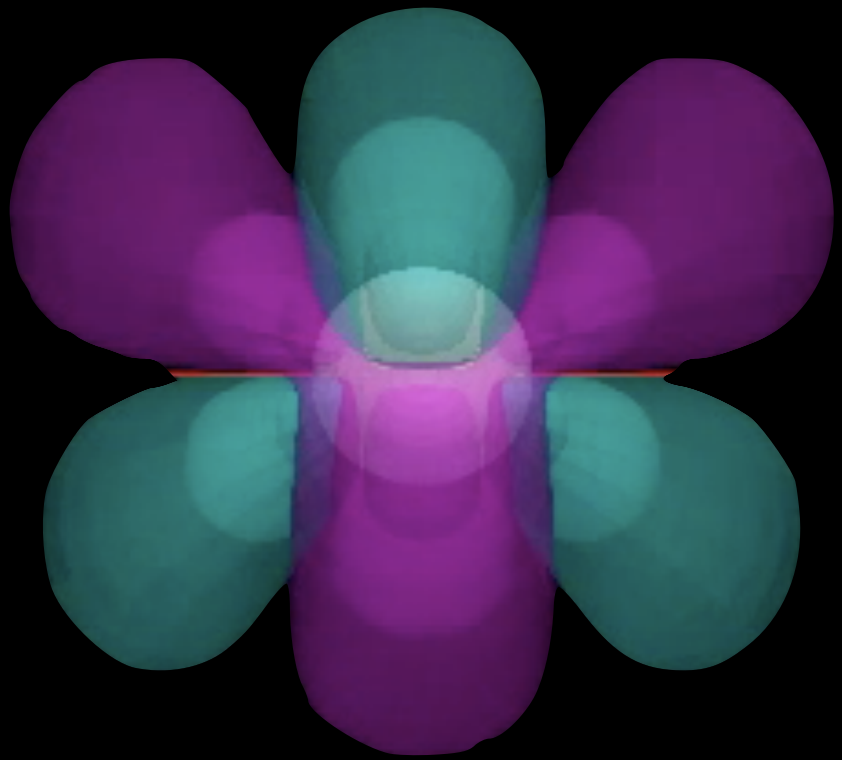
Momentum flow in numerical simulations of binary black hole mergers



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in collaboration with

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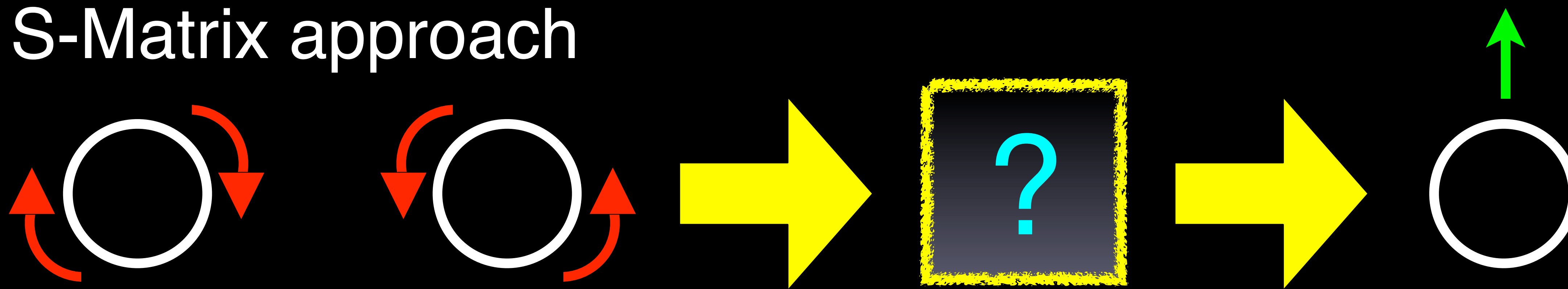
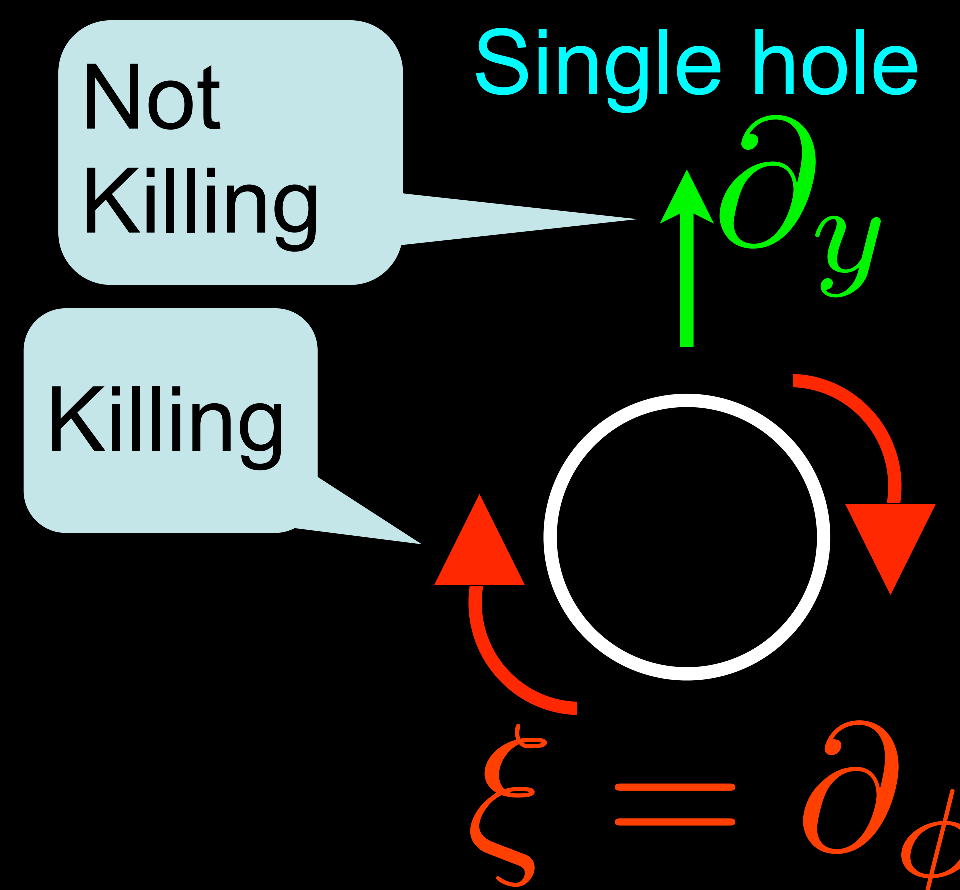
Dynamics of binary black hole (BBH) mergers

