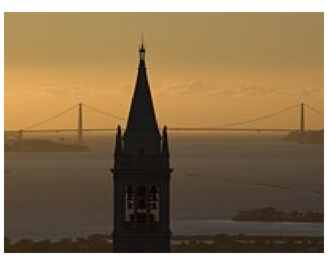
Orbits of Black holes by Test Particles

Interactive simulations in Mathematica

Acceleration of PCs using GPUs

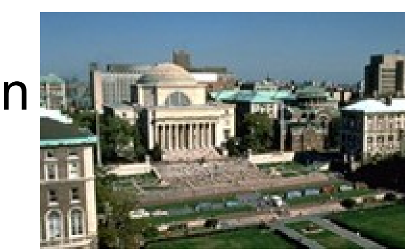


by

David Saroff

supervised by
Professor Ori Ganor

Berkeley, LBNL



drawing on work by

Gabe Perez-Giz
Becky Grossman
Glenna Clifton

supervised by
Professor Janna Levin

Columbia

Presented at ECGM25 June 2009

I've moved to RIT
The RIT mascot is a Tiger.

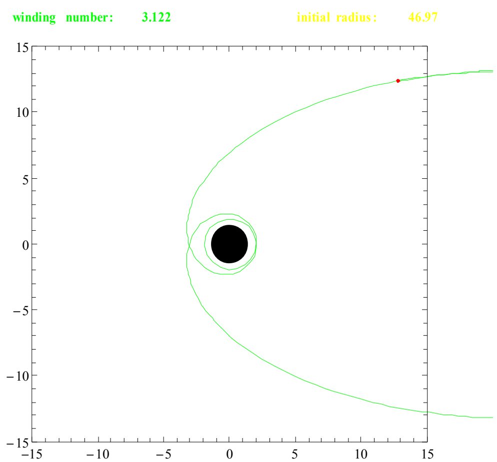
Columbia	RIT	Berkeley
		

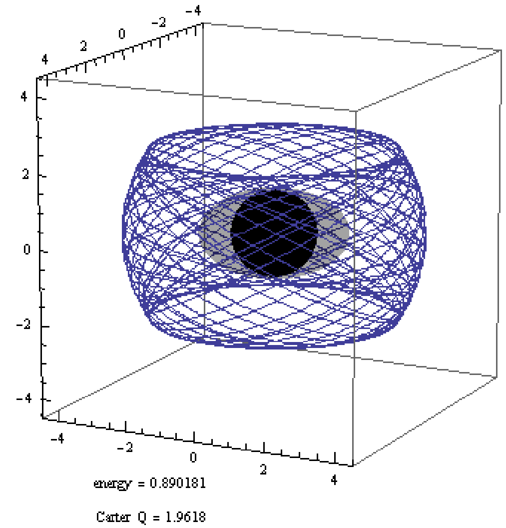
Lions and Tigers and Bears

Oh my!

ECGM25 June 2009

We wrote interactive Mathematica programs that simulate test particle orbits around a spinning black hole, in the Kerr geometry. These are toys, because there is no radiation back reaction or gravity wave generation.





Can realistic orbits that include gravity waves be simulated on PC's? No.

Can PC's be enhanced inexpensively to perform better? Yes.

Can realistic orbits that include radiation reaction be simulated on enhanced PC's? Maybe.

Graphics Processing Unit chips on graphics boards are useful accelerators.

We can learn and experiment with GPUs on the cheap
then migrate to a cluster with tested programs.

NCSA's Innovative Systems Laboratory maintains a 32-node cluster that combines both GPU (graphics processing units) and FPGA (field-programmable gate array) technology in order to explore the potential of these novel architectures to accelerate scientific computing.

