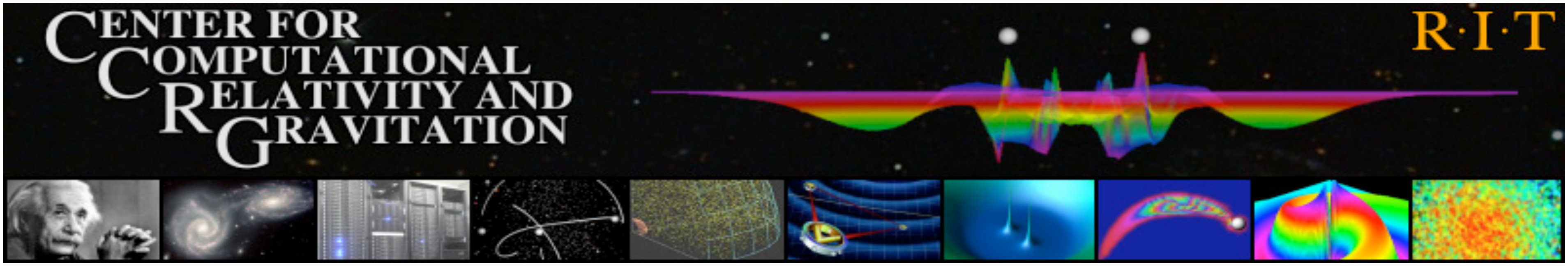


# Binary Black Hole Mergers and Gravitational Recoils

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R·I·T

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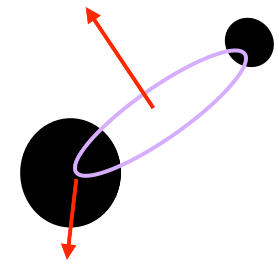
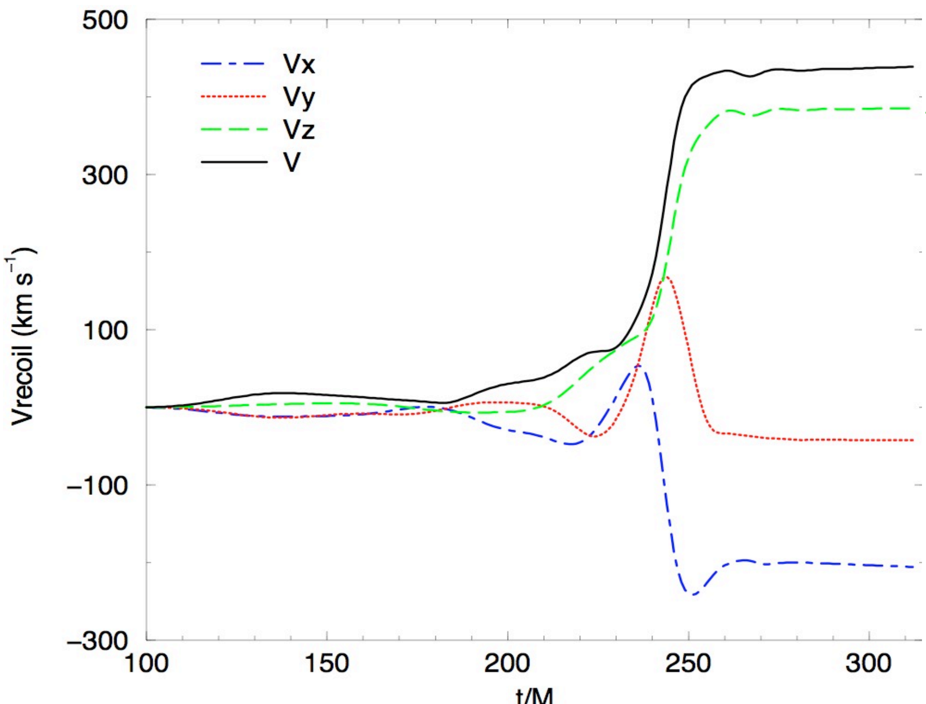
# Overview

1. **Review of the Discovery of large recoils**
2. **Modeling of recoils (news)**
3. **Non-leading corrections**
4. **Final mass and spin of the remnant**

