Algebraic Classification of Numerical Spacetimes

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Outline

Motivation

Numerical Relativity can be used to answer some important open questions in GR. RIT has been very interested in several of these:

- Cosmic Censorship: Can BHB produce naked singularities (Campanelli et al. PRD 74, 041501 (2006))
- Gravitational Recoil: How large can the recoil be? (Campanelli et al. PRL 98, 23110 (2007))
- No-Hair theorem: Is the final state of a BHB merger Kerr? (Campanelli et al. PRD 79, 084012 (2009))
  - Determine if spacetime has same algebraic type as Kerr
  - Now much simpler to show spacetime is Kerr via invariants

Petrov Types

Principle Null Directions k^μ

$$k^\nu k^\rho k_{[\tau} C_{\mu]\nu\rho[\sigma} k_{\chi]} = 0. \tag{1}$$

- 4-distinct null vectors $\rightarrow$ Type I
- 3-distinct null vectors (1 pair + 1 + 1) $\rightarrow$ Type II
- 2-distinct null vectors (1 triplet + 1) $\rightarrow$ Type III
- 2-distinct null vectors (1 pair + 1 pair) $\rightarrow$ Type D
- 1-distinct null vectors (1-quartet) $\rightarrow$ Type N
- $C_{\mu\nu\rho\sigma} = 0 \rightarrow$ Type O

PNDs and Weyl Scalars

Tetrad $(l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$

- $\psi_0 = C_{\mu\nu\rho\sigma} l^\mu m^\nu l^\rho m^\sigma$
- $\psi_0 = 0 \iff l^\mu$ is a PND
- So finding the choices of l^μ that give $\psi_0 = 0$ is equivalent to finding the PNDs
  - Start in a generic (non-aligned) tetrad where $\psi_4 \neq 0$
  - Solve: $\tilde{\psi}_0 = \psi_0 + 4\lambda\psi_1 + 6\lambda^2\psi_2 + 4\lambda^3\psi_3 + \lambda^4\psi_4 = 0$
  - Multiplicity of the roots gives the algebraic classification.
  - If 1 root is repeated then we can also have $\psi_1 = 0$
  - If root has multiplicity 2 (3) then $\psi_2 = 0$ (and $\psi_3 = 0$)
  - For Type D, we can have $\psi_i = 0$ $i \neq 2$

Algebraically Special Spacetimes

A spacetime is algebraically special if two or more PNDs are aligned

- At least one root of the quartic equations has multiplicity > 1
- $\mathrm{I}^3 = 27J^2$ $(S = 27J^2/\mathrm{I}^3 = 1)$
  - $$\mathrm{I} = \frac{1}{2}\tilde{C}_{\alpha\beta\gamma\delta}\tilde{C}^{\alpha\beta\gamma\delta} = 3{\psi_2}^2 - 4\psi_1\psi_3 + \psi_4\psi_0$$
  - $$J = -\frac{1}{6}\tilde{C}_{\alpha\beta\gamma\delta}\tilde{C^{\gamma\delta}}_{\mu\nu}\tilde{C}^{\mu\nu\alpha\beta} = \psi_2^3 + \psi_0\psi_4\psi_2 + 2\psi_1\psi_3\psi_2 - \psi_4\psi_1^2 - \psi_0\psi_3^2 \quad (2)$$

Finding the Roots

$$
\begin{aligned}
K &= \psi_1\psi_4^2 - 3\psi_4\psi_3\psi_2 + 2\psi_3^3, && (3)\\
L &= \psi_2\psi_4 - \psi_3^2, && (4)\\
N &= \psi_4^2 I - 3L^2 \\
&= {\psi_4}^3\psi_0 - 4\,{\psi_4}^2\psi_1\psi_3 + 6\,\psi_4\psi_2{\psi_3}^2 - 3\,{\psi_3}^4 && (5)
\end{aligned}
$$

$$
\begin{aligned}
D &= J^2 - (I/3)^3, \\
A &= (-J + \sqrt{D})^{1/3}, \quad B = (-J - \sqrt{D})^{1/3}, \\
y_1 &= A + B, \\
y_2 &= -\frac{1}{2}(A+B) + i\frac{\sqrt{3}}{2}(A-B), \\
y_3 &= -\frac{1}{2}(A+B) - i\frac{\sqrt{3}}{2}(A-B), && (6)
\end{aligned}
$$