- Goal: extract science from numerical simulations
- S-Matrix approach

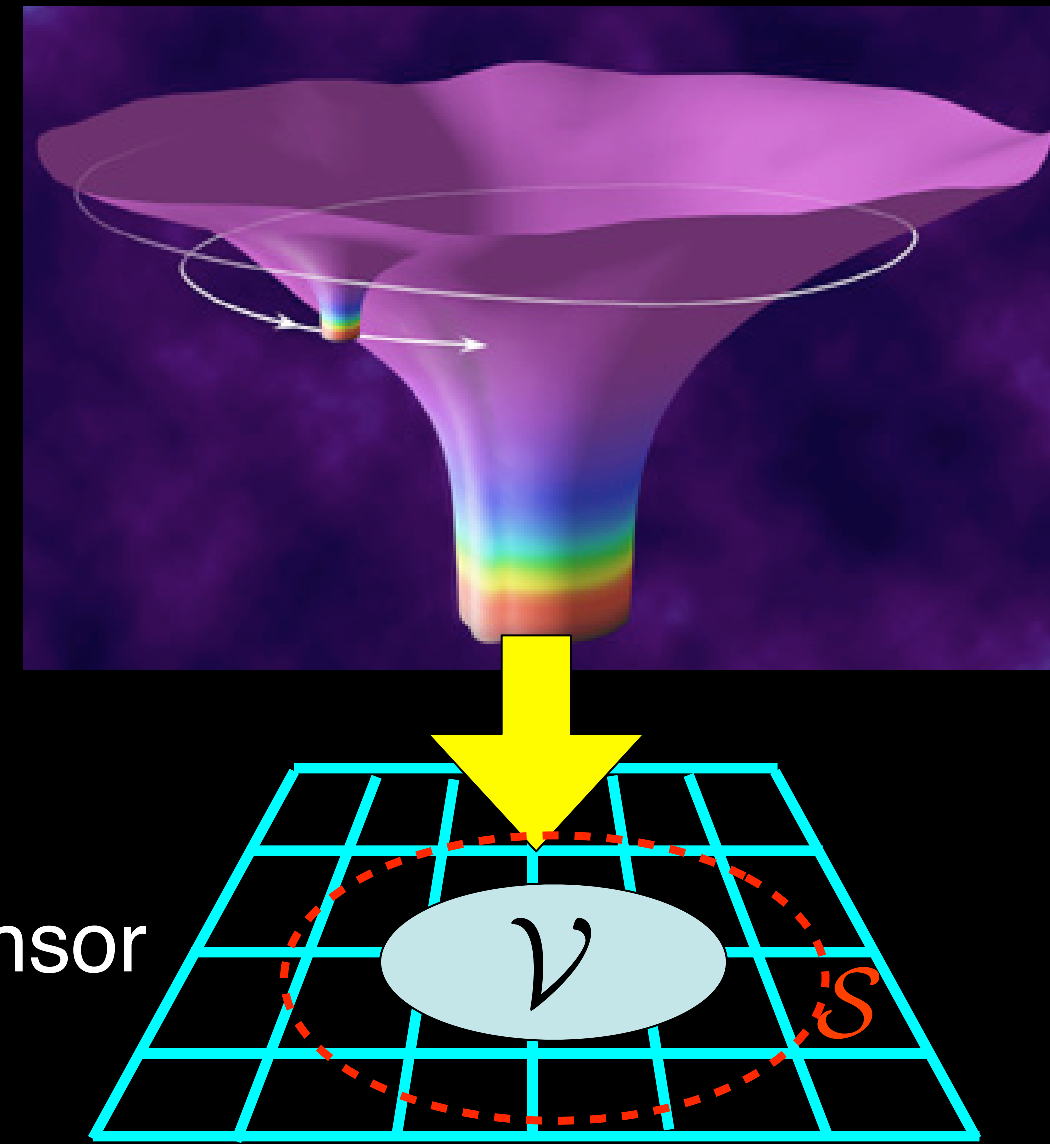


- Explore nonlinear dynamics of spacetime
  - Momentum
    - E.g., integrate on horizon, not ∞
    - Angular: approx. symmetry
    - Linear: Krishnan, Lousto, Zlochower (2008): “ADM-like” integrals
  - This talk: momentum flow
    - Field theory in flat spacetime
    - Head-on BBH merger with recoil
    - Gauge dependence



Momentum flow

- Field theory in flat spacetime
  - Use "preferred" coords. to define auxiliary flat spacetime (AFS)
  - General relativity as field-theory on AFS (Landau & Lifshitz 1962)



$$\mathfrak{g}^{\mu\nu} := \sqrt{-g}\, g^{\mu\nu}$$

- Linear momentum
  - Density: energy-momentum pseudotensor

$$P^{\mu} = \int_{\mathcal{V}} d^3x (-g) t_{LL}^{0\mu}$$

$$t^{\mu\nu} \sim \left(\partial_{\gamma} \mathfrak{g}^{\mu\nu}\right)^2$$

  - Enclosed by a surface

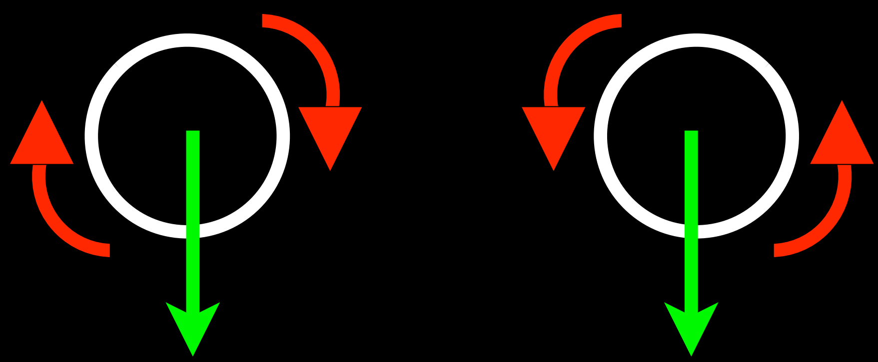
$$P^{\mu} = \oint_{S} d^2x_j H^{\mu\alpha 0 j}{}_{,\alpha}$$

$$H^{\mu\alpha\nu\beta} := \mathfrak{g}^{\mu\nu}\mathfrak{g}^{\alpha\beta} - \mathfrak{g}^{\alpha\nu}\mathfrak{g}^{\mu\beta}$$

$$v_{LL}^{i} := \frac{P^i}{P^0}$$

Simulations of head-on collisions

- Frame dragging picture



Post-Newtonian approx: acceleration to $O(10^3)$ km/s or more before merger

- Simulated wave flux: recoil velocity O(10) km/s

Simulation	Initial separation / M_{ADM} = *"total mass"*	Initial spin (dimensionless)	Recoil speed of final hole (km/s)
Goddard (Choi et al 2008)	8	0.79 (0.5)	32 (~21)
Rochester (Krishnan, Lousto, Zlochower 2008)	7	0.56	20
Caltech/Cornell spectral (this talk)	7.8	0.5	23