# Large merger recoils from precessing quasi-circular binaries

Generic binary displaying significant precession of spin axis is observed to produce a large recoil kick at merger [Campanelli et al, APJ Lett 2007]


SP6: q = 0.5, $a_1$= 0.885, $a_2$ = 0




*"... the spin component to the recoil velocity may produce the leading contribution. This is suggested by the fact that the z-component of the recoil, which is not present for non-spinning binaries, is the dominant component ..."*

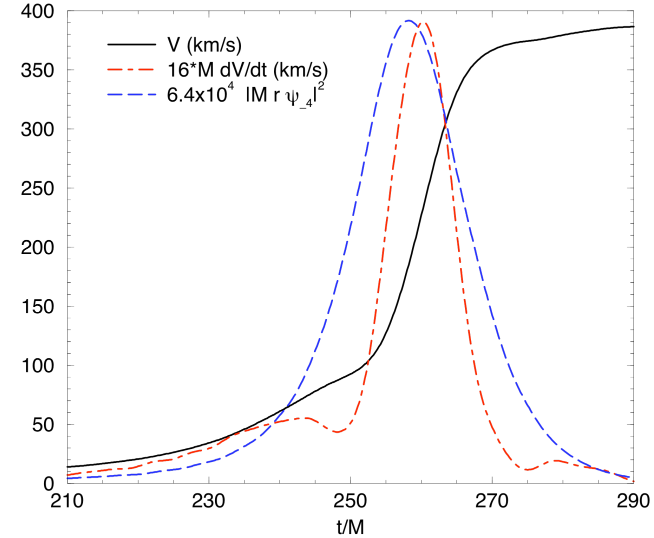
## Maximum kick configuration: equal-mass circular binaries with opposite in-plane spins produce large out-of-plane kicks

- Following:

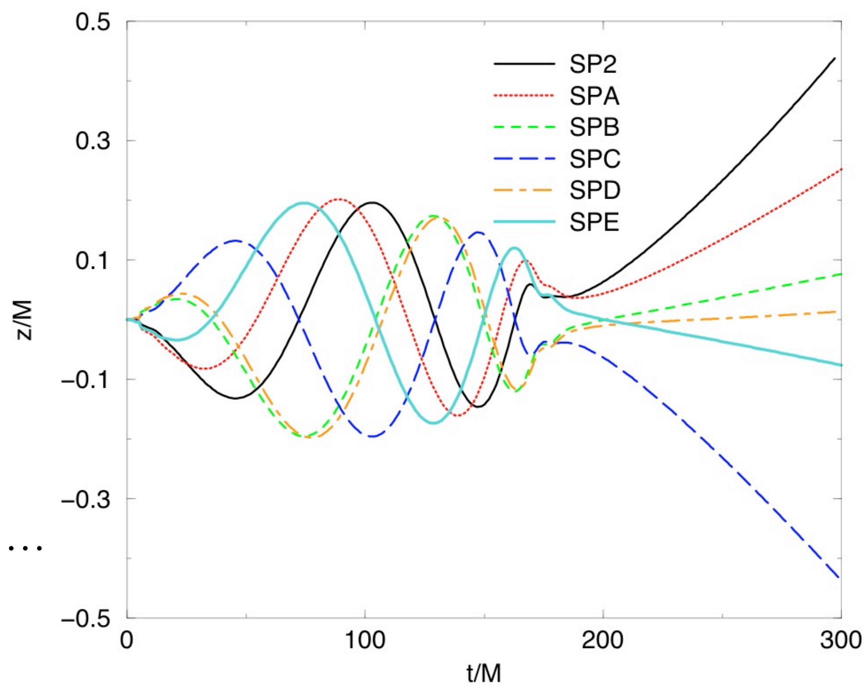
  [Gonzalez et al, Phys. Rev. Lett, 2007] calculate kick of 2500 km/s
  [Campanelli et al, Phys. Rev. Lett, 2007] predicts kicks up to 4000 km/s
  [Dain, et al, Phys. Rev. D 2008] calculate 3300 km/s for nearly maximal spins

# Understanding the merger recoil

- As the binary merges, asymmetrical radiation leads to an oscillation in the momentum of the center-of-mass.
- Since inspiral motion is quasi-periodic, this effect cancels out almost completely, except after merger, when the total recoil built-up …
- In the superkick configurations (equal-mass, equal-and-opposite spins in the orbital plane), the center-of-mass oscillates upward/downward alternatively (“bobbing”) [Campanelli et al, 2007]
- The merged hole gets kicked up or down with a final speed that depends on the velocity of the black holes at merger (relative to spins)
- The maximum bobbing amplitude $\delta z/M$ agrees very well with 1.5PN calculations [Keppel, Nichols, Chen and Thorne, arXiv:0902.4077]
- In the more general case, this bobbing effect is superimposed by a precession of the orbital plane itself …




The recoil speed, the time derivative of the recoil speed and the magnitude of ψ4 for the SP6 configuration.