For more information about ISL's Accelerator cluster, contact:
Mike Showerman
mshow@ncsa.uiuc.edu
217-244-5478

http://iacat.uiuc.edu/resources/cluster/

The 32 compute nodes feature:

* Two dual-core 2.4 GHz AMD Opterons, 8 GB of memory
* One NVIDIA Tesla S1070, containing four GT200 GPUs each with 4 GB of memory
* Nallatech H101-PCIX Xilinx Virtex-4 **FPGA** accelerator, 16 MB SRAM, 512 MB SDRAM

Professor Janna Levin at Columbia and her student Gabe Perez-Giz wrote about test particle black hole orbits. They invented a clever taxonomy for orbits in the equatorial plane. Their paper is

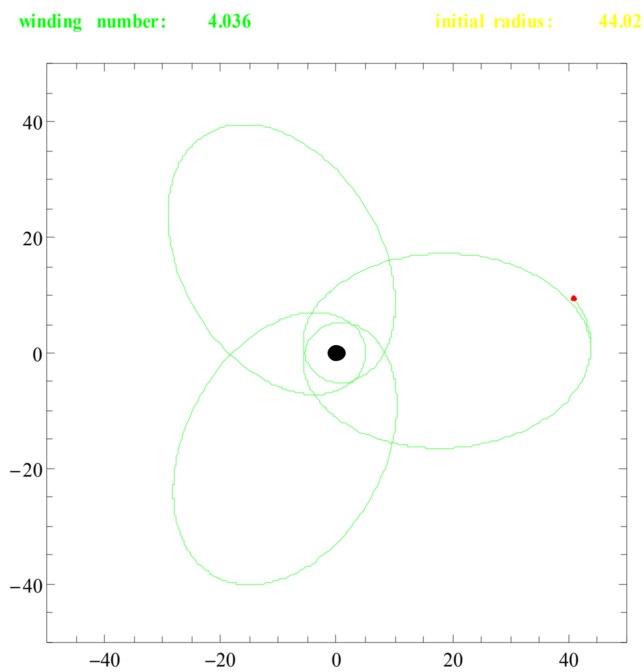
A Periodic Table for Black Hole Orbits

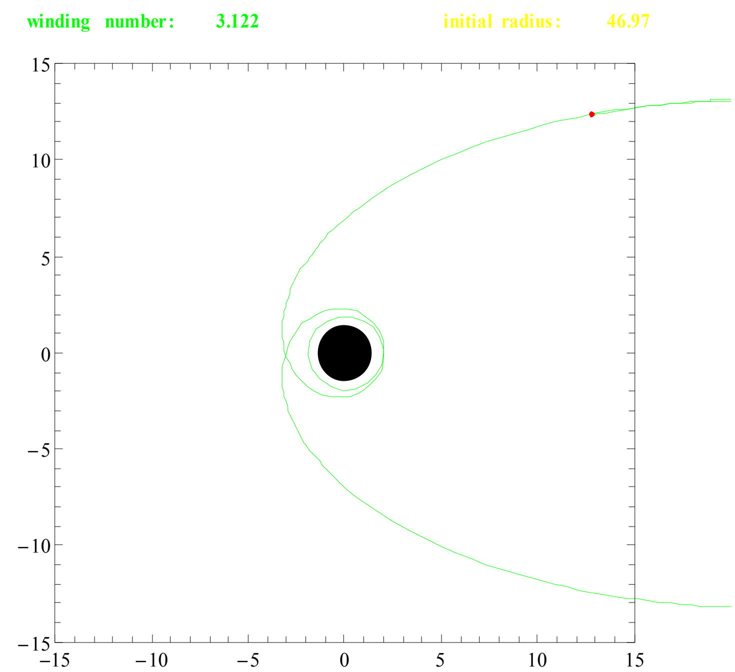
arXiv:0802.0459v1 [gr-qc] 4 Feb 2008

Orbit triplet	Orbit rational
(leaves,whirls,advance)	q = whirls + advance / leaves

