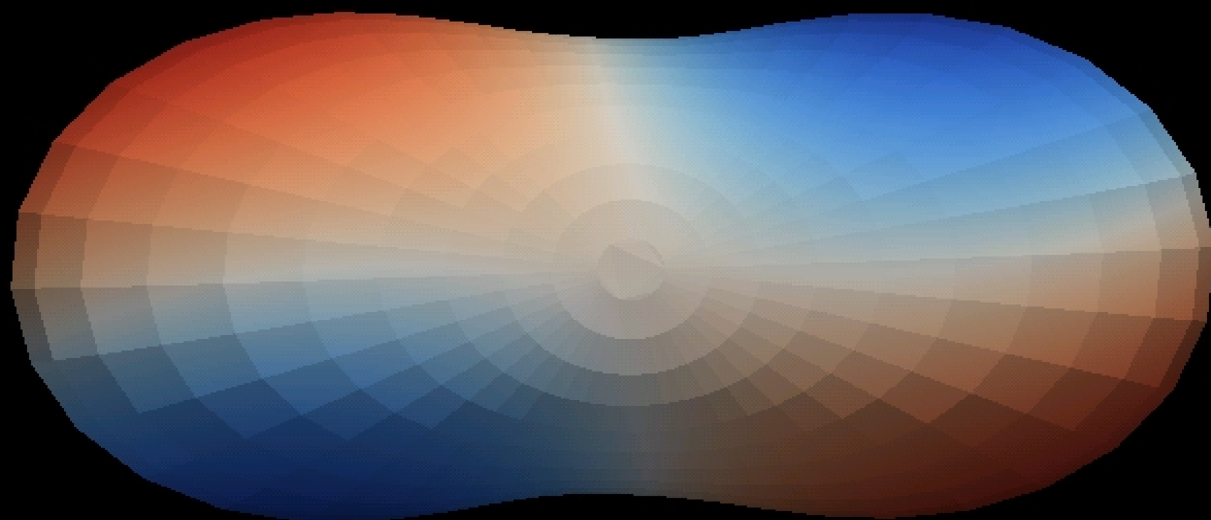
Multipolar analysis of numerical black hole mergers.



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EGM12, RIT

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Numerical Relativity can now merge black holes.

This presents the intriguing possibility of studying, in detail, the strong-field nonlinear behavior of curved spacetime.

Much of this behavior can be expected to be encoded in quasilocal structures on interacting black holes.

But in the strong-field regime, gauge ambiguity is very strong, and must be understood and controlled.

Example: spin angular momentum.

A reasonably well-developed formalism for computing spin angular momentum has appeared over the last few years, beginning with the formula:

$$J = \frac{1}{8\pi} \oint_{\mathcal{H}} \phi^B \omega_B \, dA,$$

where ω_B is a connection on the normal bundle and ϕ^B is a suitably defined "approximate Killing vector" of the apparent horizon $\mathcal{H}$.

Can this be extended to other multipole moments?

Ashtekar et al.:

gr-qc/0401114
CQG **21** 2549 (2004)

It is natural to define *Mass*- and *Current- multipoles* on axisymmetric isolated horizons as:

$$I_\alpha := \oint_{\mathcal{H}} y_\alpha R \, dA,$$
$$L_\alpha := \oint_{\mathcal{H}} y_\alpha^B \omega_B \, dA$$

(up to multiplicative factors), where R is the intrinsic scalar curvature of the two-dimensional surface $\mathcal{H}$, and y_α and y_α^A are scalar and vector spherical harmonics, respectively.

Spherical harmonics:

Ashtekar et al. made use of extra structure that appears in axisymmetry to single out preferred coordinates in which these spherical harmonic projections could be taken.

In general, this extra structure won't exist.

So instead, we don't introduce coordinates. We define the spherical harmonics *spectrally*.

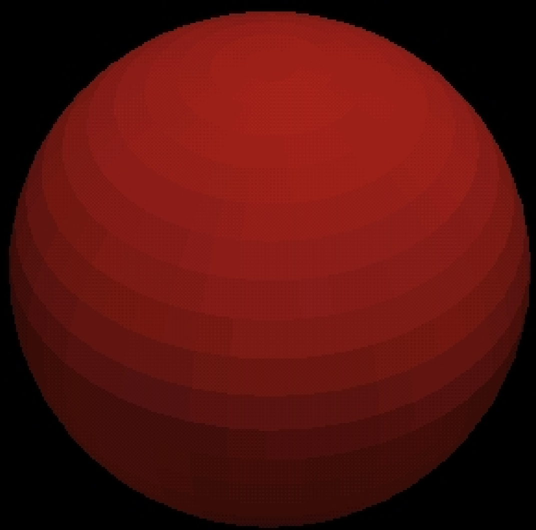
$$
\begin{aligned}
\Delta y_\alpha &= \lambda_{(\alpha)} y_\alpha . \\
\Delta^2 z_\alpha + R \Delta z_\alpha + \nabla^A R \nabla_A z_\alpha &= \mu_{(\alpha)} \Delta z_\alpha ,
\end{aligned}
$$

where

$$
y_\alpha^A := \epsilon^{AB} \nabla_B z_\alpha .
$$

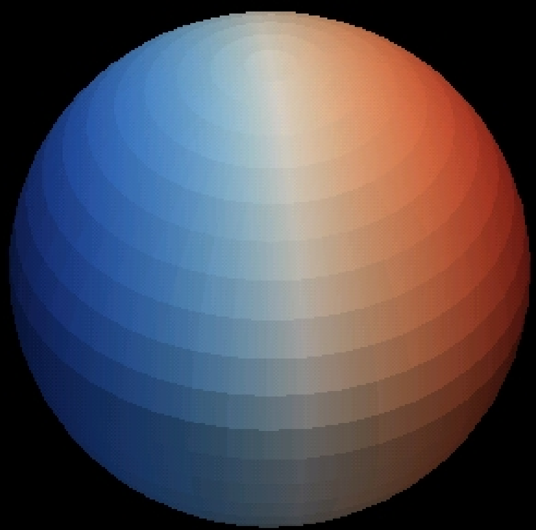
Pictures:

Kerr monopole harmonic:



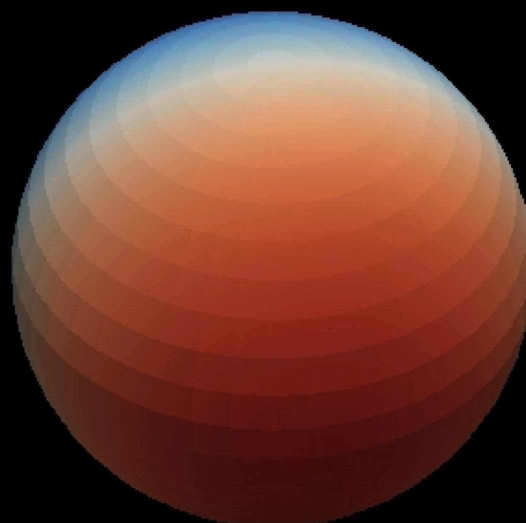
Pictures:

Kerr dipole harmonic 1:



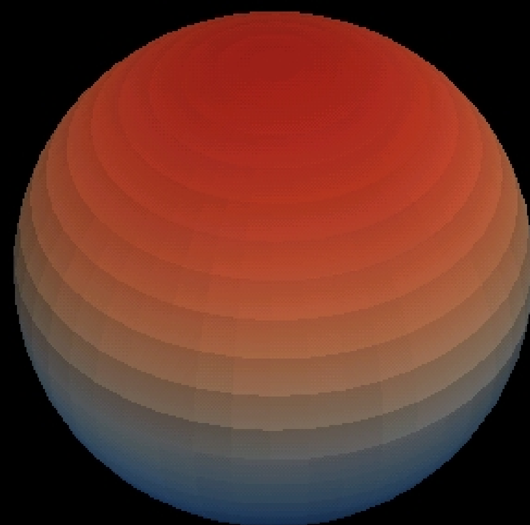
Pictures:

Kerr dipole harmonic 2:



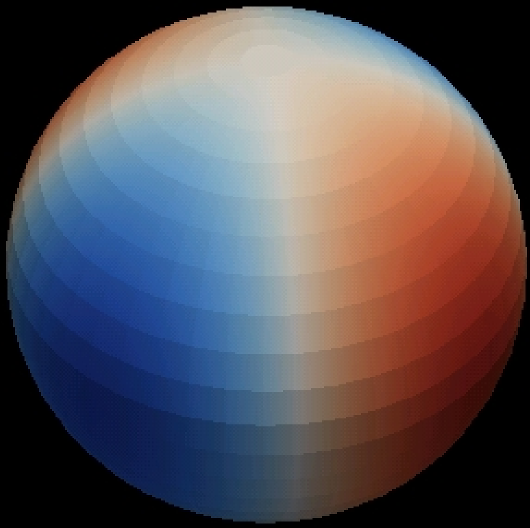
Pictures:

Kerr dipole harmonic 3:



Pictures:

Kerr quadrupole harmonic 1:



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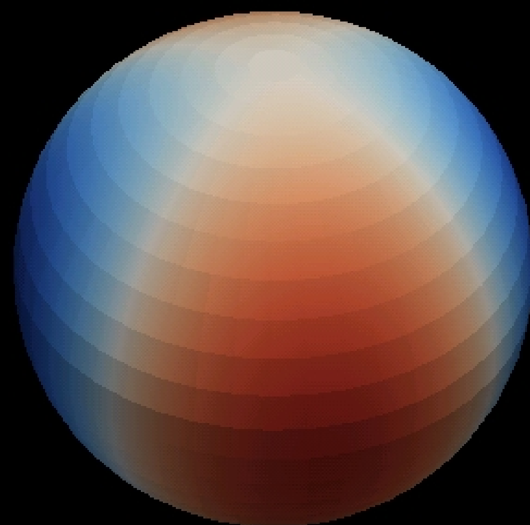
Kerr quadrupole harmonic 2:



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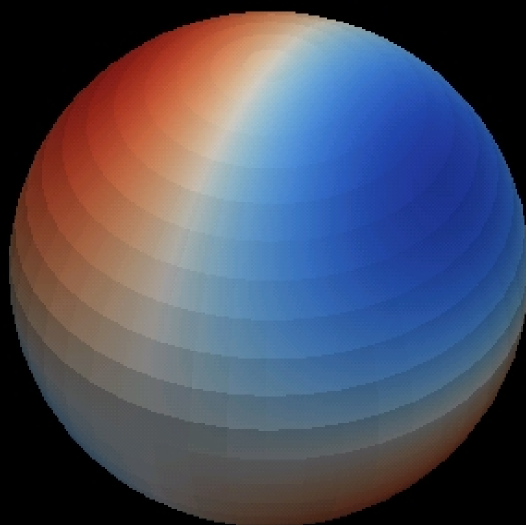
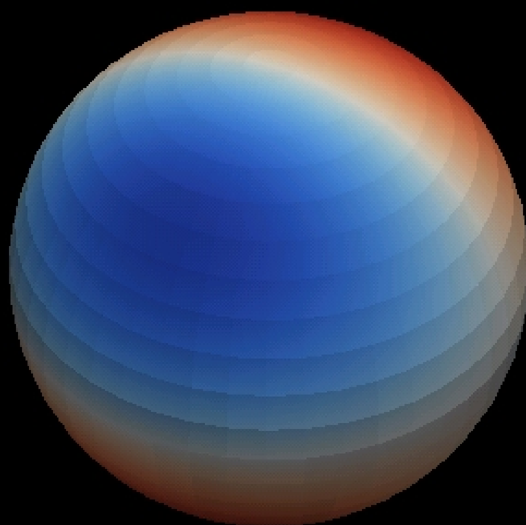