# An empirical Formula for the merger kick

Empirical formula [Campanelli et al '07] for the radiation recoil of generic binary black-hole mergers originally motivated by PN formula [Kidder 1995]

$$\vec{V}_{\rm recoil}(q, \vec{a}_i) = v_m\,\hat{e}_1 + v_\perp(\cos(\xi)\,\hat{e}_1 + \sin(\xi)\,\hat{e}_2) + v_\parallel\,\hat{e}_z,$$

$$q = m_1/m_2, \quad \eta = q/(1+q)^2, \quad \vec{a}_i = \vec{S}_i/m_i^2$$

$$v_m = A\eta^2\sqrt{1-4\eta}(1+B\eta)$$

in-plane kick < 175 km/s [Fitchett '83, Gonzalez et al 07]

$$v_\perp = H\frac{\eta^2}{(1+q)}\left(a_2^\parallel - q a_1^\parallel\right)$$

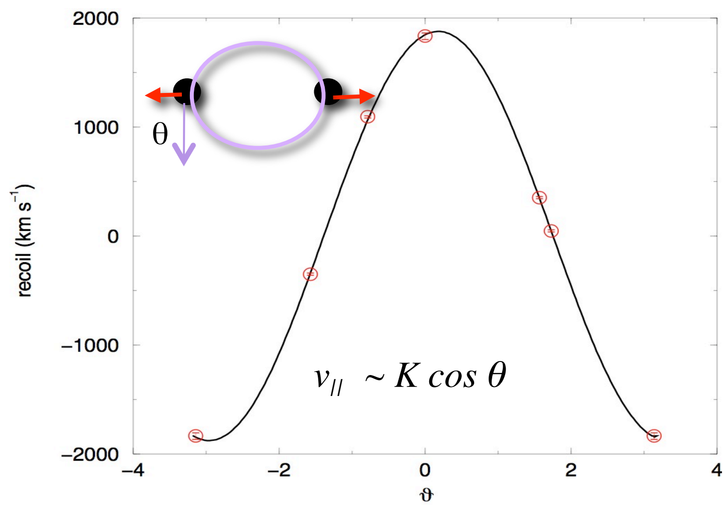
in-plane kick < 500 km/s [Baker et al 07, Hermann et al '07, Koppitz et al '07]. See [Pollney et al 2007] for quadratic corrections in the spins. Also, work in progress by RIT …

$$v_\parallel = K\frac{\eta^2}{(1+q)}\cos(\Theta - \Theta_0)\left|\vec{a}_2^\perp - q\vec{a}_1^\perp\right|$$

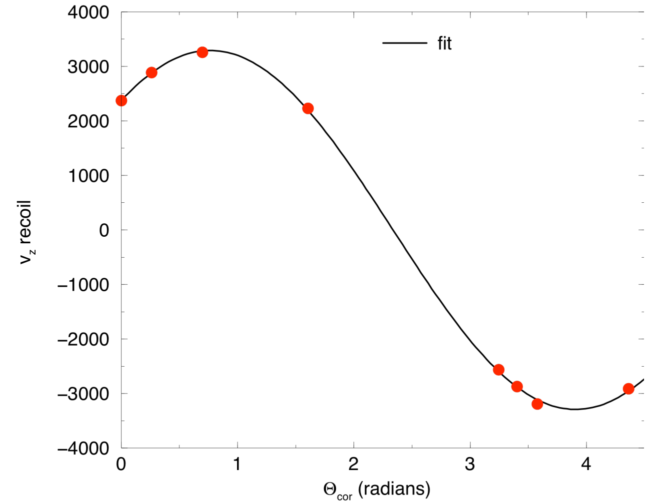
out-of-plane kick < 4,000 km/s [Campanelli et al 07, Lousto et al '08]. See also [Baker et al '08] for a modification of mass scaling ($\eta^2 \rightarrow 4\eta^3$) …

