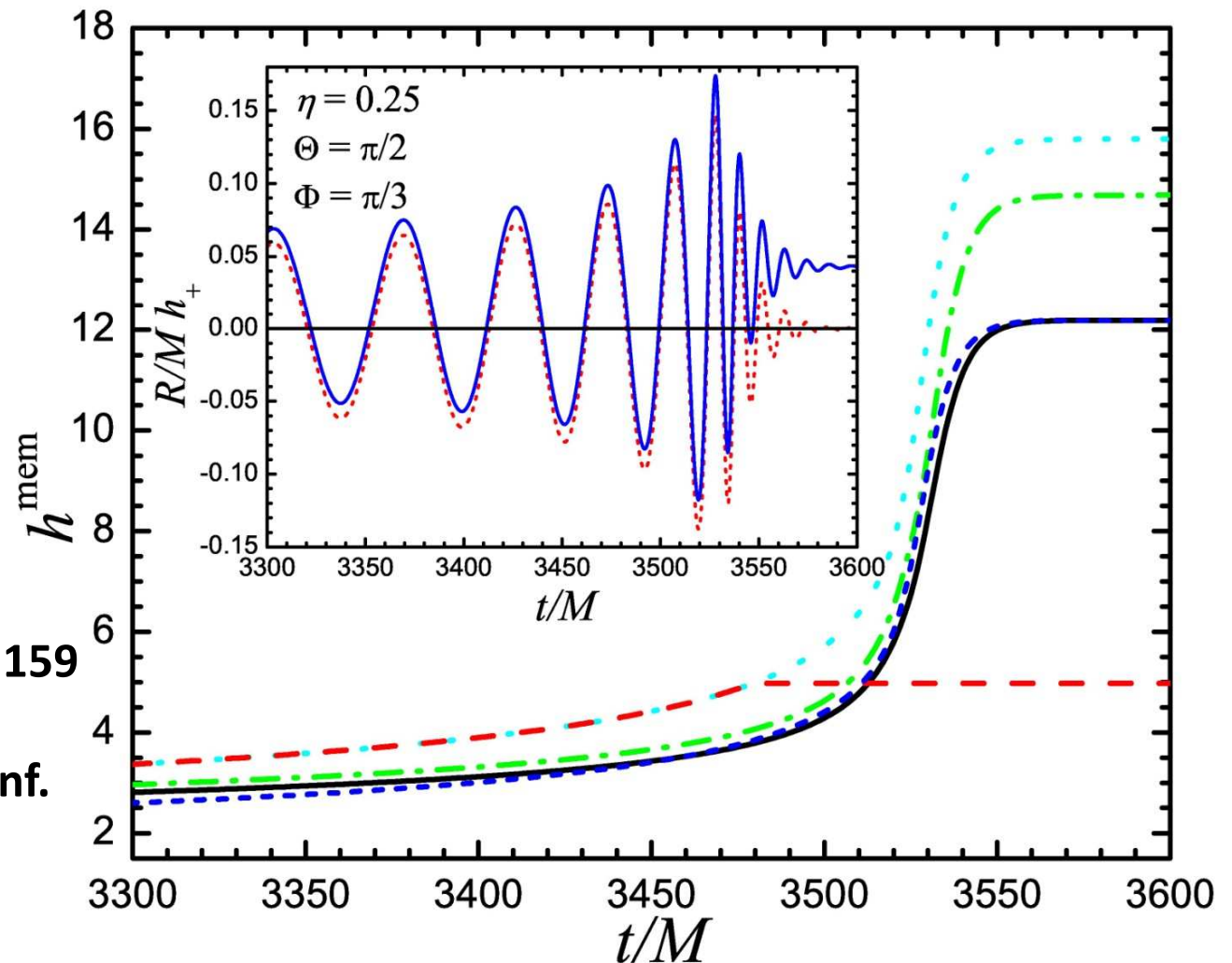
$$h_{20}^{\text{NR}}(T_R) = \frac{4}{7} \sqrt{\frac{5\pi}{6}} \frac{\eta M}{R} \left(\frac{M}{r} - \frac{M}{r_0} \right)$$

$$\frac{|\delta h_{20}^{\text{NR}}|}{h_{20}} \approx \frac{r}{r_0}$$

- Errors from gauge effects and finite extraction radius can further contaminate NR waveforms and swamp a small memory signal

EOB calculation of memory from BH mergers:

- Use EOB formalism calibrated to NR simulations to compute (2,2) mode.
- Feed this into post-Newtonian calculation of the memory modes in terms of the (2,2) mode



For details see:

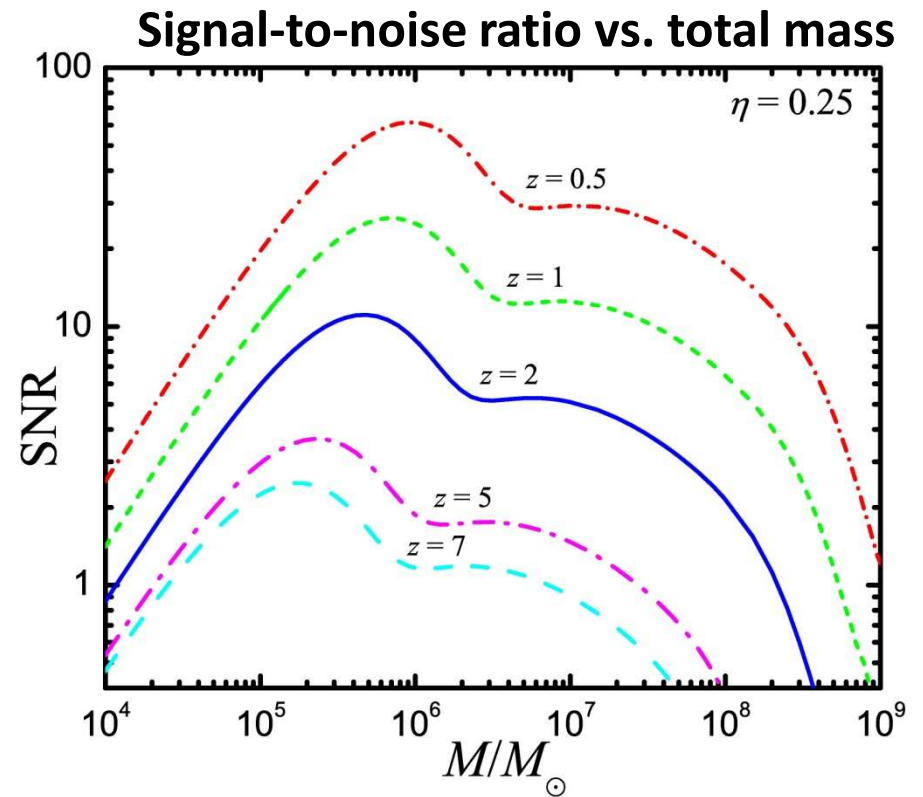
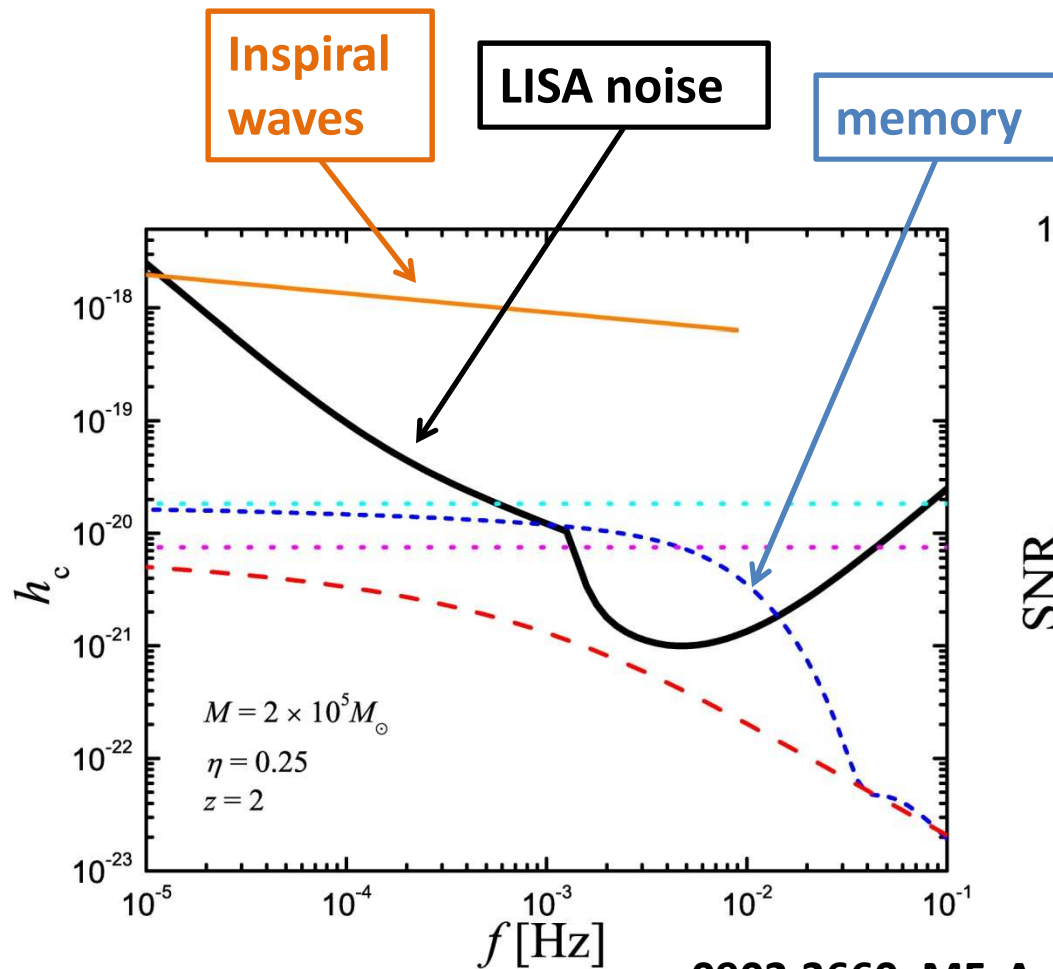
0902.3660, MF, ApJL, 696, 159

0811.3451, MF, J. Phys. Conf.