Assume kick linear in spin

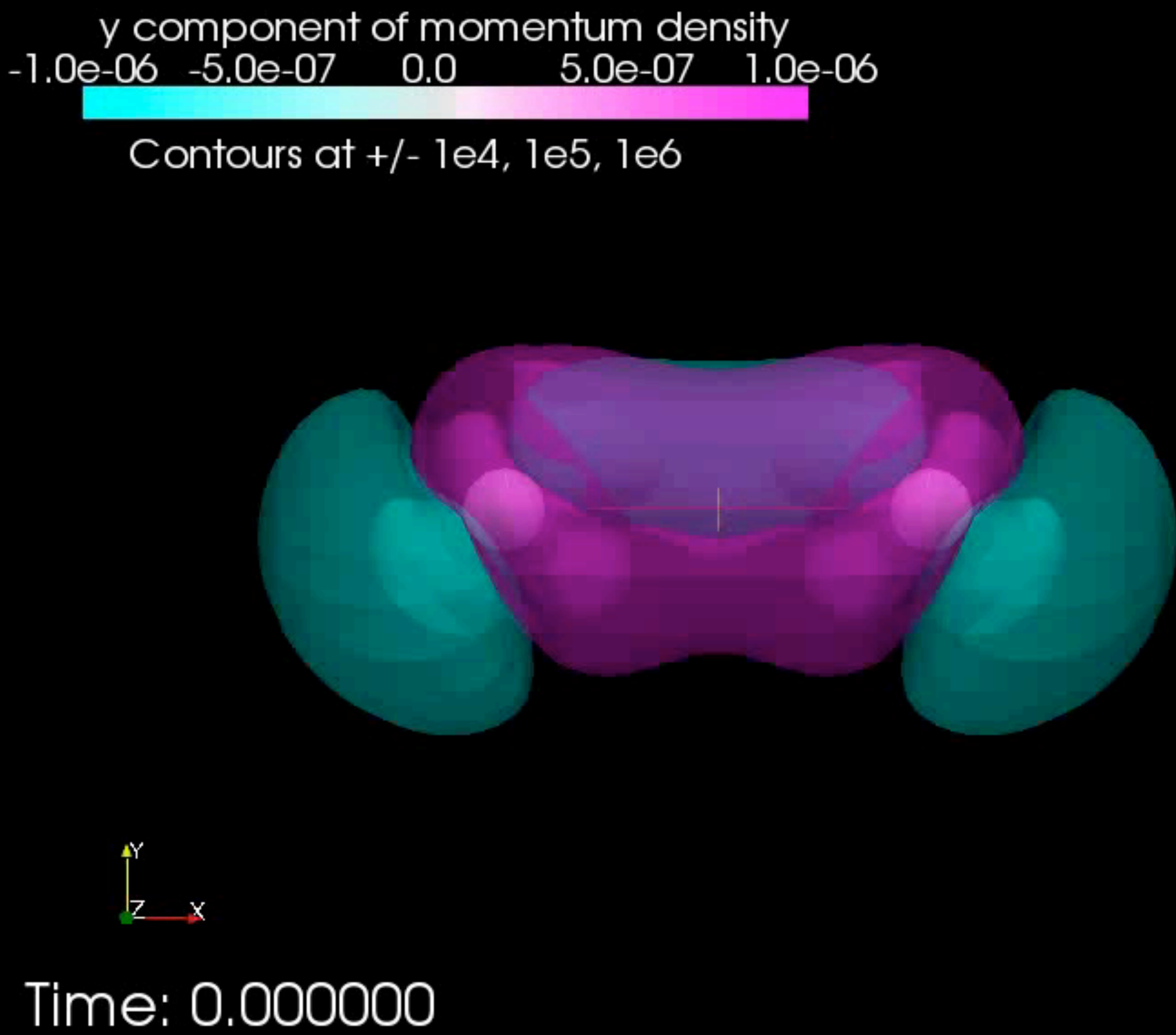
Quasilocal, wave flux agree well at late times

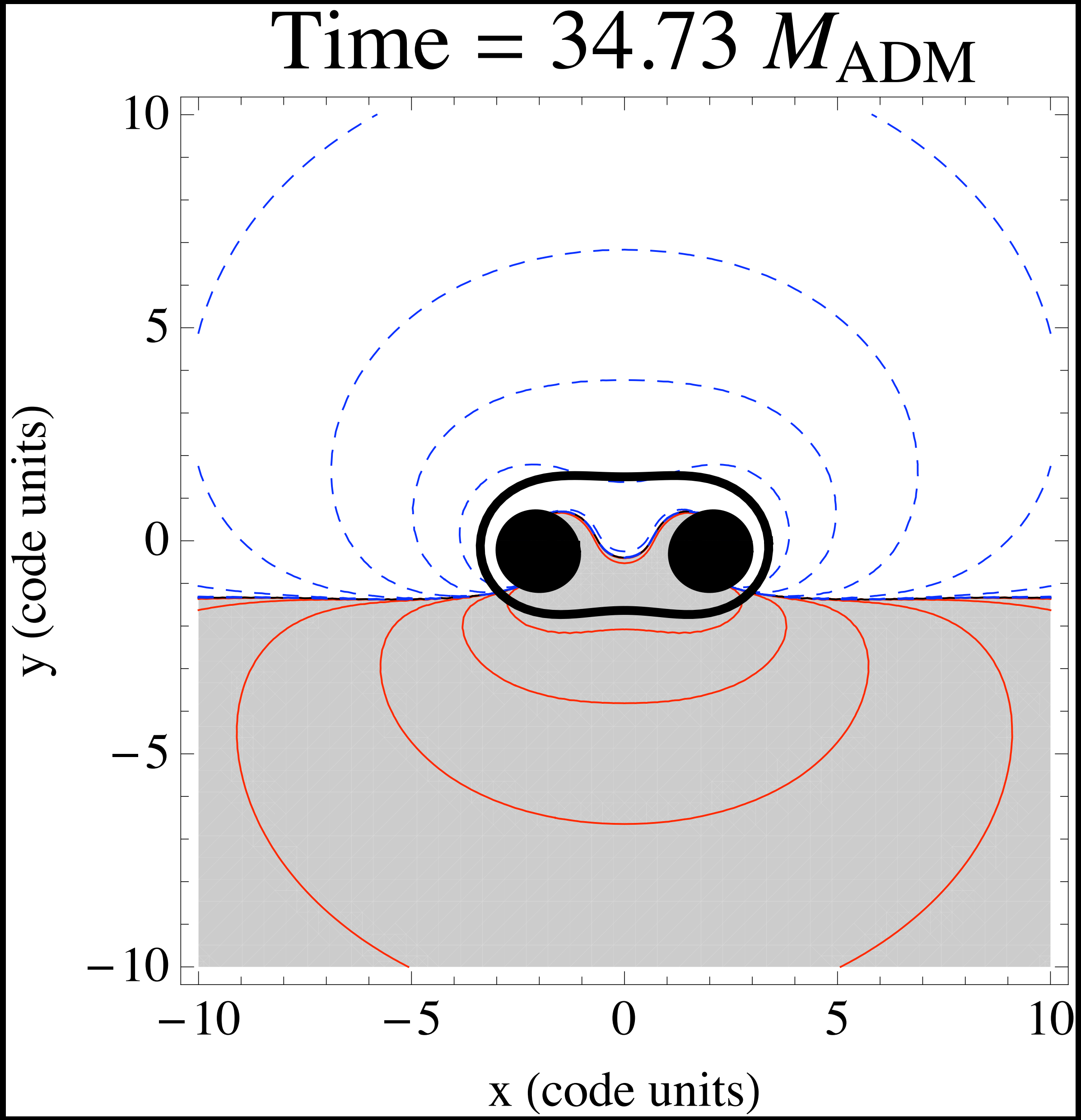
– *Total* initial momentum (holes + field) ≈ 0

