The Effect of Eccentricity in Searches for Gravitational Waves from Compact Binaries.

Peter Zimmerman

Syracuse University

Overview

- Astrophysical Motivation
- Post-Newtonian Model
- Eccentric Waveforms
- Template Bank Results

Astrophysical Motivation

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AdLIGO Event Rates
~1-100 per year

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- We use the 'x-based' formalism developed by Hinder et al., which has been shown to agree with numerical simulations of eccentric binaries.
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  - In the zero eccentricity limit this formalism reduces to the TaylorT4 approximant.
    - The T4 has been shown to agree with NR, with phase differences of ~0.3 radians at 2pN and ~0.08 radians at 3.5pN after 30-cycles. [M. Boyle et al., PRD. 76 124038 (2007)]

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  - At t ~ 1800*M* the phase difference between the x-model and NR is approximately 0.8 radians.
  - At t ~1800, the phase difference between the n-model and NR is approximately 20 radians. [I. Hinder, F. Herrmann, P. Laguna, and D. Shoemaker, arXiv:0806.1037v1 (2008)]

The post-Newtonian Model

$$x \equiv (M\omega)^{2/3} \quad \omega \equiv \frac{2\pi + \Delta\phi}{P}$$

- The expression for $P = \frac{2\pi}{n}$, $\Delta\phi$, and e_t are known functions of E and J, which allows us to express everything terms of x and e_t.

$$Mn = x^{3/2} + n_{1\mathrm{PN}} x^{5/2} + n_{2\mathrm{PN}} x^{7/2} + n_{3\mathrm{PN}} x^{9/2} + \mathcal{O}(x^{11/2})$$

The post-Newtonian Model

Conservative pN Equations:

$$M\dot{\phi} = \frac{\sqrt{1-e_t^2}}{(1-e_t\cos u)}x^{3/2} + \dot{\phi}_{1\mathrm{PN}}x^{5/2} + \dot{\phi}_{2\mathrm{PN}}x^{5/2} + \dot{\phi}_{3\mathrm{PN}}x^{5/2} + \mathcal{O}(x^{11/2})$$

$$\frac{r}{M} = (1-e_t\cos u)\,x^{-1} + r_{1\mathrm{PN}} + r_{2\mathrm{PN}}x + r_{3\mathrm{PN}}x^2 + \mathcal{O}(x^3)$$

$$l = u - e_t\sin u + l_{2\mathrm{PN}}x^2 + l_{3\mathrm{PN}}x^3 + \mathcal{O}(x^4)$$

Radiation Reaction pN Equations:

$$M\dot{x} = \frac{2\eta}{15\,(1-e_t^2)^{7/2}}\left(96 + 292e_t^2 + 37e_t^4\right)x^5 + \dot{x}_{1\mathrm{PN}}x^6 + \dot{x}_{1.5\mathrm{PN}}x^{13/2} + \dot{x}_{2\mathrm{PN}}x^7 + \mathcal{O}(x^{15/2})$$

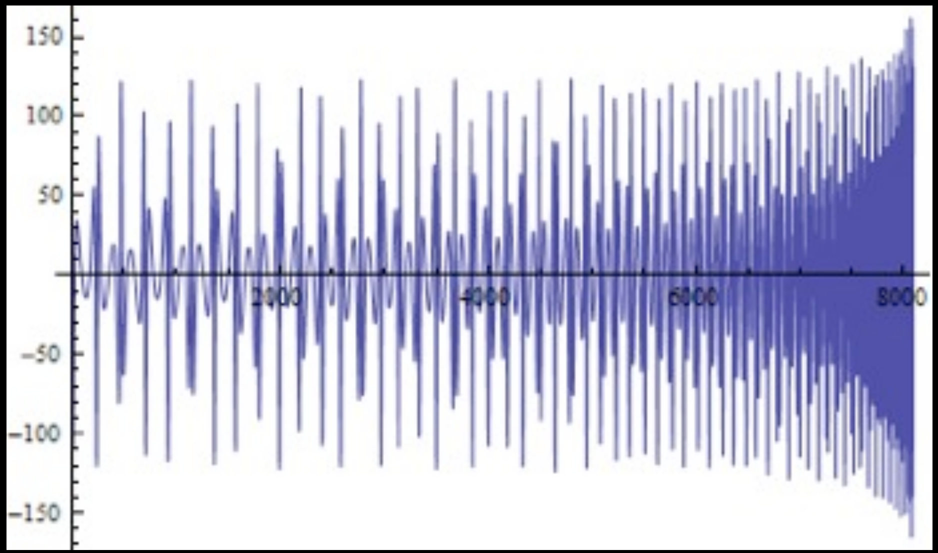
$$M\dot{e} = \frac{-e\eta}{15\,(1-e_t^2)^{5/2}}\left(304 + 121e_t^2\right)x^4 + \dot{e}_{1\mathrm{PN}}x^5 + \dot{e}_{1.5\mathrm{PN}}x^{11/2} + \dot{e}_{2\mathrm{PN}}x^6 + \mathcal{O}(x^{13/2})$$