Ser. 154, 012043

Detectability of the memory:

- will be difficult to observe w/ Advanced LIGO
- likely to be visible by LISA out to redshift $z \lesssim 2$



0902.3660, MF, ApJL, 696, 159

Work in progress:

- **memory never calculated for inspiralling, eccentric binaries**
 - eccentricity increases in the past
 - potentially important due to its hereditary nature
- **improvements to computing memory from merger/ringdown**
 - hybrid PN/NR calculation using Caltech/Cornell merger waveform
 - ✓ memory is about 27% smaller (preliminary)
 - combine with full LISA response function to better assess detectability
 - check if memory improves parameter estimation
 - extensions to other mass ratios & spins...

Conclusions

- **The Christodoulou memory is a unique manifestation of the nonlinearity of GR that affects the waveform amplitude starting at Newtonian order.**
- **PN corrections to the inspiral memory have been computed; this completes the waveform to 3PN order**
- **Via EOB techniques, numerical simulations are now helping to guide and calibrate analytic studies of BH mergers---including the study of quantities not yet easily calculated with numerical relativity**
- **Using an EOB approach I've computed the final saturation value of the memory.**
- **Prospects for detecting the Christodoulou memory:**
 - **Initial LIGO: forget it**
 - **Advanced LIGO: only if we get very lucky**
 - **LISA: should be detectable from SMBH mergers out to $z \sim 2$.**
- **Binaries that recoil after merger also show a memory effect, as well as a Doppler shift of the QNM frequencies (in progress).**

extra slides

Memory from the merger: minimal waveform model

- First use analytic toy model: bare-bones EOB

- Inspiral moments given by leading-order OPN expansion:

$$I_{2\pm 2}^{\text{insp}(q)} = 2\sqrt{\frac{2\pi}{5}}\eta M r^2 (\mp 2i\omega)^q e^{\mp 2i\varphi}$$

$$\omega \equiv \dot{\varphi} = (M/r^3)^{1/2}$$

$$r = r_m(1 - T/\tau_{\text{rr}})^{1/4}$$

$$\varphi = \varphi_m + (r_m/M)^{5/2}[1 - (1 - T/\tau_{\text{rr}})^{5/8}]/(32\eta)$$

- Ringdown given by quasi-normal mode (QNM) sum:

$$I_{2\pm 2}^{(2+p),\text{ring}} = \sum_{n=0}^{n_{\text{max}}} (-\sigma_{22n})^p A_{22n} e^{-\sigma_{22n}T}$$

$$\sigma_{lmn} = i\omega_{lmn} + \tau_{lmn}^{-1}$$

$$\tau_{\text{rr}} = (5/256)(M/\eta)(r_m/M)^4$$

$$T = t - t_m$$

- QNMs given by Berti, Cardoso, Will '06; final BH mass and spin determined from numerical relativity fits (Baker et al. '08)
- A_{22n} determined by matching derivatives at $t=t_m$ at Schwarzschild light-ring $r_m = 3M$.

- Result is the following simple analytic function:

$$\hat{h}_{\text{MWM}}^{\text{mem}} = \frac{8\pi M}{r(T)} H(-T) + H(T) \left\{ \frac{8\pi M}{r_m} + \frac{1}{\eta M} \sum_{n,n'=0}^{n_{\text{max}}} \frac{\sigma_{22n} \sigma_{22n'}^* A_{22n} A_{22n'}^*}{\sigma_{22n} + \sigma_{22n'}^*} [1 - e^{-(\sigma_{22n} + \sigma_{22n'}^*)T}] \right\}$$

Memory from the merger: full-EOB model

- Use EOB method applied in Damour, Nagar, Hannam, Husa, Bruggmann'08
 - “flexibility” parameters in EOB formalism were calibrated to Caltech/Cornell inspiral waveforms and Jena merger waveforms

- Inspiral moments modeled as:

$$I_{2\pm 2}^{(q)} = 2\sqrt{\frac{2\pi}{5}} (\mp 2i)^q M^{3-q} (r_\omega \Omega)^{3q-4} e^{\mp 2i\varphi} F_{22} f_{22}^{\text{NQC}}$$

- Solve EOB equations for r , φ , p_{r^*} , p_φ
- Ringdown same as minimal-waveform model, but with 5 QNMs
- Matching done for $I_{22}^{(2)}$ at 5 points near the EOB-deformed light ring
- Construct $I_{22}^{(3)}$; plug into previous leading-order expression for the memory; numerically integrate from $r_0=15M$ using leading-PN order memory for the initial condition