– Kick in direction opposite frame dragging

Linear momentum density

- y component (kick direction)

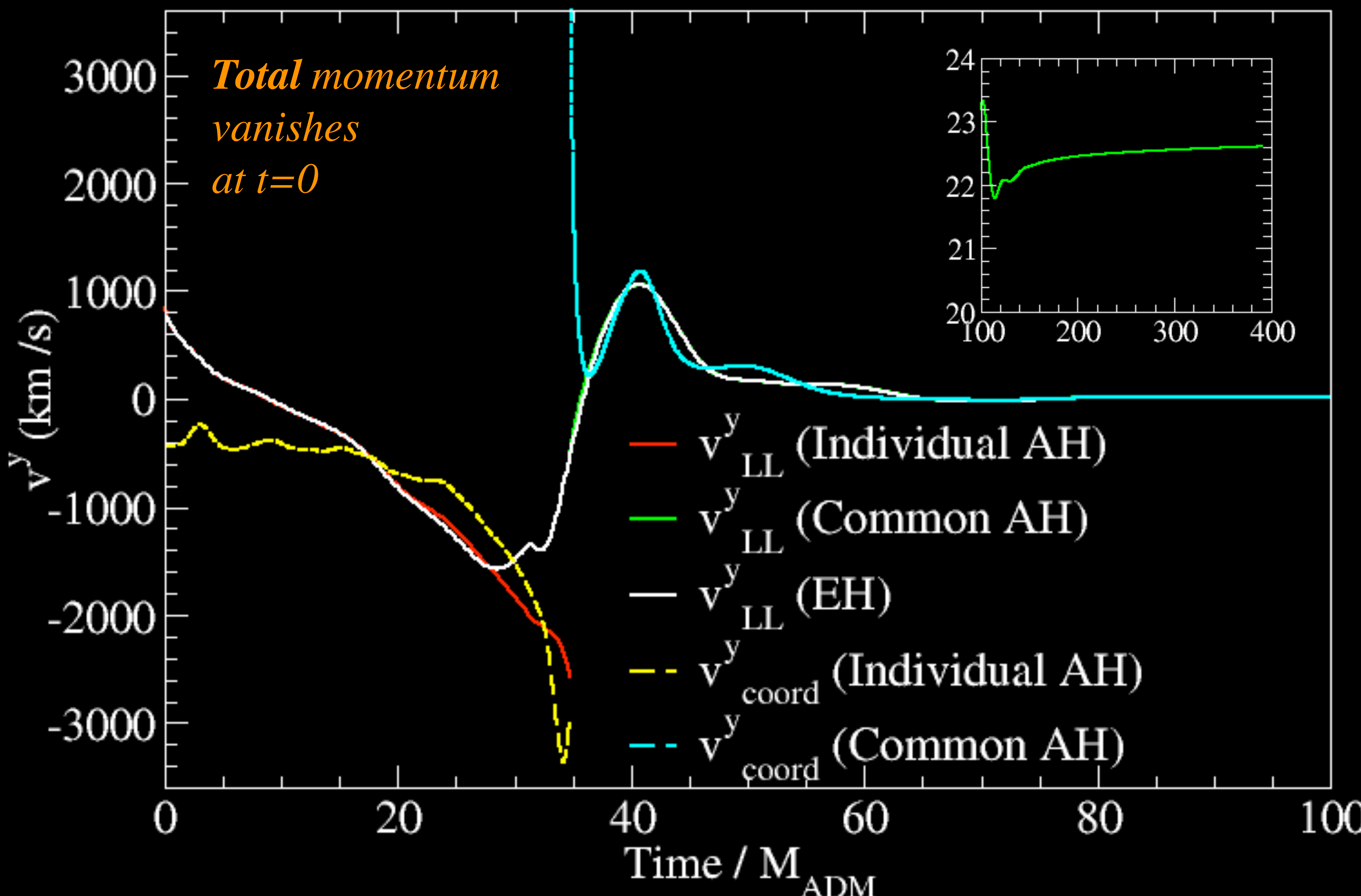


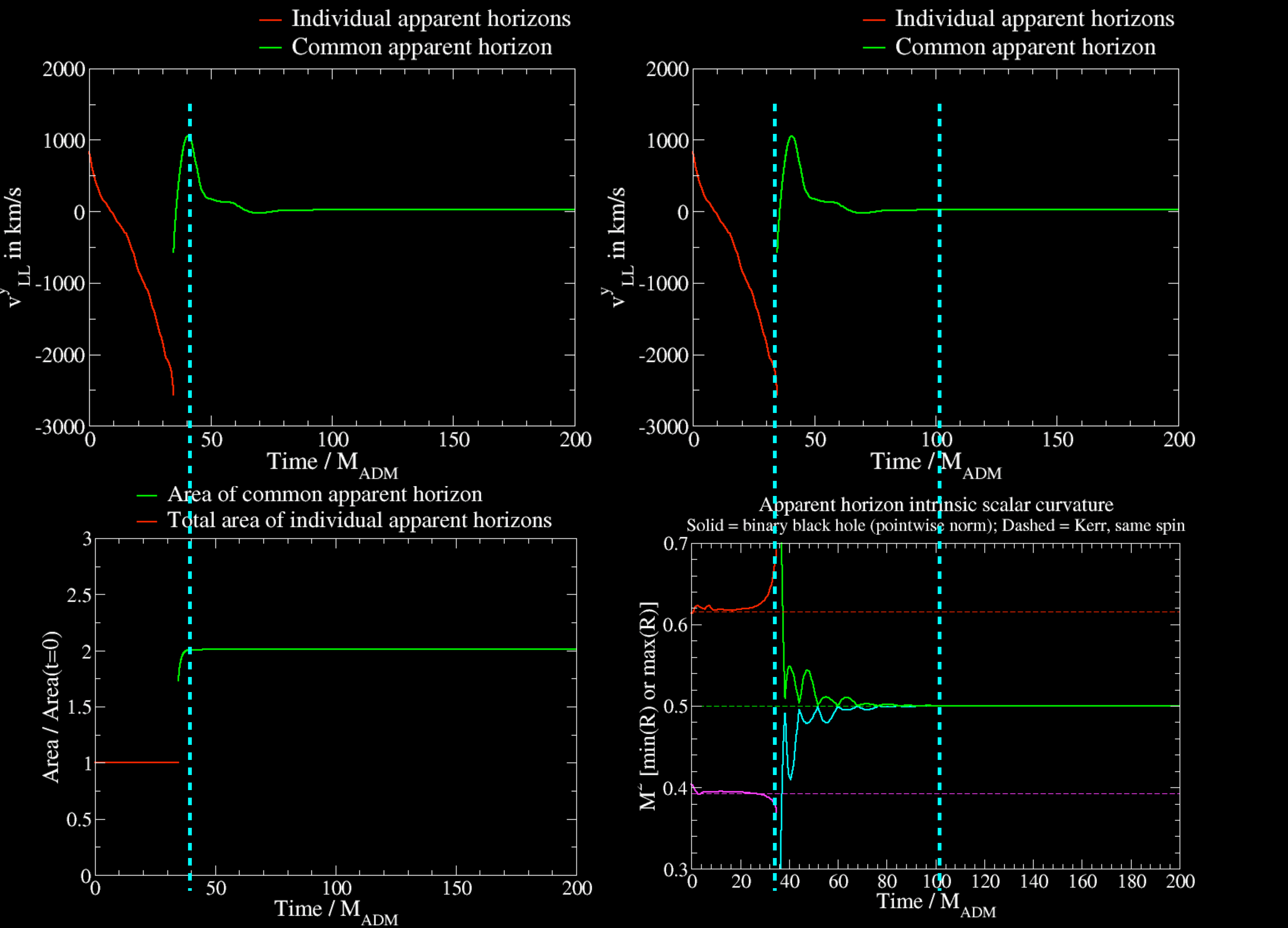


Velocities measured on horizons

AH = Apparent horizon
EH = Event horizon

Event horizon curve courtesy Jeff Kaplan



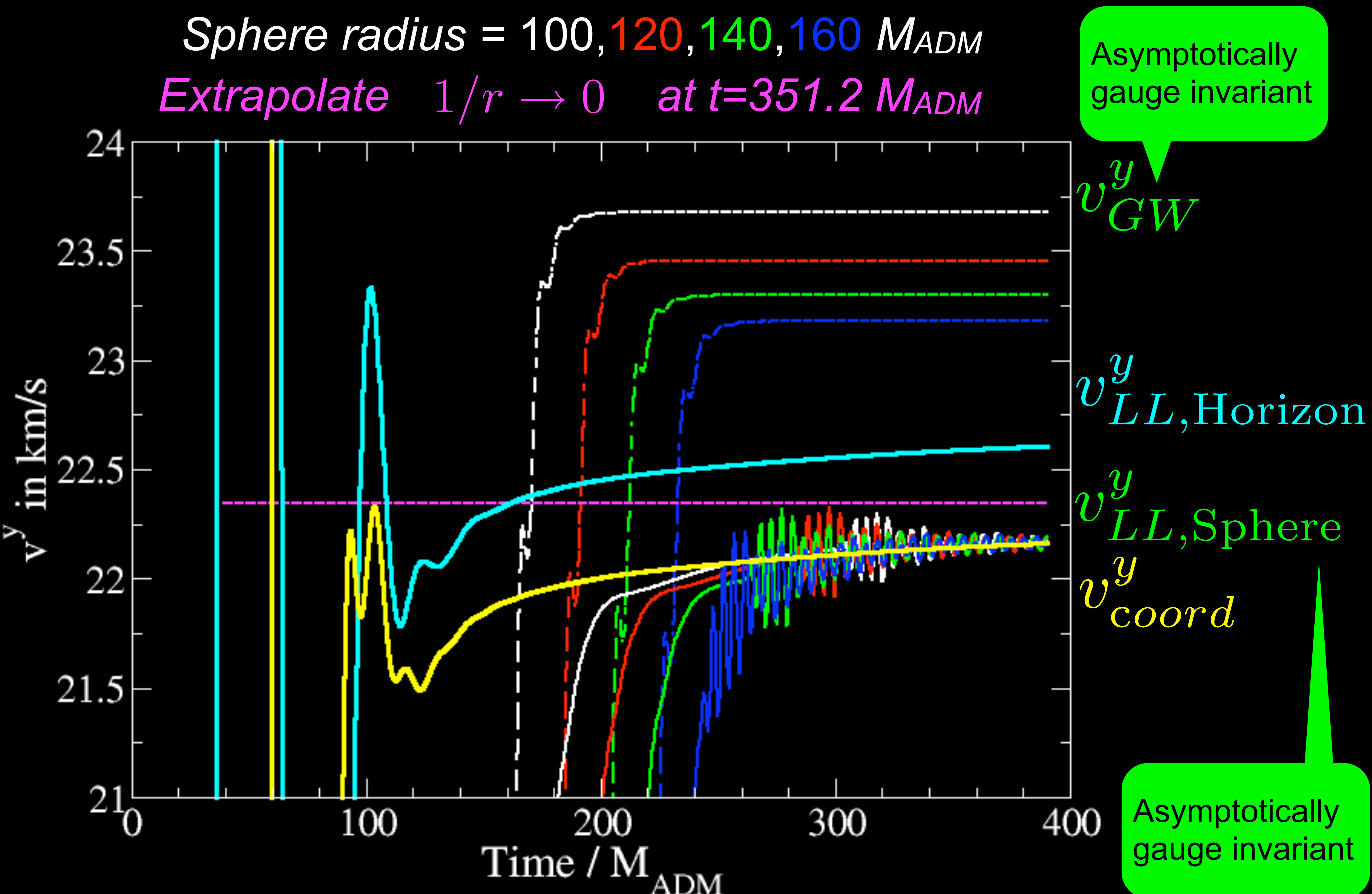


Individual apparent horizons
Common apparent horizon
v^y_{LL} in km/s
Time / M_{ADM}
Individual apparent horizons
Common apparent horizon
v^y_{LL} in km/s
Time / M_{ADM}
Area of common apparent horizon
Total area of individual apparent horizons
Area / Area(t=0)
Time / M_{ADM}
Apparent horizon intrinsic scalar curvature
Solid = binary black hole (pointwise norm); Dashed = Kerr, same spin
M^2 [min(R) or max(R)]
Time / M_{ADM}