(3,0,1) q = 0 + 1 / 3 = 1/3

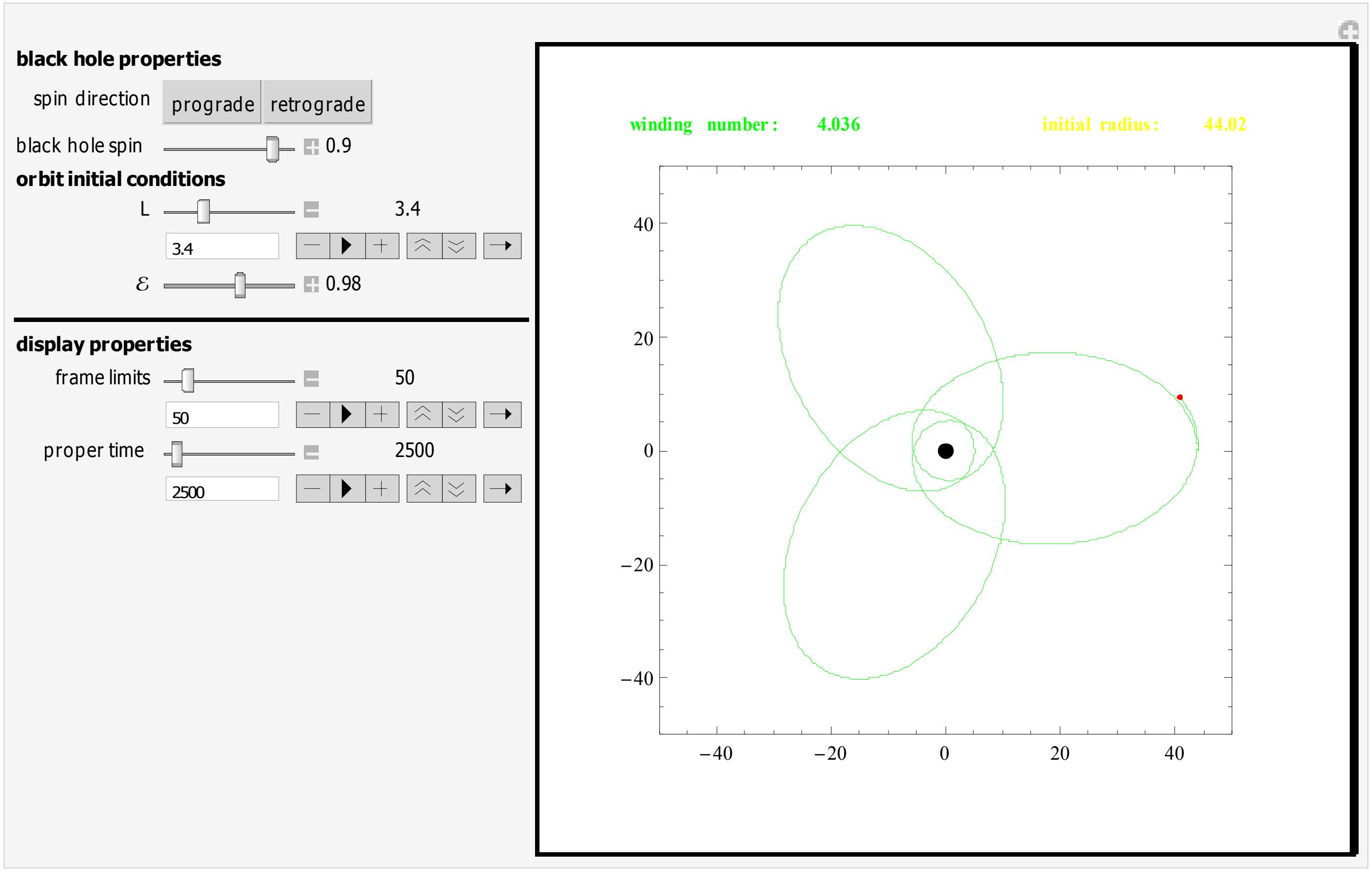
(1,2,0) q = 2 + 0 / 1 = 2





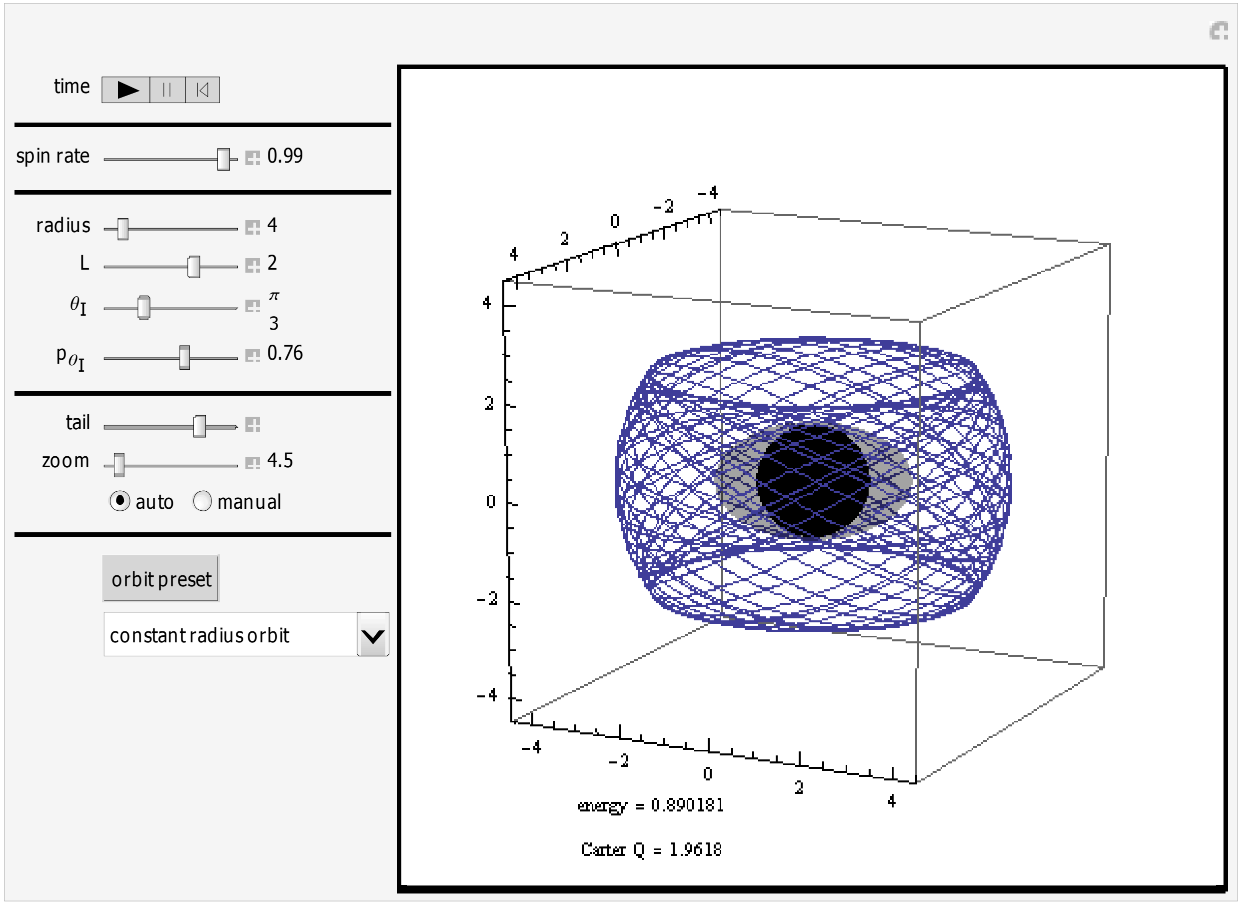
Professor Levin, her student Glenna Clifton, and I wrote a program simulating test particle orbits in the Kerr equatorial plane in Mathematica. The simulator is interactive. It made the orbits on the previous slide.