$S = 1 \Longrightarrow y_2 = y_3$

Finding the Roots: Cont

$$\begin{aligned} z_1 &= 2\psi_4\, y_1 - 4L, \\ z_2 &= 2\psi_4\, y_2 - 4L, \\ z_3 &= 2\psi_4\, y_3 - 4L. \end{aligned} \tag{7}$$

$$\begin{aligned} \lambda_1 &= \left[-\psi_3 + \frac{1}{2}(\sqrt{z_1} + \sqrt{z_2} + \sqrt{z_3})\right] / \psi_4, \\ \lambda_2 &= \left[-\psi_3 + \frac{1}{2}(\sqrt{z_1} - \sqrt{z_2} - \sqrt{z_3})\right] / \psi_4, \\ \lambda_3 &= \left[-\psi_3 + \frac{1}{2}(-\sqrt{z_1} + \sqrt{z_2} - \sqrt{z_3})\right] / \psi_4, \\ \lambda_4 &= \left[-\psi_3 + \frac{1}{2}(-\sqrt{z_1} - \sqrt{z_2} + \sqrt{z_3})\right] / \psi_4, \end{aligned} \tag{8}$$

where the signs of the $\sqrt{z_i}$ are chosen such that $(\sqrt{z_1}\sqrt{z_2}\sqrt{z_3}) = -4K$.

Generic Type D metric In Vacuum

$$ds^2 = \frac{1}{\Omega^2}\left\{\frac{Q}{\rho^2}\left[dt - \left(a\sin^2\theta + 4\ell\sin^2\frac{\theta}{2}\right)d\phi\right]^2 - \frac{\rho^2}{Q}dr^2 \right.$$
$$\left. -\frac{P}{\rho^2}\left[adt - \left(r^2 + (a+\ell)^2\right)d\phi\right]^2 - \frac{\rho^2}{P}\sin^2\theta\, d\theta^2\right\}, \quad (9)$$

where

$$\Omega = 1 - \alpha(\ell + a\cos\theta)\, r, \quad (10)$$
$$\rho^2 = r^2 + (\ell + a\cos\theta)^2, \quad (11)$$
$$P = \sin^2\theta\,(1 - a_3\cos\theta - a_4\cos^2\theta), \quad (12)$$
$$Q = k - 2mr + \epsilon r^2 - 2\alpha n r^3 - \alpha^2 k r^4, \quad (13)$$
$$a_3 = 2\alpha a m - 4\alpha^2\, a\,\ell\, k, \quad (14)$$
$$a_4 = -\alpha^2 a^2 k \quad (15)$$
$$\epsilon = \frac{k}{a^2 - \ell^2} + 4\alpha\ell m - (a^2 + 3\ell^2)\alpha^2 k, \quad (16)$$
$$n = \frac{k\,\ell}{a^2 - \ell^2} - \alpha(a^2 - \ell^2)\, m + (a^2 - \ell^2)\ell\alpha^2 k, \quad (17)$$
$$\left(\frac{1}{a^2 - \ell^2} + 3\alpha^2\ell^2\right) k = 1 + 2\alpha\ell m. \quad (18)$$

NUT charge and Acceleration: cont

- If acceleration $\alpha \neq 0$ then
  - $\mathrm{I} = 3(m + i\ell)^2 \alpha^6 p^6 + \mathcal{O}(1/r)$) $(p = \ell + a\cos\theta)$
  - Look at asymptotic form of I to check α
- If $\alpha = 0$
  - $\Im(\mathrm{I})/\Re(\mathrm{I}) = \frac{2m\ell}{m^2 - \ell^2} + \mathcal{O}(1/r)$
  - Look at asymptotic form of $\Im(\mathrm{I})/\Re(\mathrm{I})$ to exclude $\ell \neq 0$

Configuration

- "Generic Binary" evolved via PN from $r = 50M$ to $r = 2.3M$.
- Evolved Numerically from with high-resolution and 8th order FD
- Tetrad is not aligned with PND