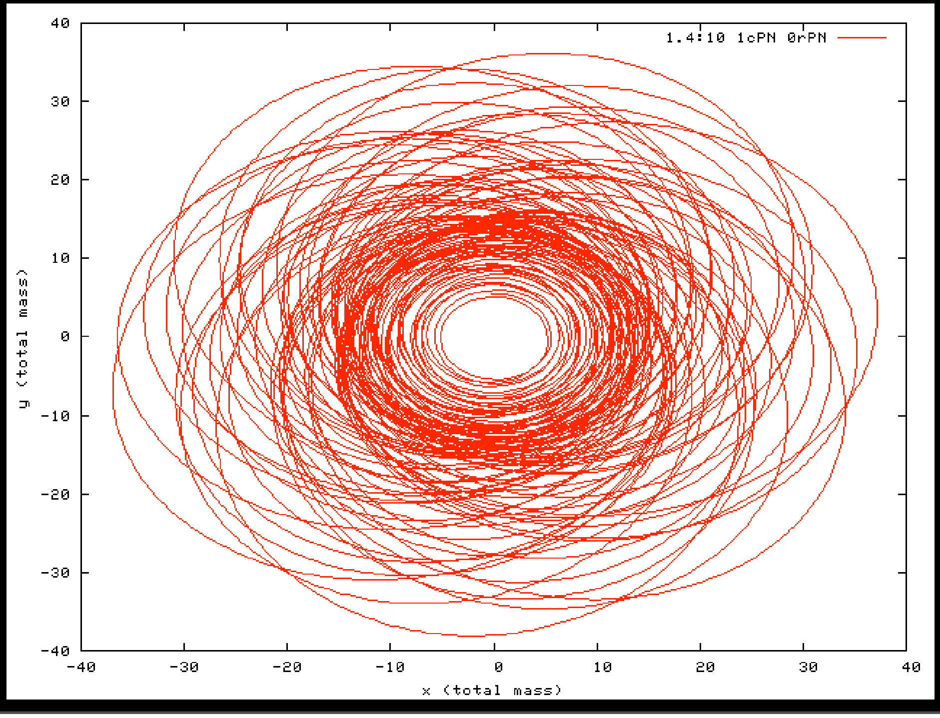
Waveforms

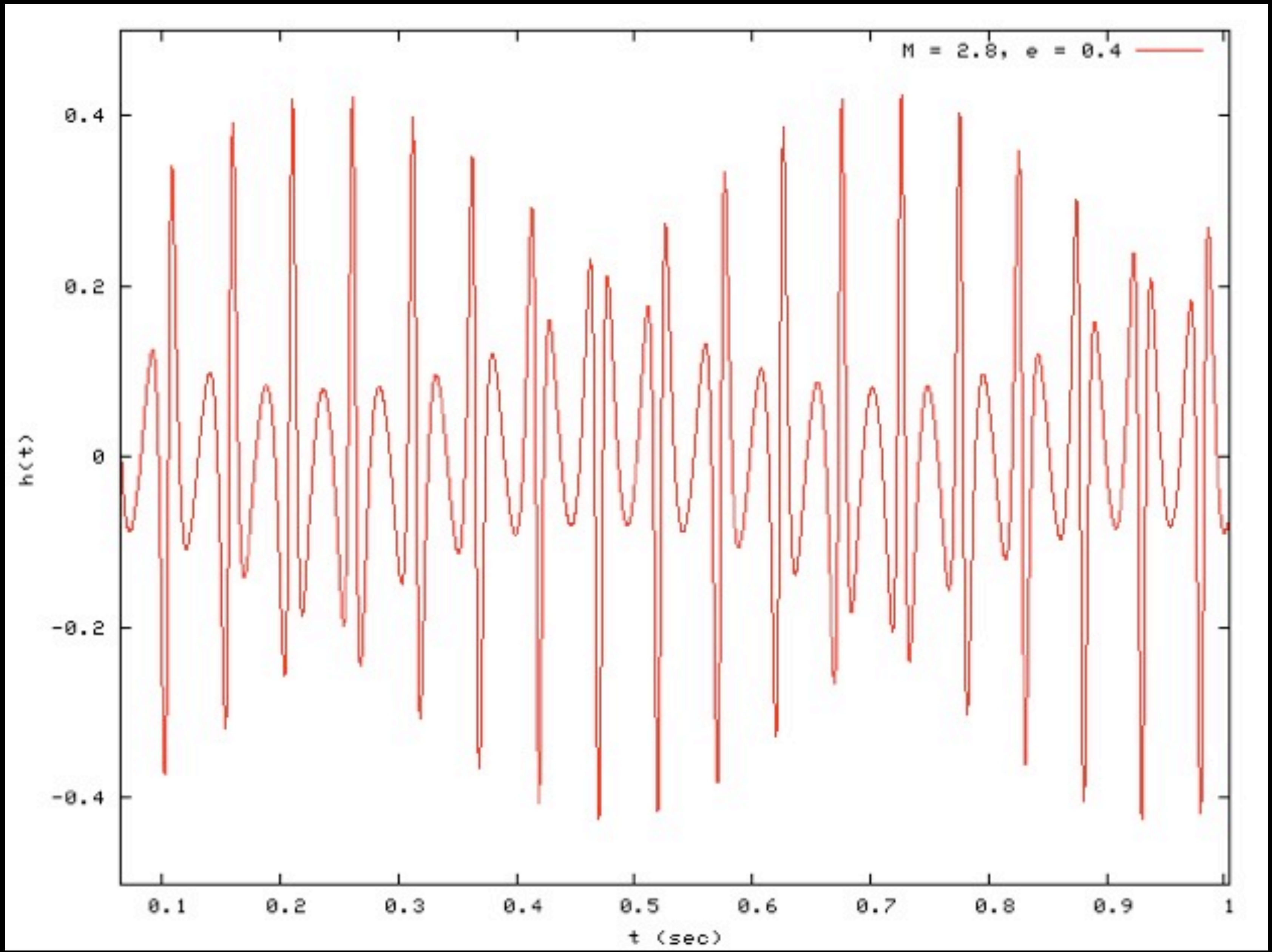
$$h_+ = \frac{-M\eta}{R}\left[(\cos^2\theta + 1)\left[\cos\phi'\left(-\dot{r}^2 + r^2\dot{\phi}^2 + \frac{M}{r}\right) + 2r\dot{r}\dot{\phi}\sin 2\phi'\right] + \left(-\dot{r}^2 - \dot{r}^2\dot{\phi}^2 + \frac{M}{r}\right)\sin^2\theta\right]$$

$$h_\times = \frac{-2M\eta}{R}\cos\theta\left[\left(-\dot{r}^2 + r^2\dot{\phi}^2 + \frac{M}{r}\right)\sin 2\phi' - 2r\dot{r}\dot{\phi}\cos 2\phi'\right]$$

[Damour, T., Gopakumar A., Iyer B. PRD 70, 1064028 (2004)]







Tuesday, June 16, 2009

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- Neglecting the conservative dynamics may reduce the overlaps significantly. [A .Gopakumar and M. Tessmer, PRD. **78** 084029 (2008)]