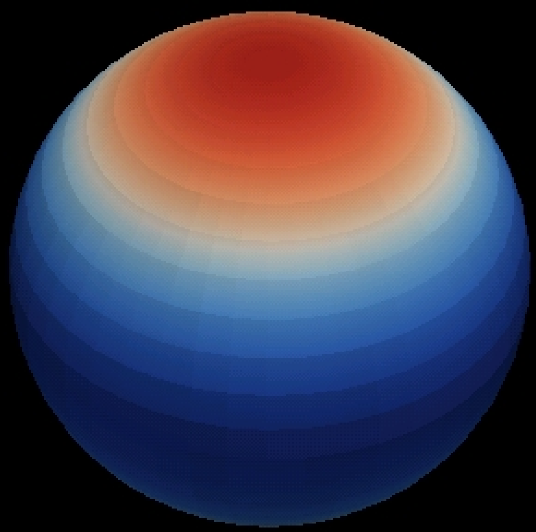
Pictures:

Kerr quadrupole harmonic 4:



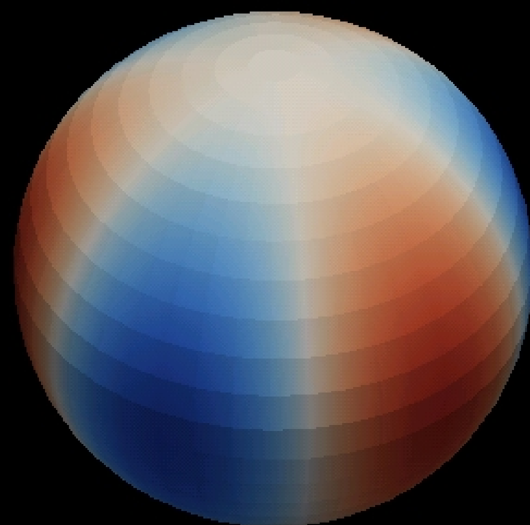
Pictures:

Kerr quadrupole harmonic 5:



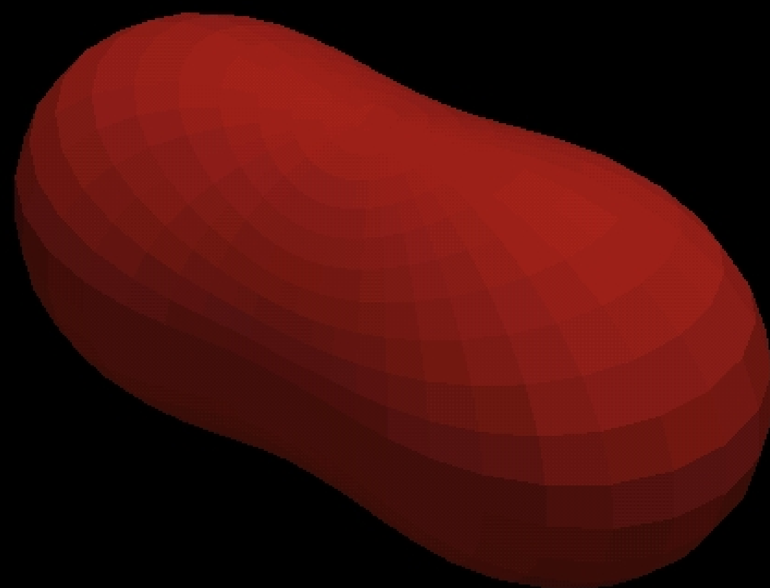
Pictures:

Kerr octupole harmonic 1:



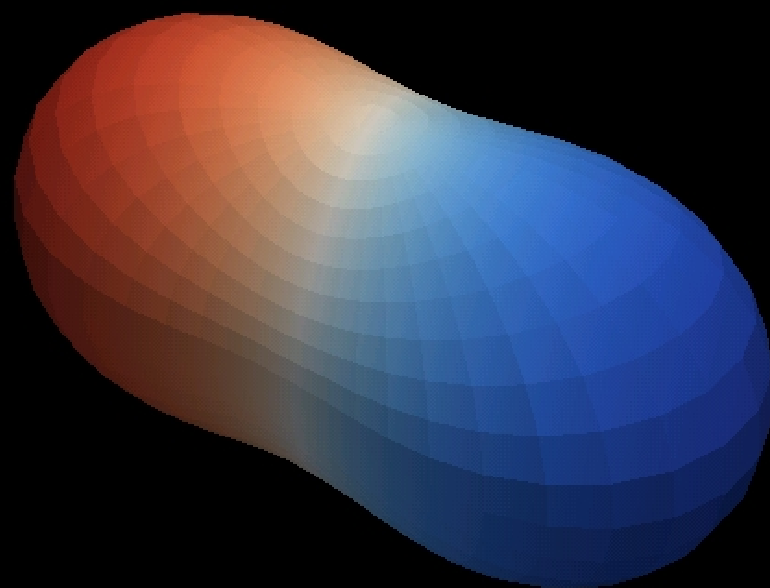
Initial common horizon of a BBH merger:

Monopole harmonic:



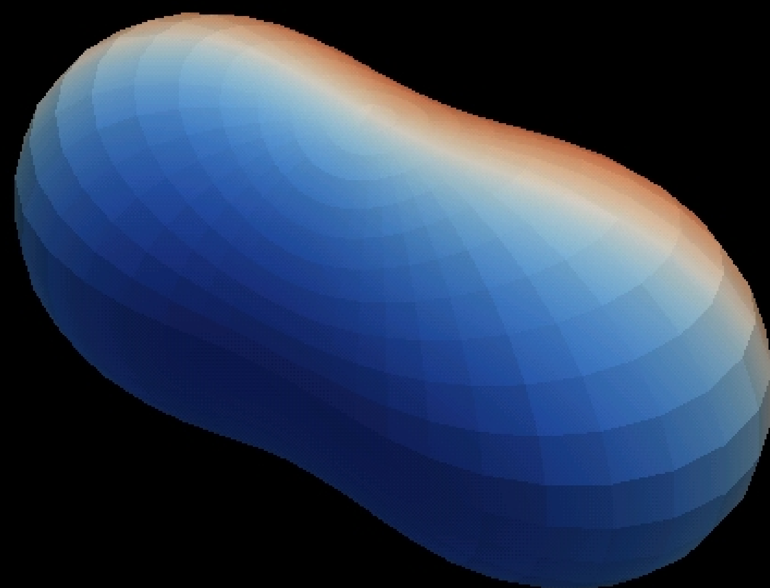
Initial common horizon of a BBH merger:

Dipole harmonic 1:



Initial common horizon of a BBH merger:

Dipole harmonic 2:



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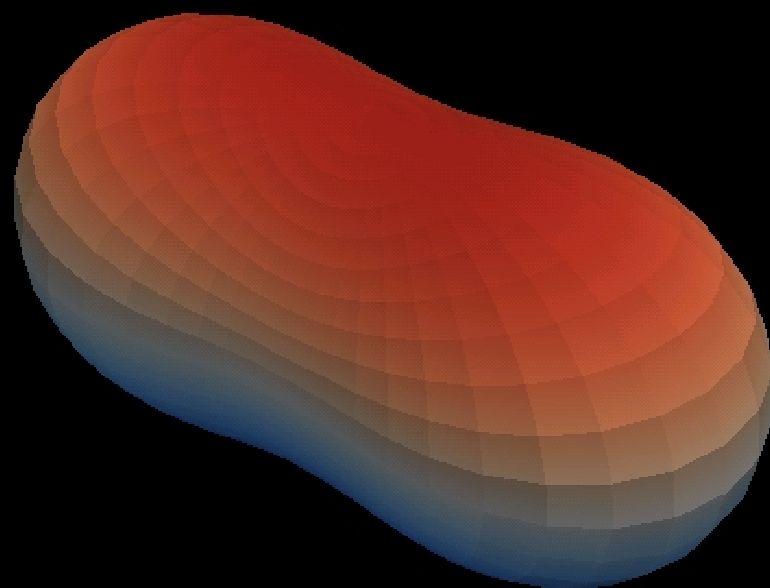
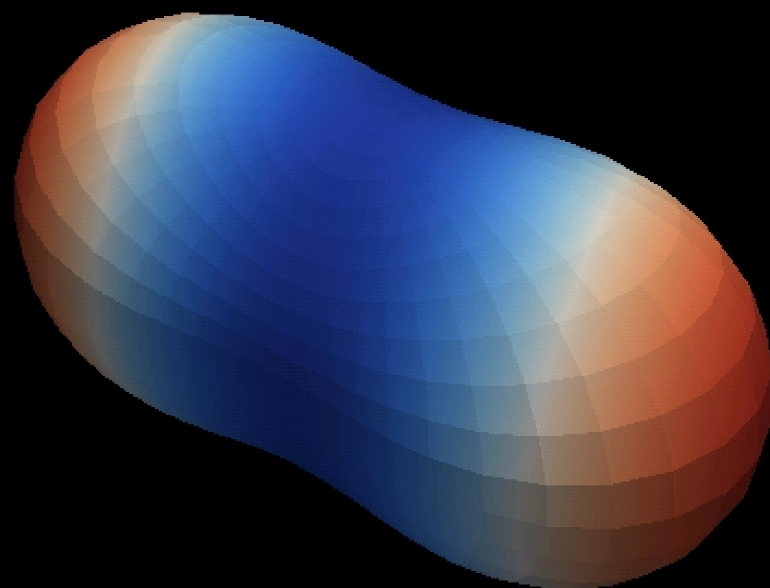