The simulator is available from The Wolfram Demonstrations Project using the URL below.



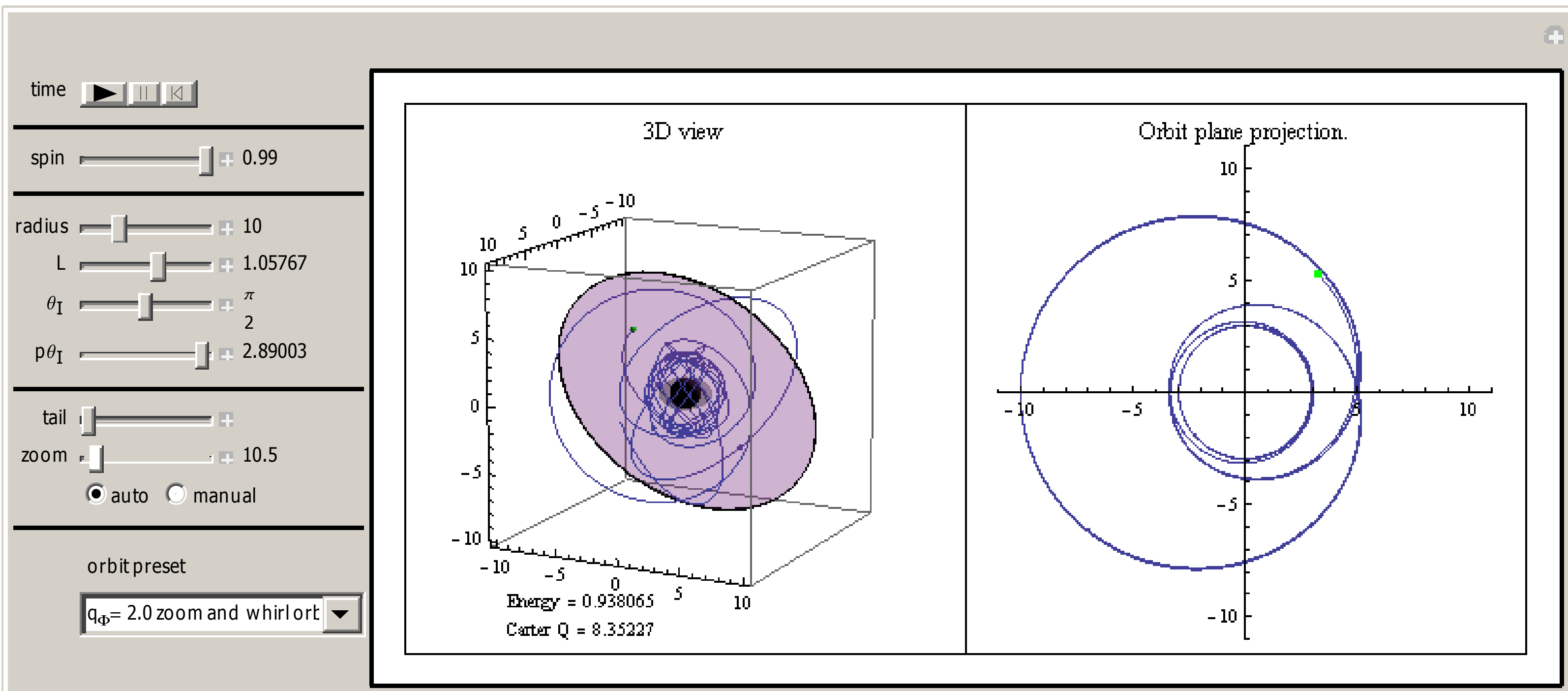
http://demonstrations.wolfram.com/OrbitsAroundASpinningBlackHole/

Jim Hartle suggested that I try simulating the test particle orbits generally (3D) in the Kerr metric using Mathematica.