- $\xi$ angle between unequal-mass and spin contributions to recoil in the orbital plane
- $\Theta$ angle between in-plane $\vec{\Delta} \equiv (m_1 + m_2)(\vec{S}_2/m_2 - \vec{S}_1/m_1)$ and infall direction at merger

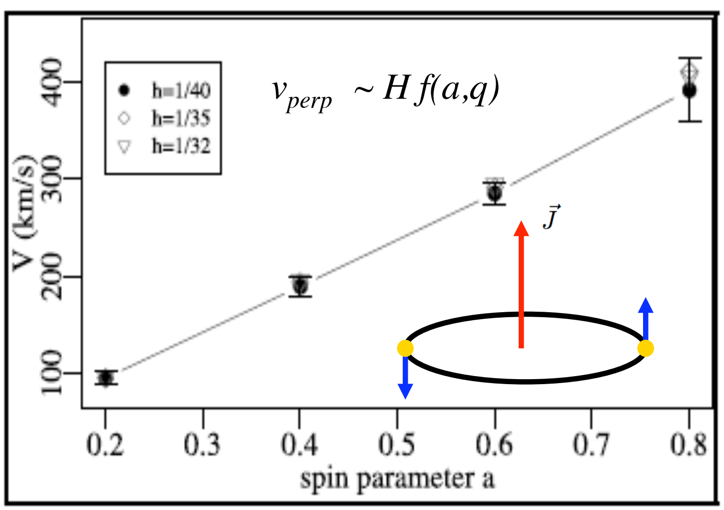
# Tests of the empirical recoil formula …



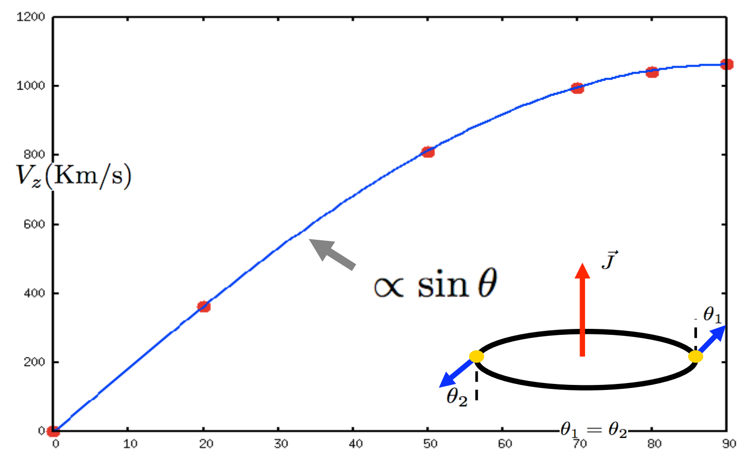

Test of the K cos θ dependence of the out-of plane component of the kick [Campanelli, Lousto & Zlochower, Phys. Rev. Lett 98: 231102, 2007.




"Extra-large" recoils (~3300km/s) from highly spinning binaries (a/M=0.92) [Lousto & Zlochower, Phys. Rev.D77: 044028, 2008; Dain, Lousto & Zlochower, Phys. Rev. D 78: 024039, 2008]




Tests of the in-plane kick for non-precessing binaries [Hermann et al'07]




Testing the decomposition of the spin related components of the recoil [Hermann et al , Phys Rev D 76, 084032, 2007]

# How does the kick depend on the mass ratio?

In a follow-up paper [Lousto & Zlochower '08] made further studies of the mass scaling:

- 45 simulations of precessing binaries with high kicks (~ 1000 km/s) and mass ratios in the range {1 - 1/8}.
  - Extreme-mass-ratio limit: Small non-spinning colliding highly spinning SMBH …

- Introduced new techniques for determining orbital plane and measure accurately spin near merger.