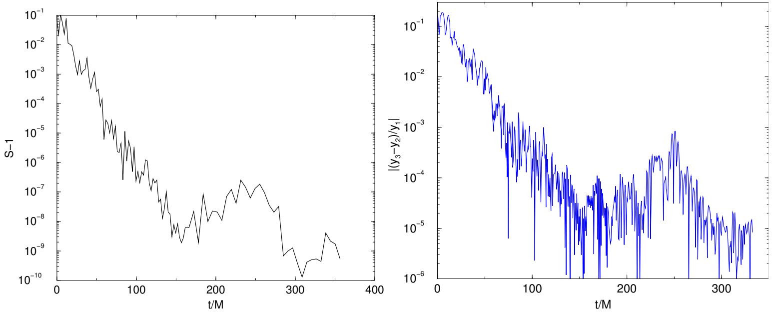
$$l^\mu = (t^\mu + r^\mu)/\sqrt{2}, \quad (19)$$

$$n^\mu = (t^\mu - r^\mu)/\sqrt{2}, \quad (20)$$

$$m^\mu = (\theta^\mu + i\phi^\mu)/\sqrt{2}, \quad (21)$$

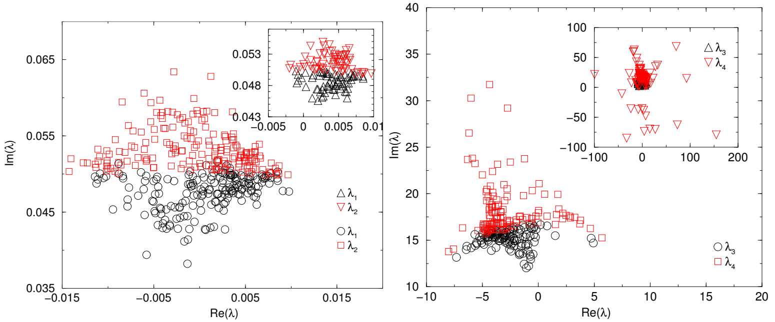
(see Baker et al. PRD 65, 044001 (2002)

Approaching Algebraic Speciality

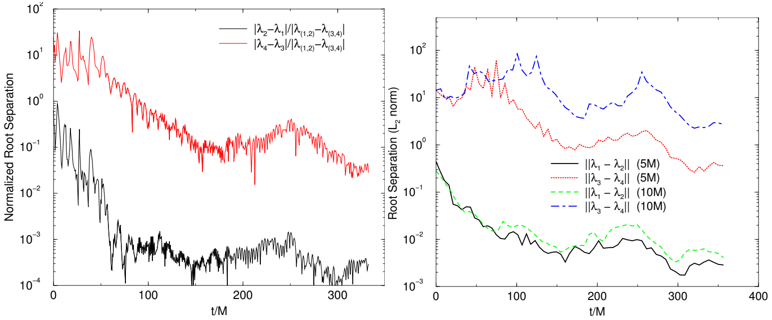


- $S \to 1$ exponentially (e-folding time $\sim 10M$)
- $y_2 \to y_3$ exponentially (e-folding time $\sim 20M$)
- Late-time oscillations due to grid noise

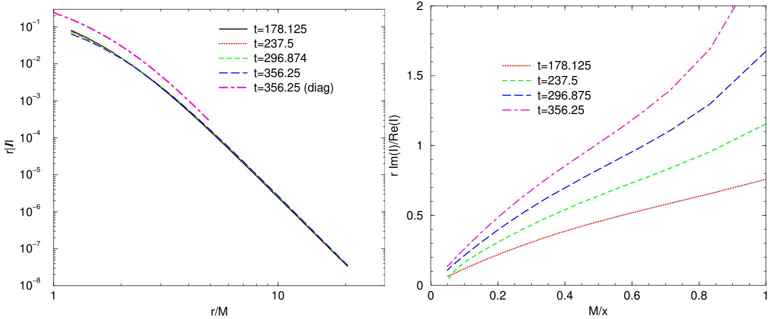
Approach to Type D



Approach to Type D Cont.



Is it Kerr ???



- $\lim_{r\to\infty} r|\mathrm{I}| = 0$ so $\alpha = 0$
- $\lim_{r\to\infty} r\Im(\mathrm{I})/\Re(\mathrm{I}) = 0$ so $l = 0$
- Spacetime $\rightarrow$ Type D with no NUT charge or acceleration (KERR)

Summary

- Spacetime approaches Type II quickly, Type D much more slowly
- Strong evidence that the final state of a generic merger is Kerr
- Also a strong test of the stability of Kerr to highly nonlinear perturbations
- Calculations challenging because ψ's are very small at late-times.
- Calculations can be improved using spectral methods