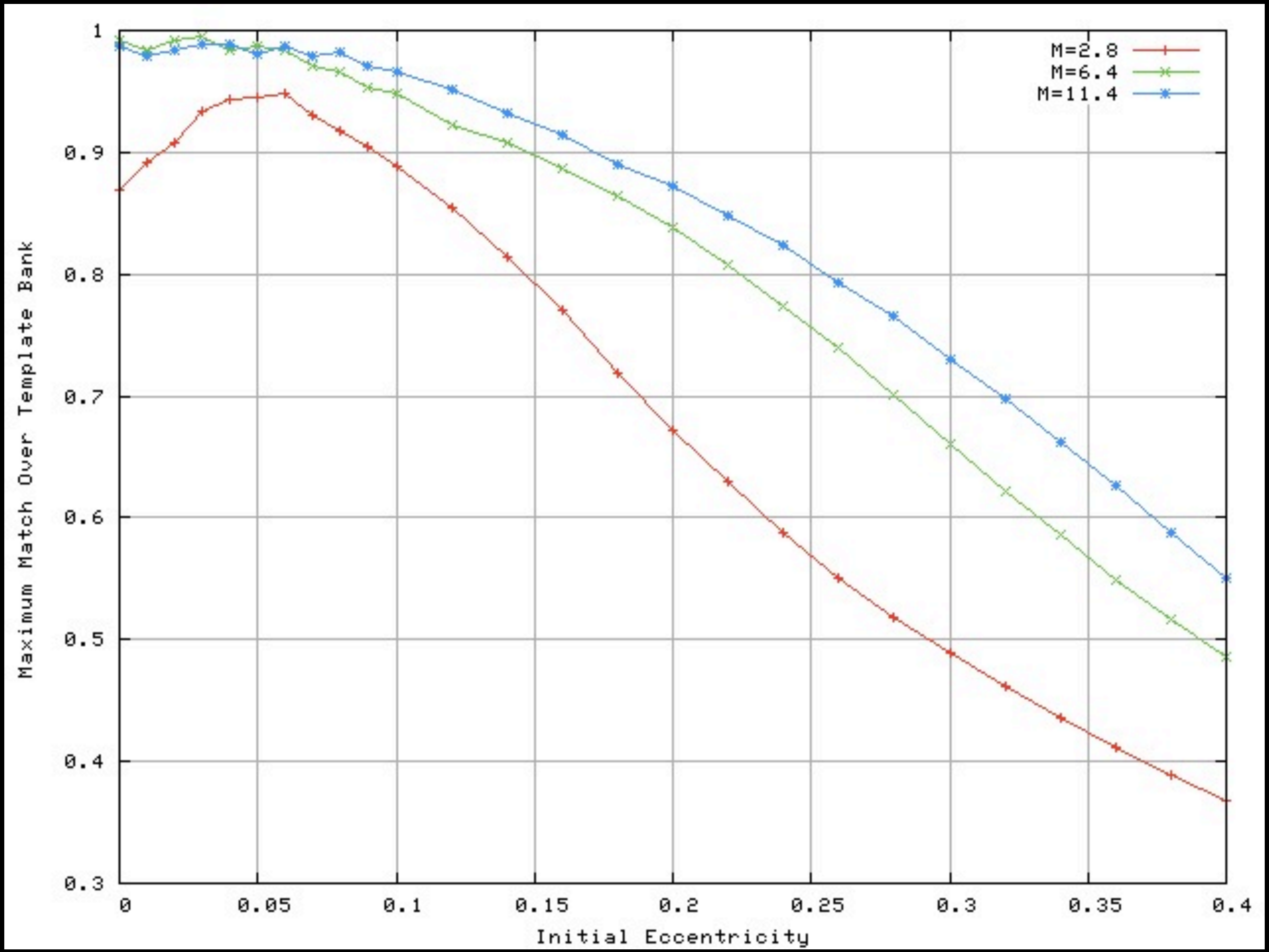
Template Bank Simulation

$$FF(s, h(\mathcal{M},\, \eta)) = \max_{\mathcal{M},\, \eta}\, \max_{t_C,\, \varphi_C} (\hat{s}|\, \hat{h}(\mathcal{M},\, \eta))$$

$$(s|h) = 2 \int_{f_0}^{f_{\rm isco}} \frac{\tilde{s^*}(f)\tilde{h}(f) + \tilde{s}(f)\tilde{h^*}(f)}{S_n(f)}\, df$$

- The Fitting Factor (*FF*) measures the loss in SNR incurred when an eccentric signal is filtered with a circular template.
- We compute the FF by maximizing over a bank of TaylorF2 SPA templates.
- N ~ 1000 eccentric signals were injected in our simulation.