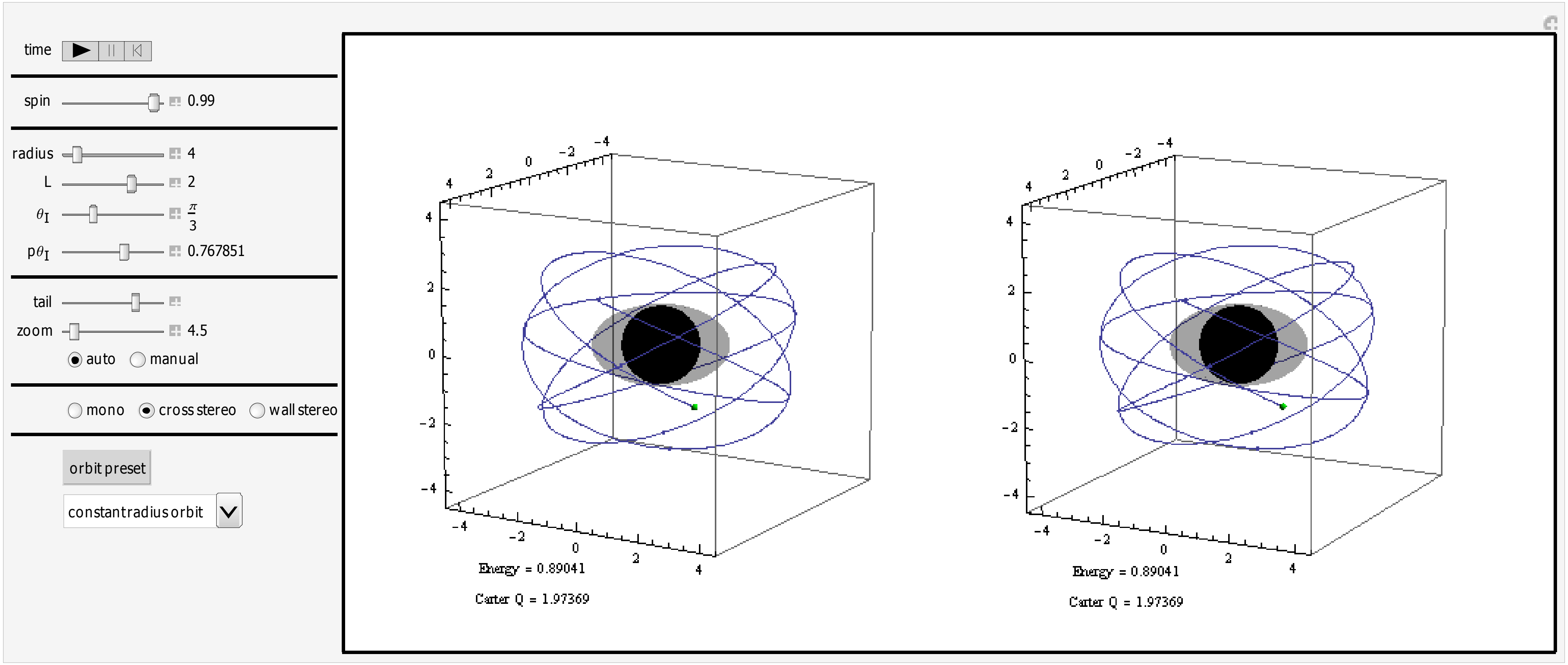


Professor Levin and her student Becky Grossman did an analysis showing that the projection of a 3D orbit on its instantaneous orbital plane has the same appealing rational order as the equatorial case.

Here is an interactive simulator to explore this numerically.