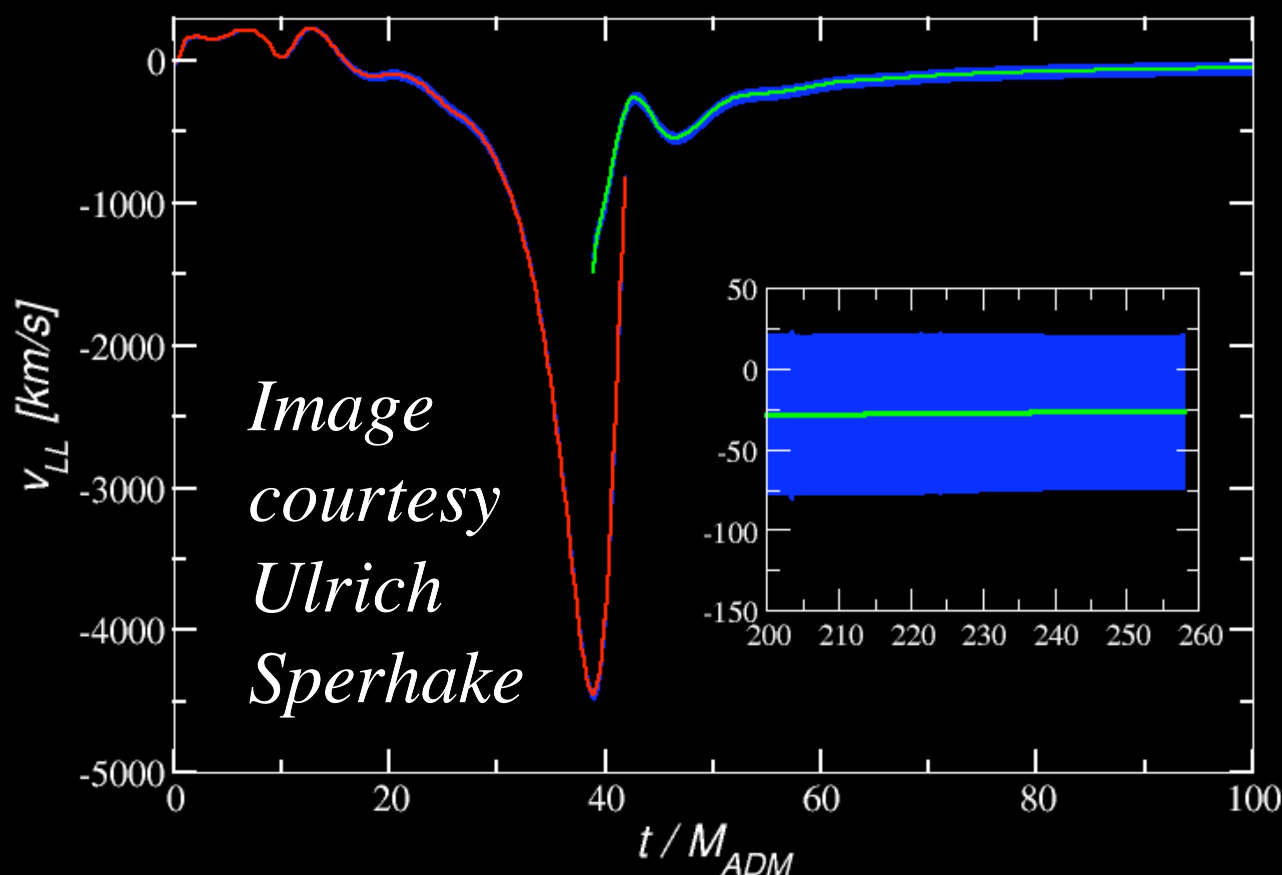
Final recoil velocity: near zone vs. wave zone