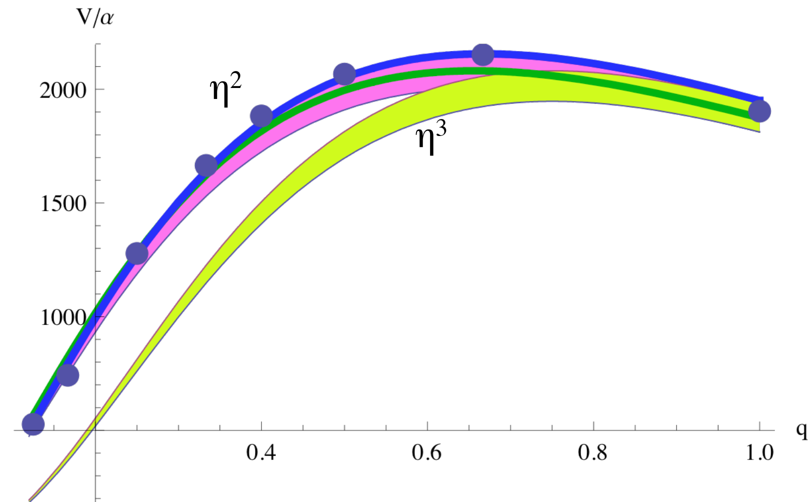
- Fits of the mass ratio dependence indicates a $\eta^2$ scaling
  - The errors in the fits (orbital plane, extraction, initial kick) are found to be insignificant …

- Recent calculations with 2PN [Racine, Buonanno & Kidder, 2008] and perturbation theory (RIT's work in progress) also indicates $\eta^2$ scaling …

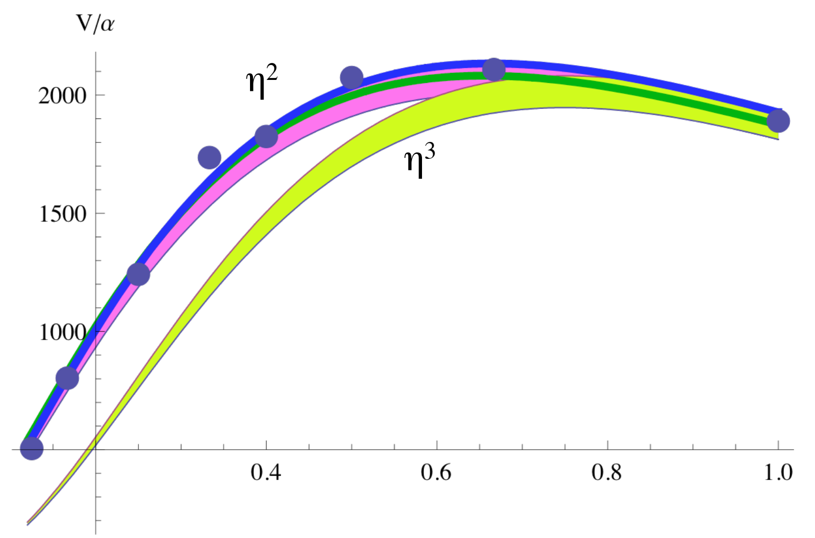
- Find that the in-plane kick is larger than expected, due to precession …

- Higher-order (non-linear) corrections may be needed in the recoil formula …

- The magnitude of the recoil is accurate.