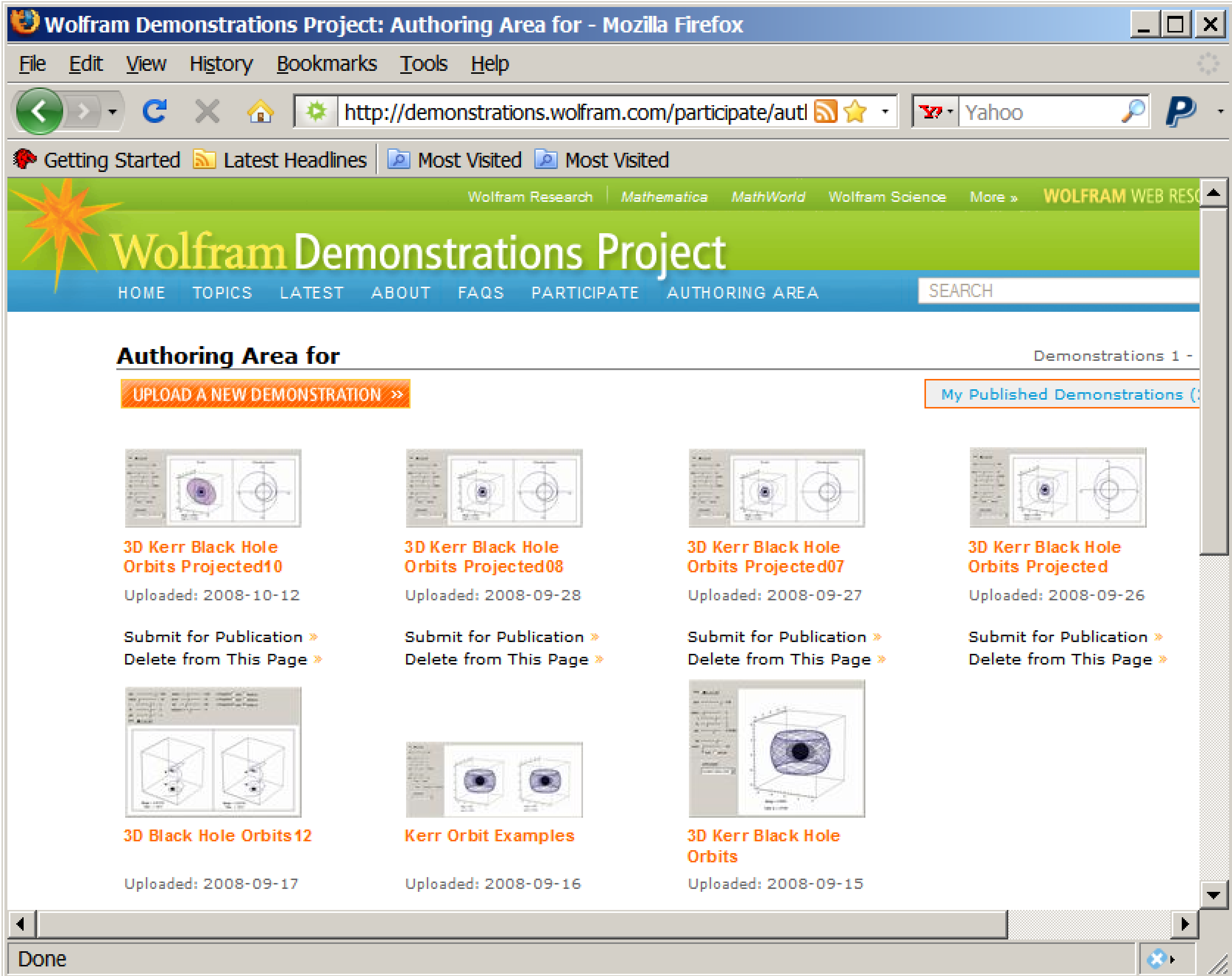


This one is fun. It is a stereo pair. If you cross your eyes slightly to fuse the two graphics, you will see 3D depth.



These three are in the public collection at the Wolfram Demonstrations Project

Please ask me for the URL if you want one of these.