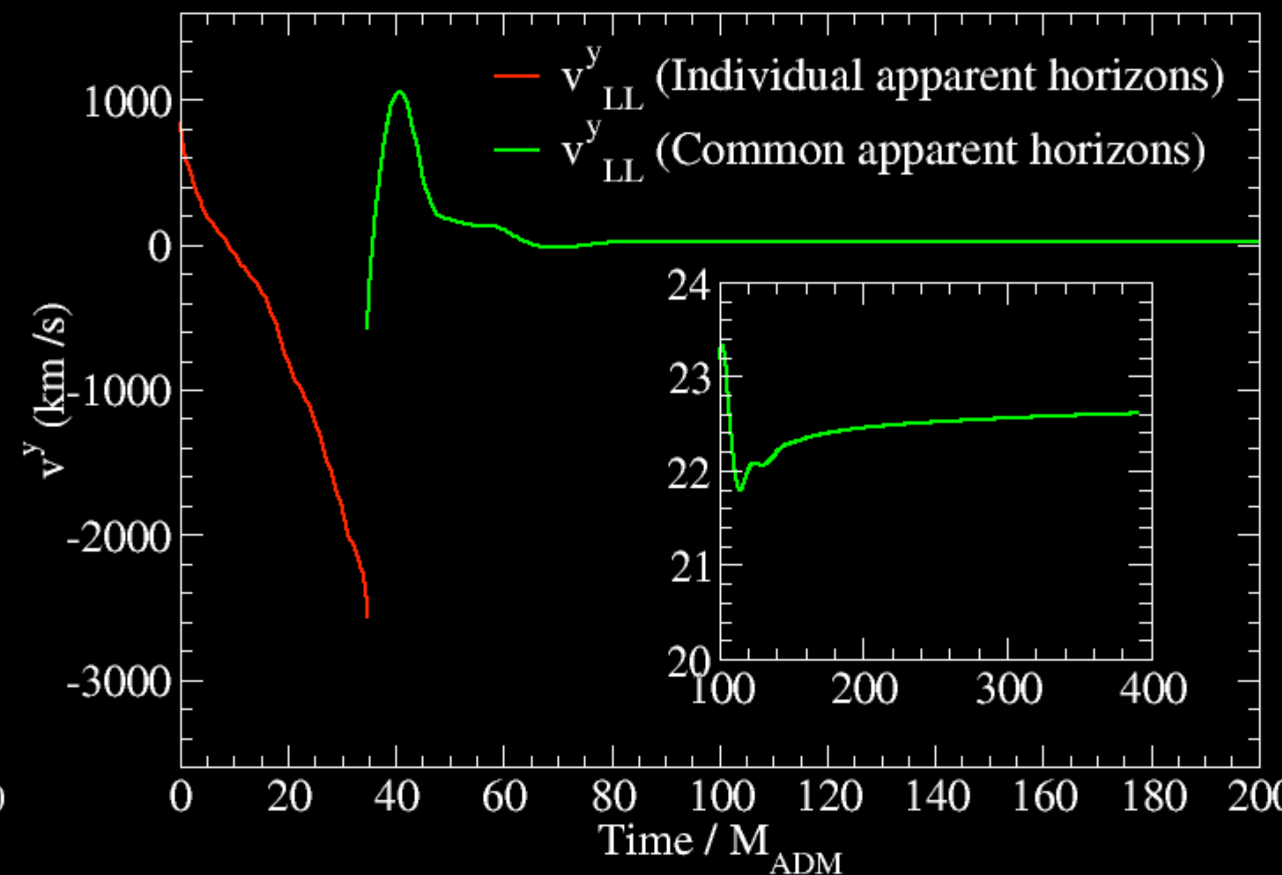
Sphere radius = 100,120,140,160 M_{ADM}

Extrapolate $1/r \to 0$ *at t=351.2* M_{ADM}

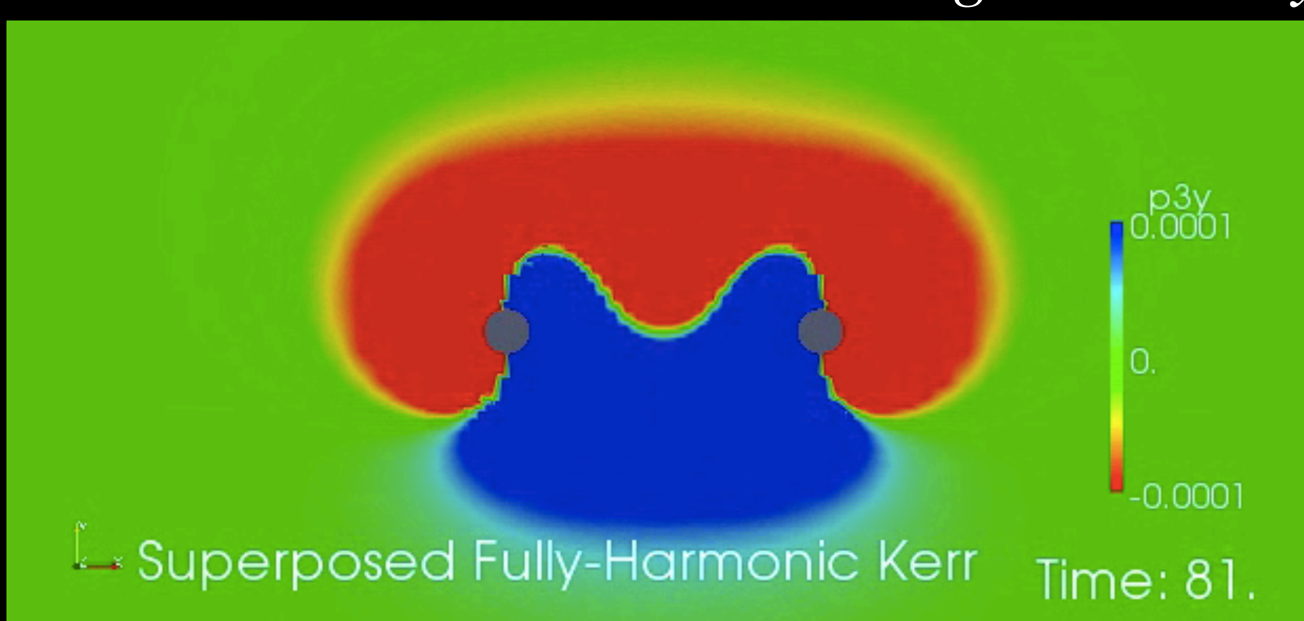


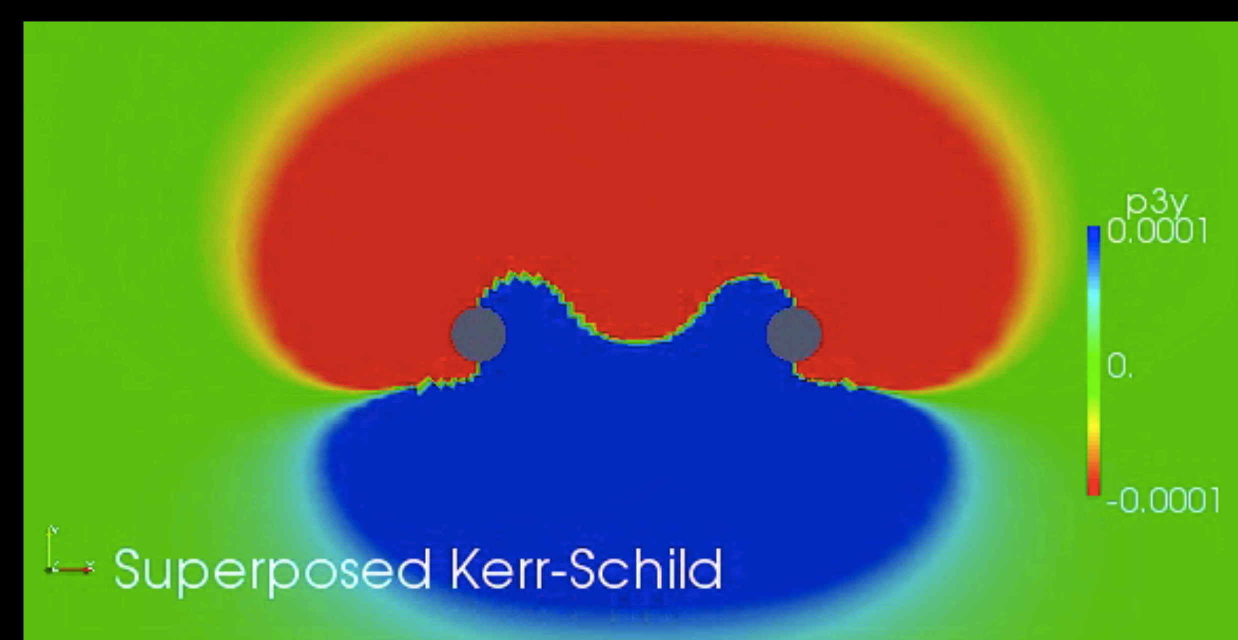
Compare mergers in different gauges





Images courtesy Keith Matthews





Compare plunge with Post-Newtonian

$(-g)\ t^{0y}{}_{LL}$ in xy plane

Post-Newtonian | **Numerical Relativity**

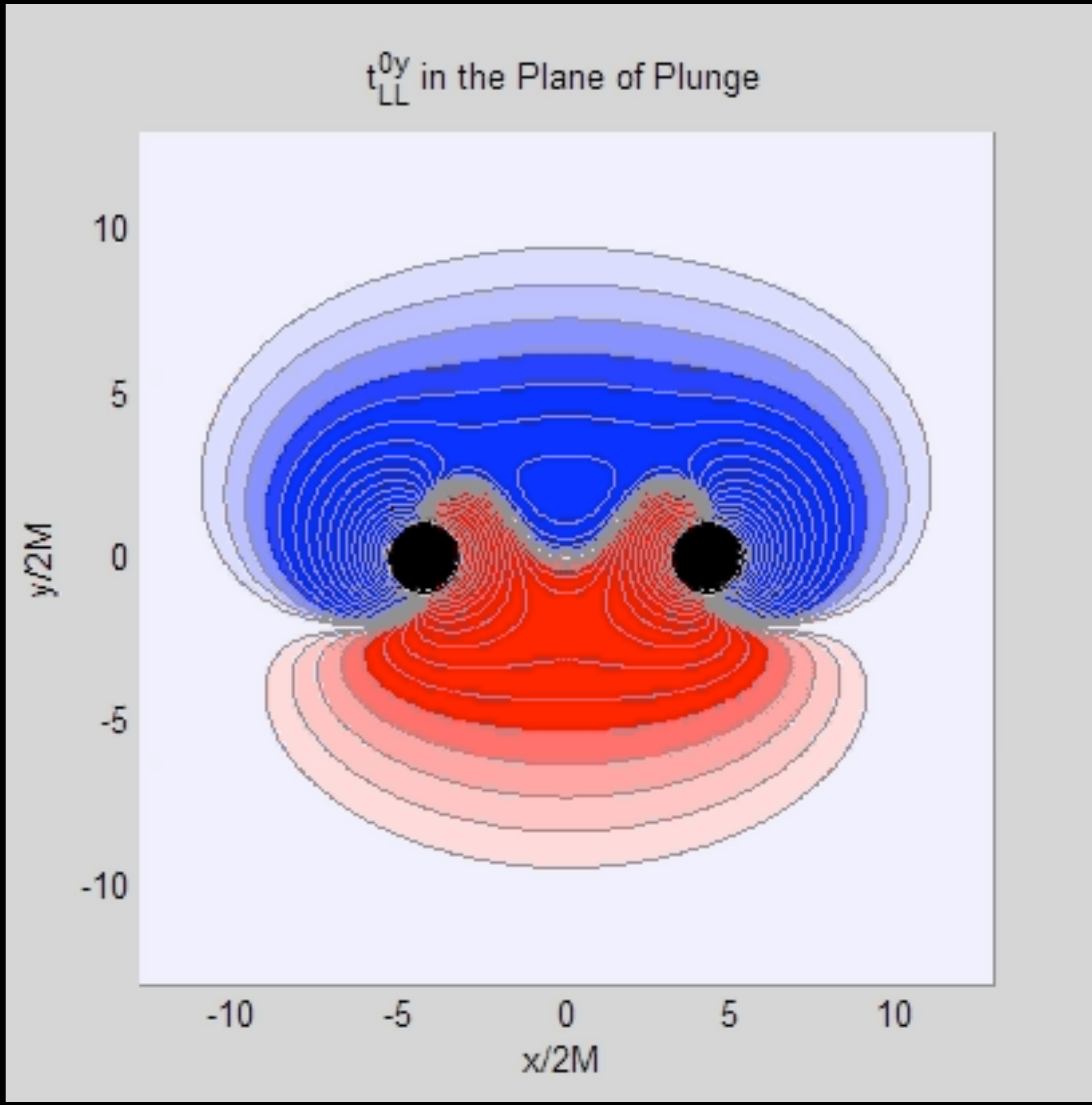
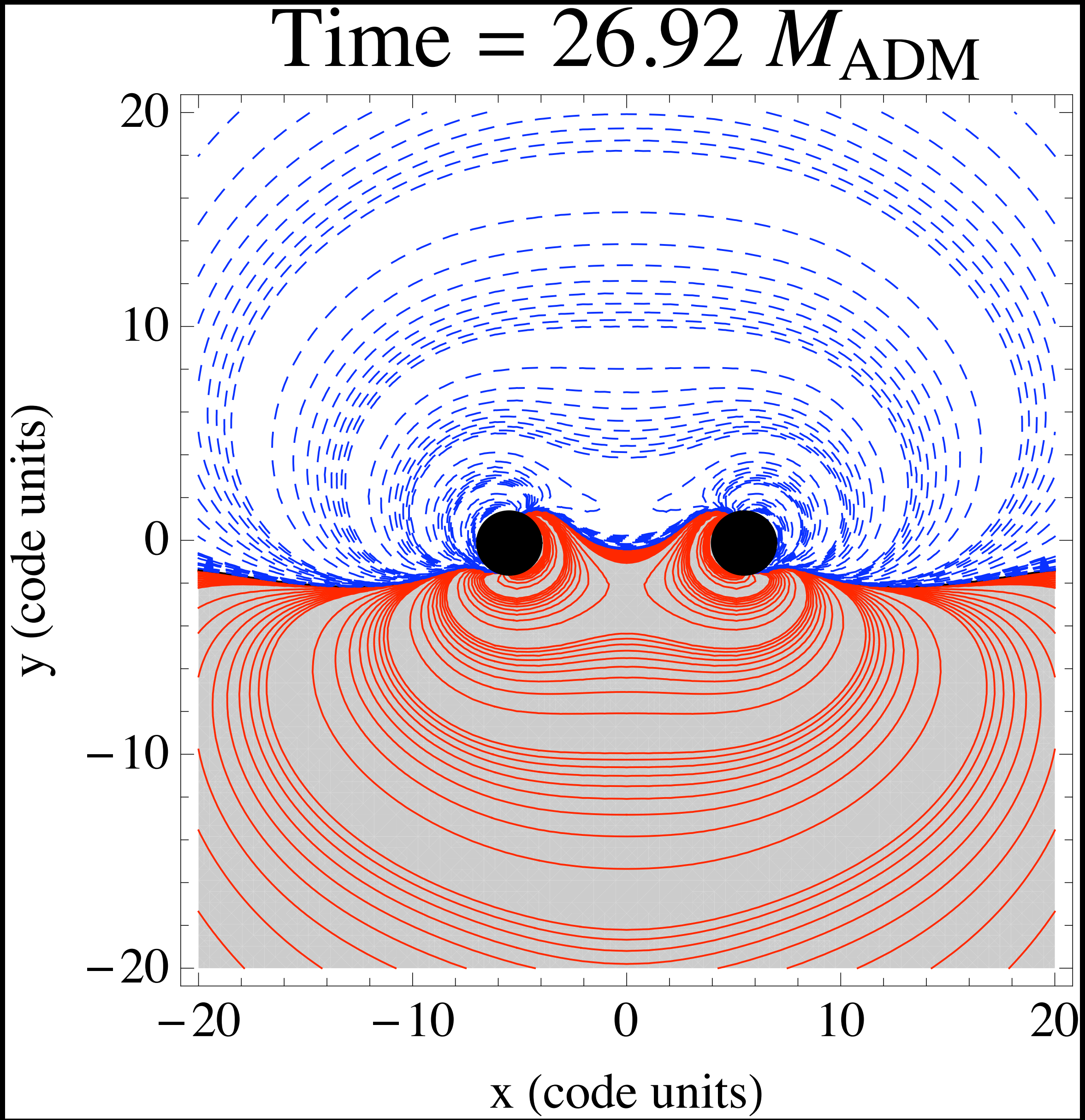


Image courtesy David Nichols



Conclusion

- Summary
  - Internal dynamics of merging binary black holes can be understood in terms of momentum flow
  - Various measures of final kick velocities agree well
  - Preliminary: gauge dependence not too severe
- Future work
  - Better understand gauge dependence of Landau-Lifshitz results
  - Quantitative comparison with post-Newtonian prediction
  - Extremal kicks (i.e. inspiral, not head-on)
  - Higher spins