Fits for two orbital plane determination methods



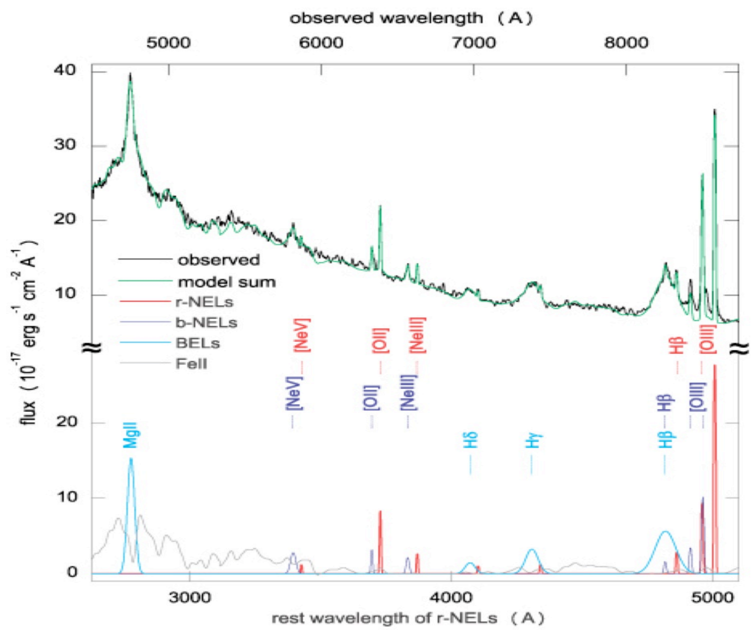

Lousto & Zlochower, Phys. Rev. D79: 064018, 2009; arXiv: 0805.0159

# Observations?

- Numerical (NR) works and generate fairly accurate predictions for kicks. The empirical formula gives an accurate estimate, at least for large kicks …

- The calculation of kicks is important for retention rates in globular clusters and galaxies …

- Can we observe them?
  - Off-set galactic nuclei, displaced active galactic nuclei, population of galaxies without SMBHs, x-rays afterglows, feedback trails, etc.
  - One possible candidate: SDSS J0927 [Komossa et al 2008];




- NR/MHD simulations could help us to find key features in the electromagnetic signals associated with recoiling black holes (and black hole mergers).

# Nonleading terms

RIT group (arXiv:0904.3541)

$$
\begin{aligned}
\vec{V}_{\rm recoil}(q,\vec\alpha) &= v_m\,\hat e_1 + v_\perp(\cos\xi\,\hat e_1 + \sin\xi\,\hat e_2) + v_\|\,\hat e_z,\\
v_m &= A\frac{\eta^2(1-q)}{(1+q)}\left[1+B\,\eta\right],\\
v_\perp &= H_\Delta\frac{\eta^2}{(1+q)}\left[(1+B_H\,\eta)\,(\alpha_2^\| - q\alpha_1^\|) + H_S\,\frac{(1-q)}{(1+q)^2}\,(\alpha_2^\| + q^2\alpha_1^\|)\right],\\
v_\| &= K_\Delta\frac{\eta^2}{(1+q)}\Big[\,(1+B_K\,\eta)\,\left|\alpha_2^\perp - q\alpha_1^\perp\right|\cos(\Theta_\Delta-\Theta_0)\\
&\quad + K_S\,\frac{(1-q)}{(1+q)^2}\,\left|\alpha_2^\perp + q^2\alpha_1^\perp\right|\cos(\Theta_S-\Theta_1)\Big],
\end{aligned}
$$

where $\eta = q/(1+q)^2$, with $q = m_1/m_2$

Based on 2PN corrections (Racine et al ’09)

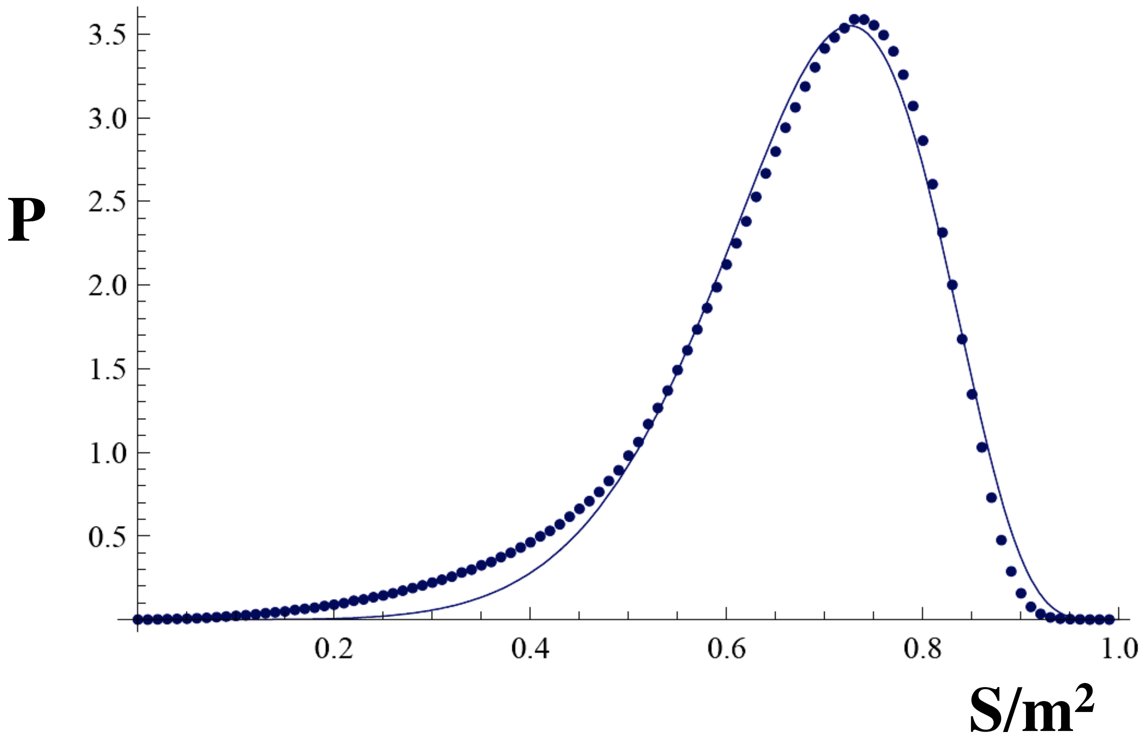
Similarly we propose a
PN-inspired parametrization
Of the *total* energy radiated