Template Bank Results



TaylorF2 2.0pN

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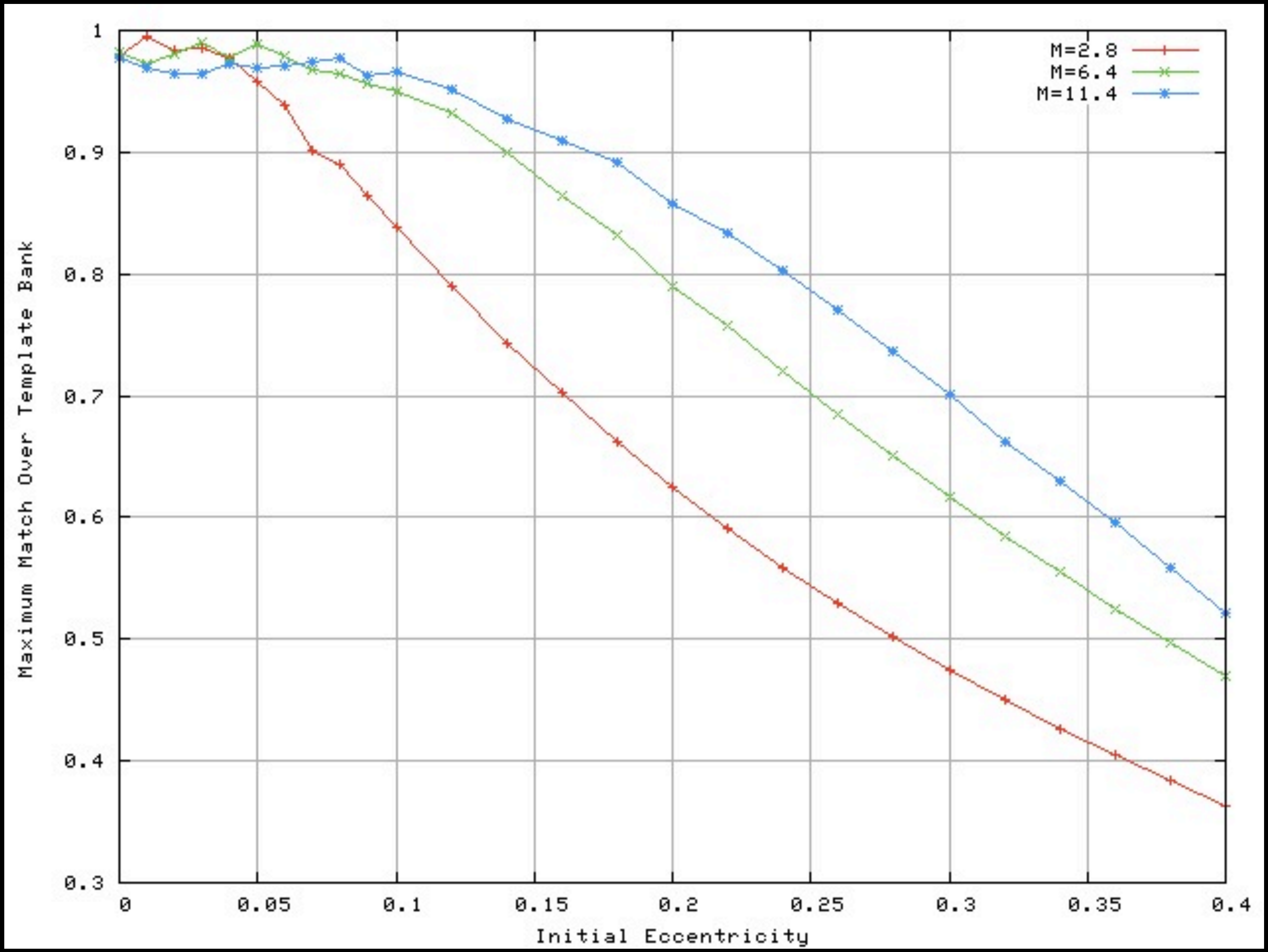