ECGM25 June 2009

Test particle orbits are easy. From the metric the geodesic equation is available. Mathematica numerically integrates the geodesic in less time than it takes to draw the graphic.

These are the equations for motion in the Kerr equatorial plane in a convenient form for numerical integration.

$$r''[\tau] == \frac{1}{2}\left(-\frac{3\left(2 L^2 M+4 a L M^2 \mathcal{E}+2 a^2 M^3 \mathcal{E}^2\right)}{r[\tau]^4}-\frac{2\left(-L^2-a^2 M^2+a^2 M^2 \mathcal{E}^2\right)}{r[\tau]^3}-\frac{2 M}{r[\tau]^2}\right)$$

$$t'[\tau] == -\frac{2 a L M^2+2 a^2 M^3 \mathcal{E}+a^2 M^2 \mathcal{E}\, r[\tau]+\mathcal{E}\, r[\tau]^3}{r[\tau]\left(a^2 M^2-2 M r[\tau]+r[\tau]^2\right)}$$

$$\phi'[\tau] == -\frac{2 L M+2 a M^2 \mathcal{E}-L\, r[\tau]}{r[\tau]\left(a^2 M^2-2 M r[\tau]+r[\tau]^2\right)}$$

Only the first equation is necessary during the integration. It is simple, constants over powers of r.

$$r''[\tau] == \frac{c1}{r[\tau]^4}+\frac{c2}{r[\tau]^3}+\frac{c3}{r[\tau]^2}$$

G = c = 1

Real orbits, where there is gravitational radiation are hard

Lee Lindblom says that the difficulty grows
as (small mass/large mass)^5

BSSN Equations

Baumgarte & Shapiro PRD 59, 024007 (1999)
Shibata & Nakamura PRD 52, 5428 (1995)

$$\partial_o \Phi = -\frac{1}{6}\alpha K$$

$$\partial_o \hat{g}_{ij} = -2\alpha \hat{A}_{ij}$$

$$\partial_o K = -\nabla_i \nabla^i \alpha + \alpha \left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}\hat{K}^2 \right)$$

$$\partial_o \hat{A}_{ij} = e^{-4\Phi}[-\nabla_i \nabla_j \alpha + \alpha R_{ij}]^{TF} + \alpha (K \hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$\partial_o \hat{\Gamma}^i = \hat{g}^{jk}\partial_{jk}\beta^i + \frac{1}{3}\hat{g}^{ij}\partial_{jk}\beta^k - 2\hat{A}^{ij}\partial_j \alpha + 2\alpha \hat{\Gamma}^i{}_{jk}\hat{A}^{jk} + 12\alpha \hat{A}^{ij}\partial_j \Phi - \frac{4}{3}\alpha \hat{g}^{ij}\partial_j K$$

Tesla C1060

78 GFlops
double precision
floating point

* Massively-parallel many-core architecture

* 240 scalar processor cores per GPU

* Integer, single-precision and double-precision floating point operations

* Hardware Thread Execution Manager enables thousands of concurrent threads per GPU

* Parallel shared memory enables processor cores to collaborate on shared information at local cache performance

* Ultra-fast GPU memory access with 102 GB/s peak bandwidth per GPU

* IEEE 754 single-precision and double-precision floating point

* Each Tesla C1060 GPU delivers 933 GFlops Single Precision and

78 GFlops Double Precision performance

Isn't it hard to program GPUs ?

No!

OpenCL

an extension of C that has been standardized by
NVIDIA, AMD/ATI, INTEL, and APPLE

The language hides most of the complexity of the GPU.

Just enough is exposed.

Become a GPU Computing Registered Developer at:
http://developer.nvidia.com/page/registered_developer_program.html

Take home thoughts:

Hardware techniques seen in supercomputers are available to the rest of us as inexpensive plug in boards to the graphics slots of our PCs.

Full scale gravity wave problems are still beyond the capabilities of an enhanced PC

Take advantage of mass market computing equipment!