$$\begin{aligned}
\delta M/M &= \eta\,\tilde{E}_{ISCO} + E_2\eta^2 + E_3\eta^3 + \\
&\frac{\eta^2}{(1+q)^2}\Big\{ E_S\,(\alpha_2^{\|} + q^2\,\alpha_1^{\|}) + E_\Delta\,(1-q)\,(\alpha_2^{\|} - q\,\alpha_1^{\|}) + \\
&E_A\,|\vec{\alpha}_2 + q\vec{\alpha}_1|^2 + \\
&E_B\,|\alpha_2^{\perp} + q\alpha_1^{\perp}|^2\,(\cos^2(\Theta_+ - \Theta_2) + E_C) + \\
&E_D\,|\vec{\alpha}_2 - q\vec{\alpha}_1|^2 + \\
&E_E\,|\alpha_2^{\perp} - q\alpha_1^{\perp}|^2\,(\cos^2(\Theta_- - \Theta_3) + E_F)\Big\},
\end{aligned} \tag{2}$$

And the final spin of the
Remnant Kerr Black Hole

$$\begin{aligned}
\vec{\alpha}_{\rm final} &= (1-\delta M/M)^{-2}\Big\{\eta\,\tilde{\vec{J}}_{ISCO} + (J_2\eta^2 + J_3\eta^3)\,\hat{n}_{||} + \\
&\frac{\eta^2}{(1+q)^2}\Big(\Big[J_A\,(\alpha_2^{\|} + q^2\,\alpha_1^{\|}) + J_B\,(1-q)\,(\alpha_2^{\|} - q\,\alpha_1^{\|})\Big]\,\hat{n}_{||} + \\
&(1-q)\,|\vec{\alpha}_2^{\perp} - q\,\vec{\alpha}_1^{\perp}|\sqrt{J_\Delta\cos[2(\Theta_\Delta - \Theta_4)] + J_{M\Delta}}\,\hat{n}_\perp + \\
&|\vec{\alpha}_2^{\perp} + q^2\,\vec{\alpha}_1^{\perp}|\,\sqrt{J_S\cos[2(\Theta_S - \Theta_5)] + J_{MS}}\,\hat{n}_\perp\Big)\Big\}.
\end{aligned} \tag{4}$$

# Probability distribution of spin magnitudes




Random q, $\mathbf{S}_1$ and $\mathbf{S}_2$

(10 million magnitudes and directions over the sphere.)

**Maximum at S ~ 0.73. Width ~ +0.1 to -0.2**

**Fit to distribution:**

$$f(x;a,b) = abx^{a-1}(1-x^a)^{b-1}, \qquad a = 6.59 \pm 0.08,\ b = 7.18 \pm 0.19.$$

# Discussion

- Energy radiated 3-10 % of total mass (brightest event in Universe!)
- Final spins submaximal (<0.96) respect the cosmic censorship hypothesis
- Astrophysical application of formulae for recoils, masses, and spins
  - N-body simulations including mergers
  - IMBH growth (Globular clusters?)
  - SMBH collisions, merger trees
  - Cosmological growth trees of BHS