TaylorF2 3.5pN

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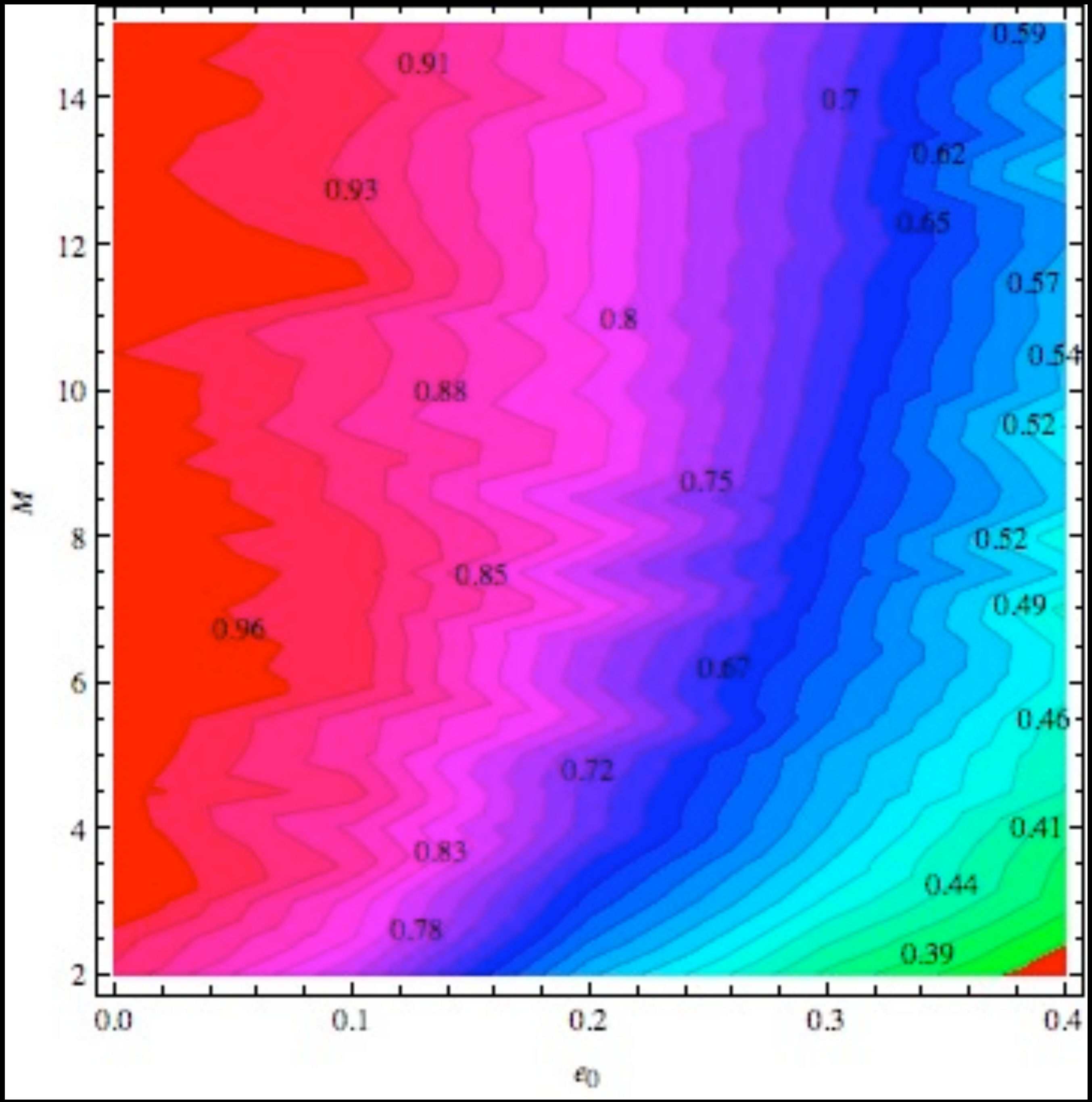
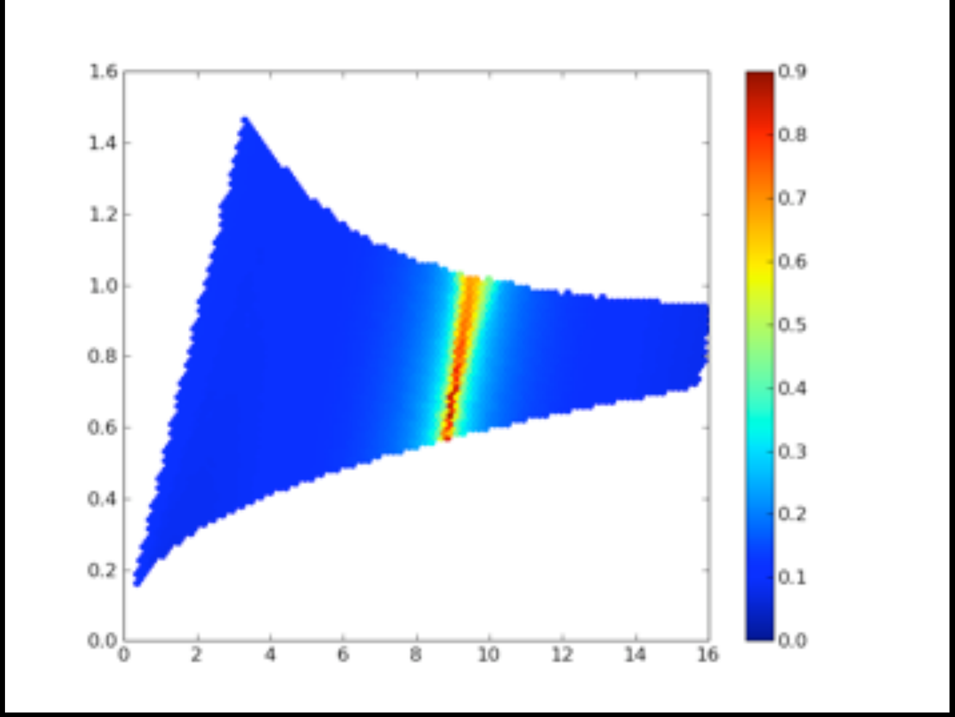
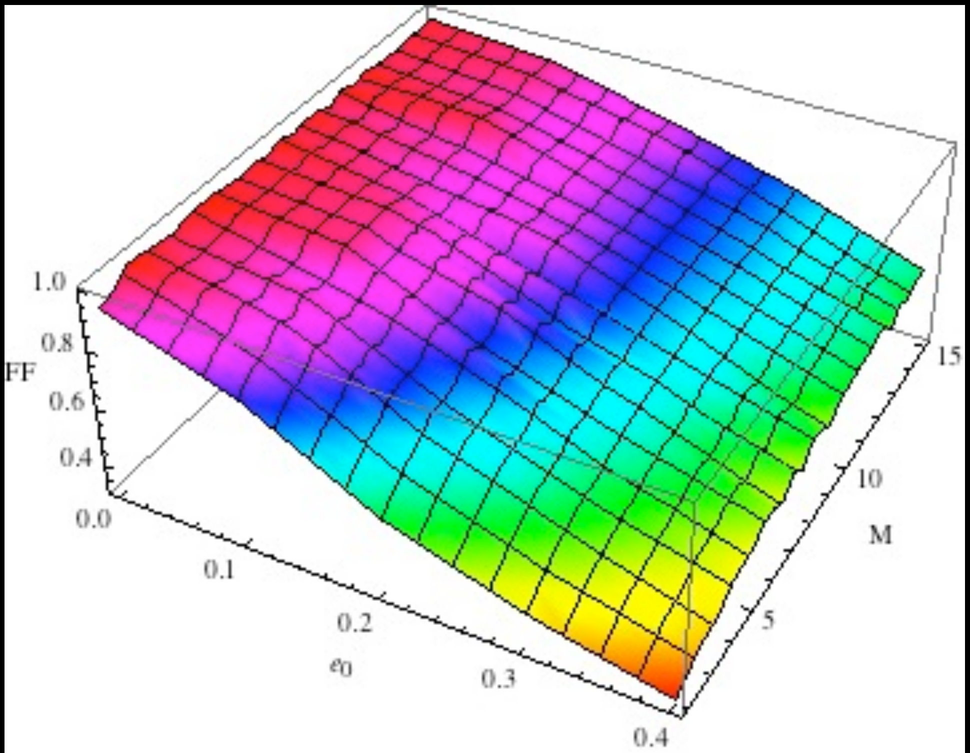


Fitting Factors over a grid of ~1000 signals



Sample Bank Simulation: M=2.8, e=0.1 in (τ_0, τ_3) coordinates.



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Conclusions

- Using circular templates as matched filters for realistic eccentric signals results in loss of SNR.
- For small eccentricities ($e<0.1$) this loss is not significant for detection purposes.
- Parameter estimation may be poor.