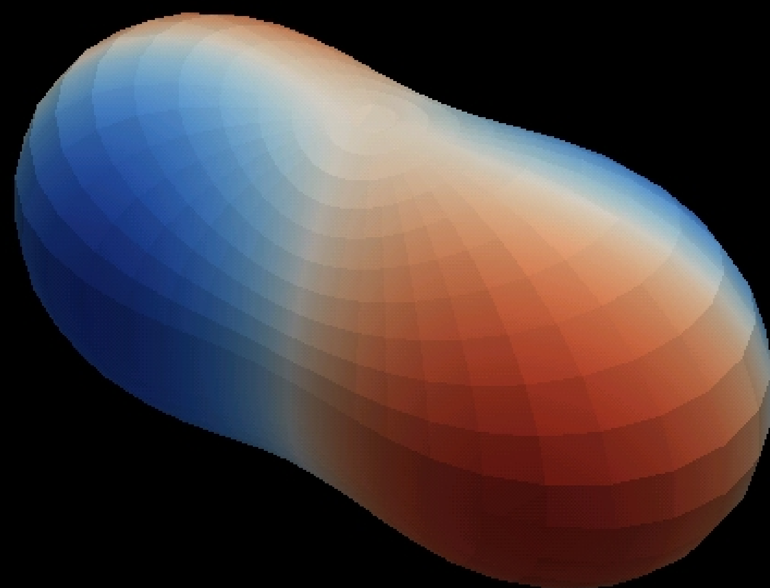
Initial common horizon of a BBH merger:

Quadrupole harmonic 1:



Initial common horizon of a BBH merger:

Quadrupole harmonic 2:



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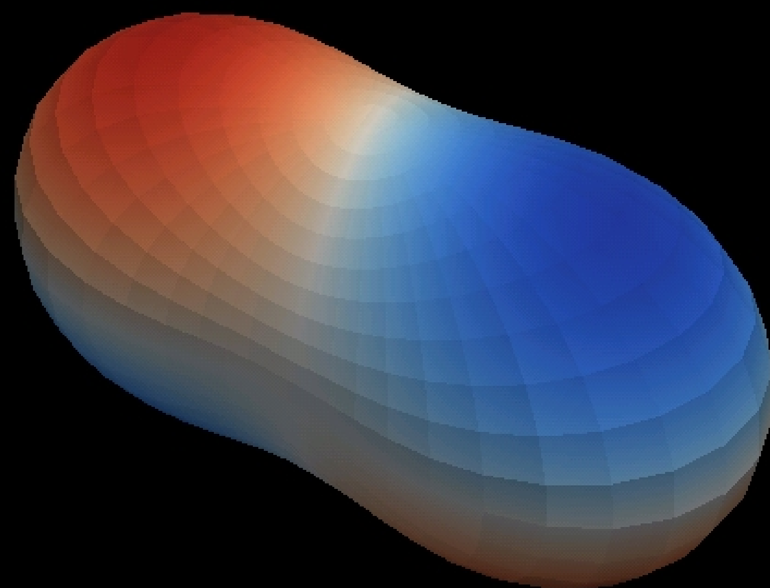
Quadrupole harmonic 3:



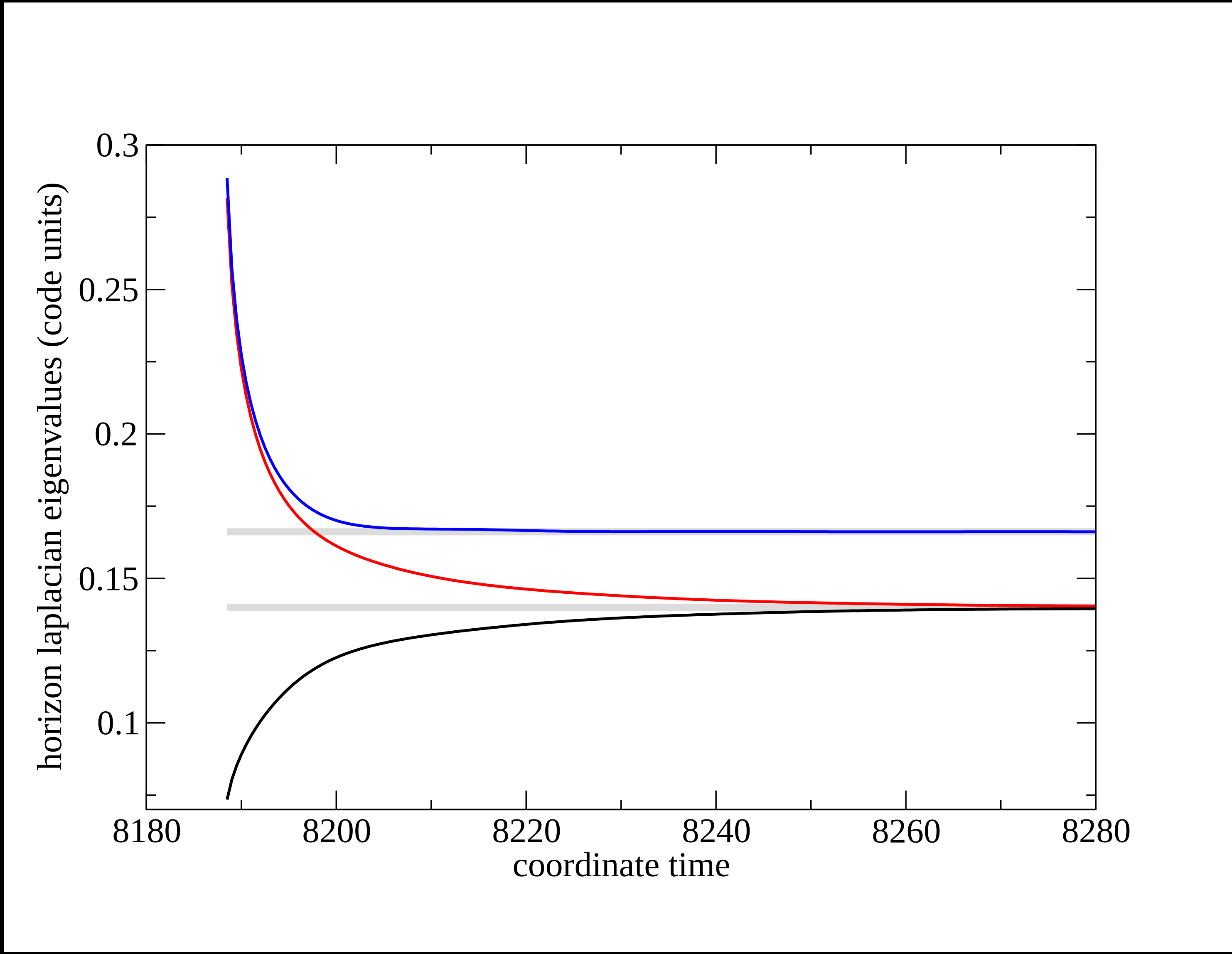
Application:

These multipoles can be used to verify, in a (spatially) coordinate-invariant manner, that the final remnant of a black hole merger is Kerr. (*See also Yosef Zlochower's talk.*)

Test case: ringdown after the merger of equal-mass, nonspinning, binary black holes after sixteen orbits of noneccentric inspiral (the data discussed in Scheel et al., arxiv:0810.1767, Phys.Rev.D **79** 024003 (2009)).

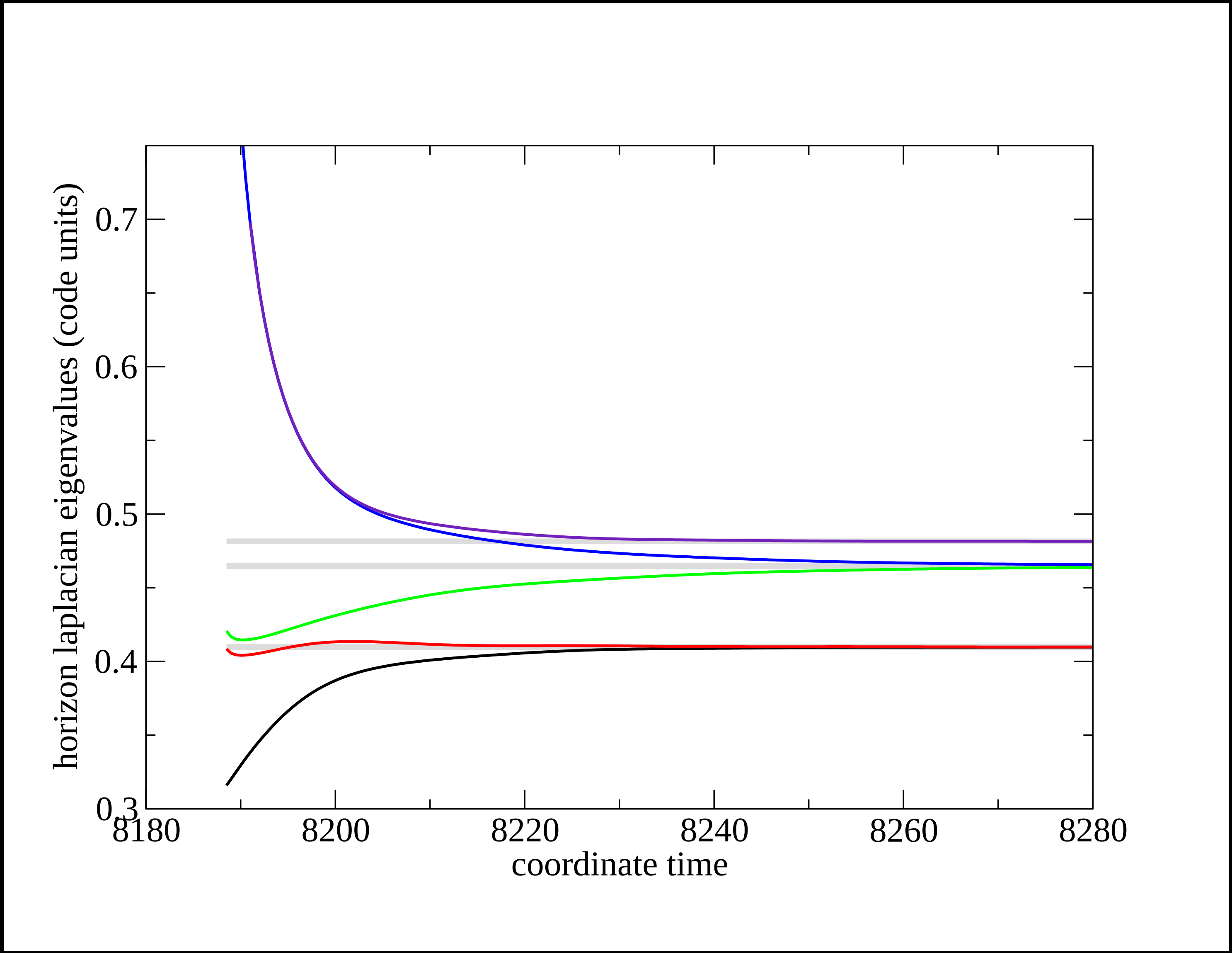
Spectrum:

"$\ell = 1$" scalar harmonics:



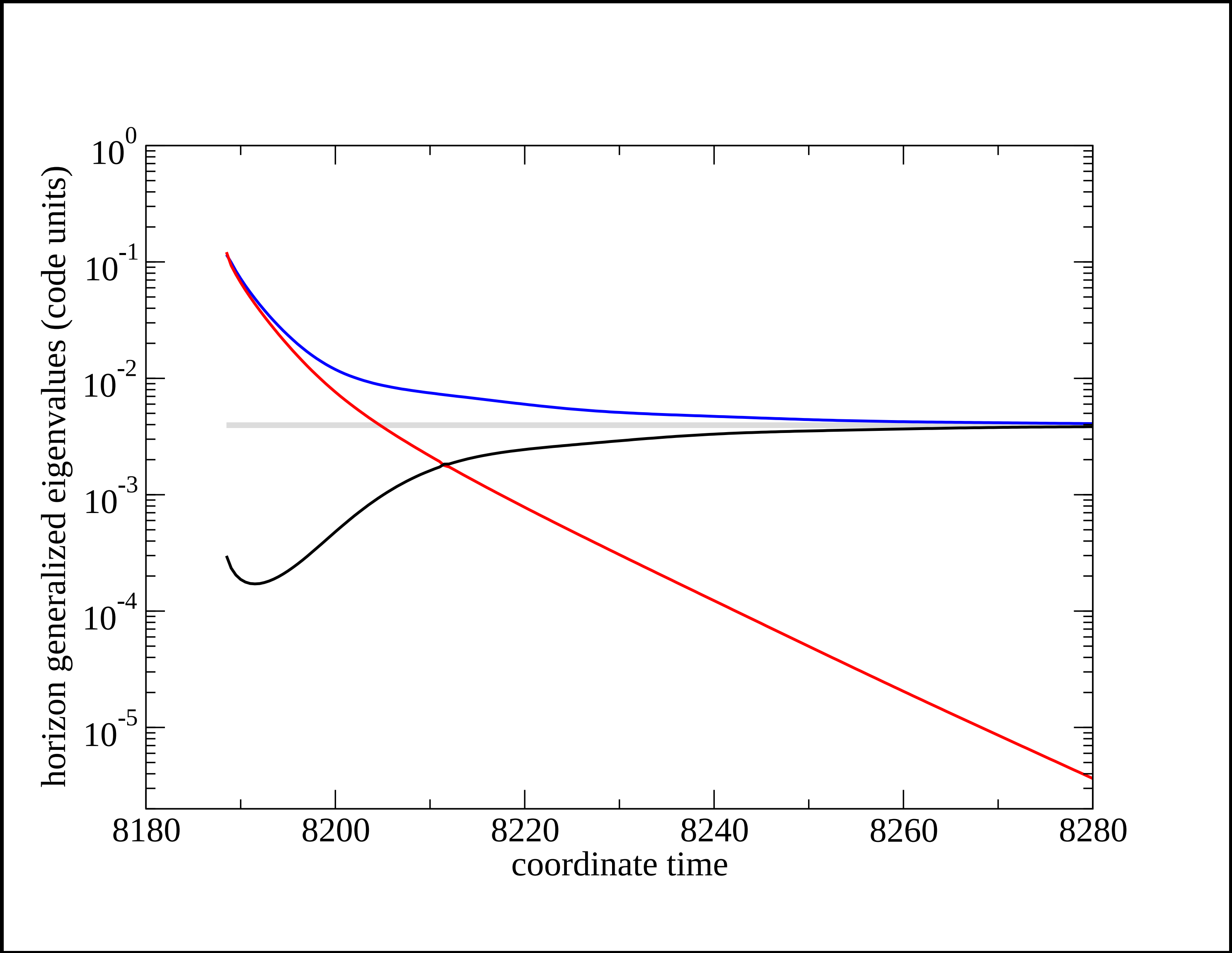
Spectrum:

"$\ell = 2$" scalar harmonics:



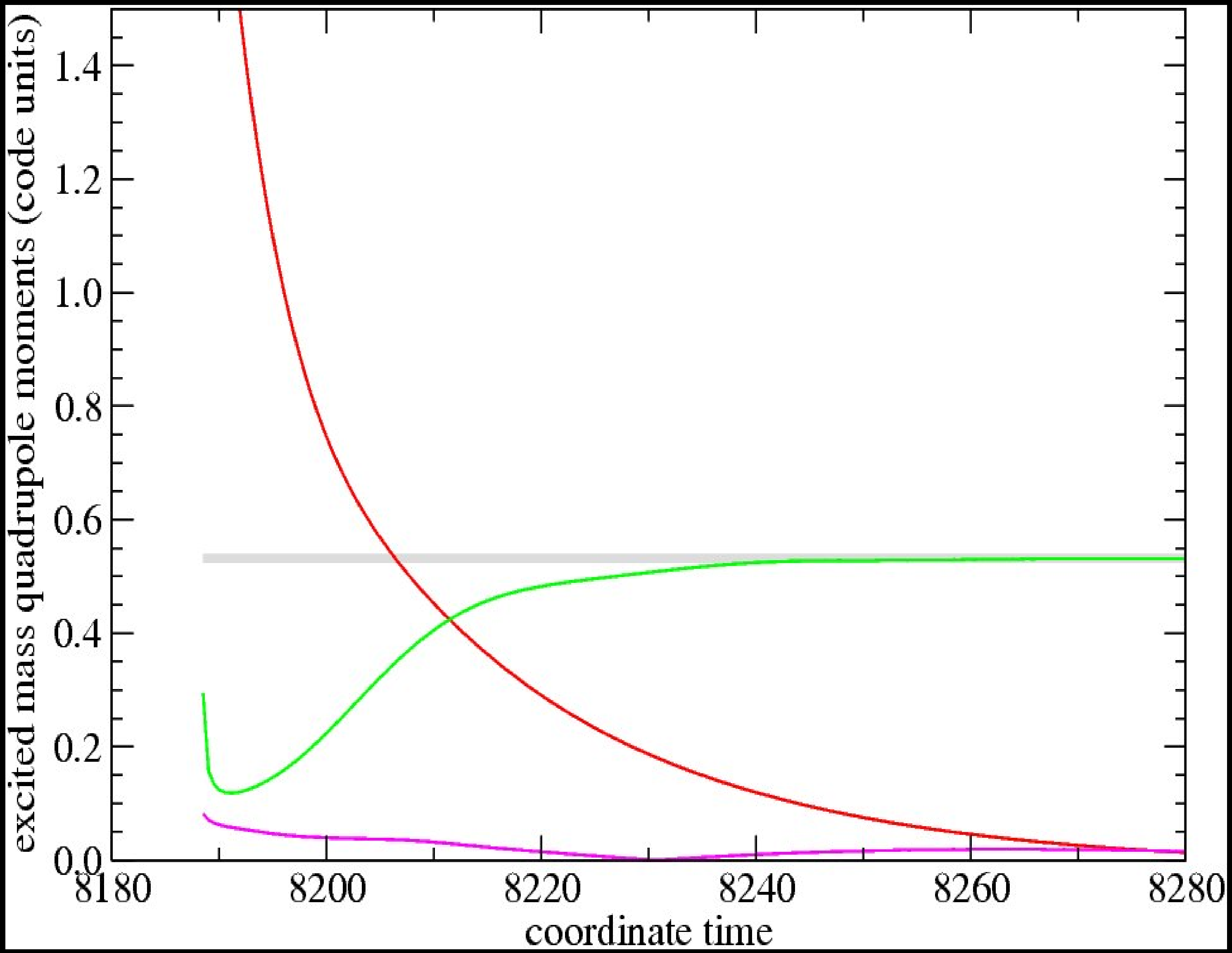
Spectrum:

"$\ell = 1$" vector harmonics:



Multipoles:

Excited mass quadrupole moments:



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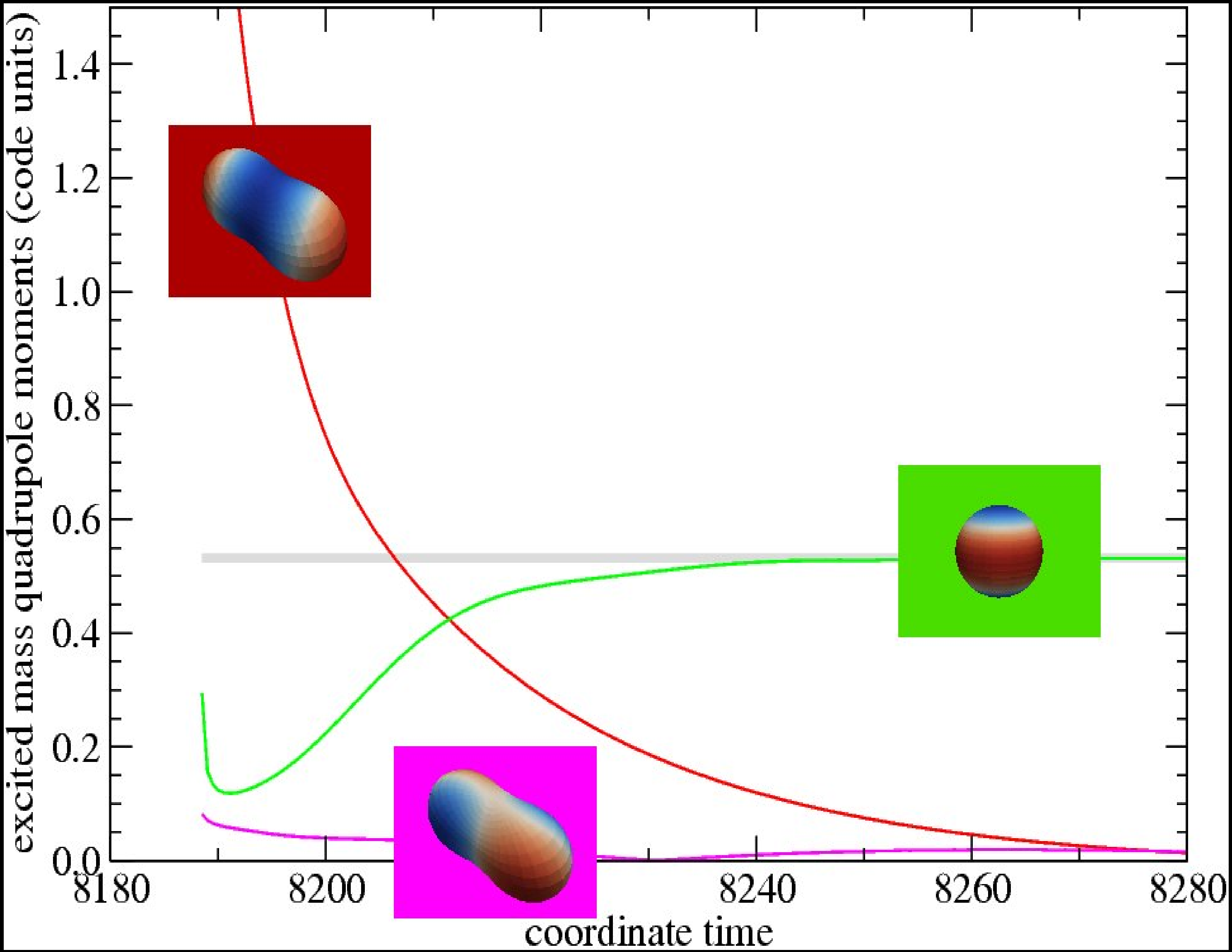
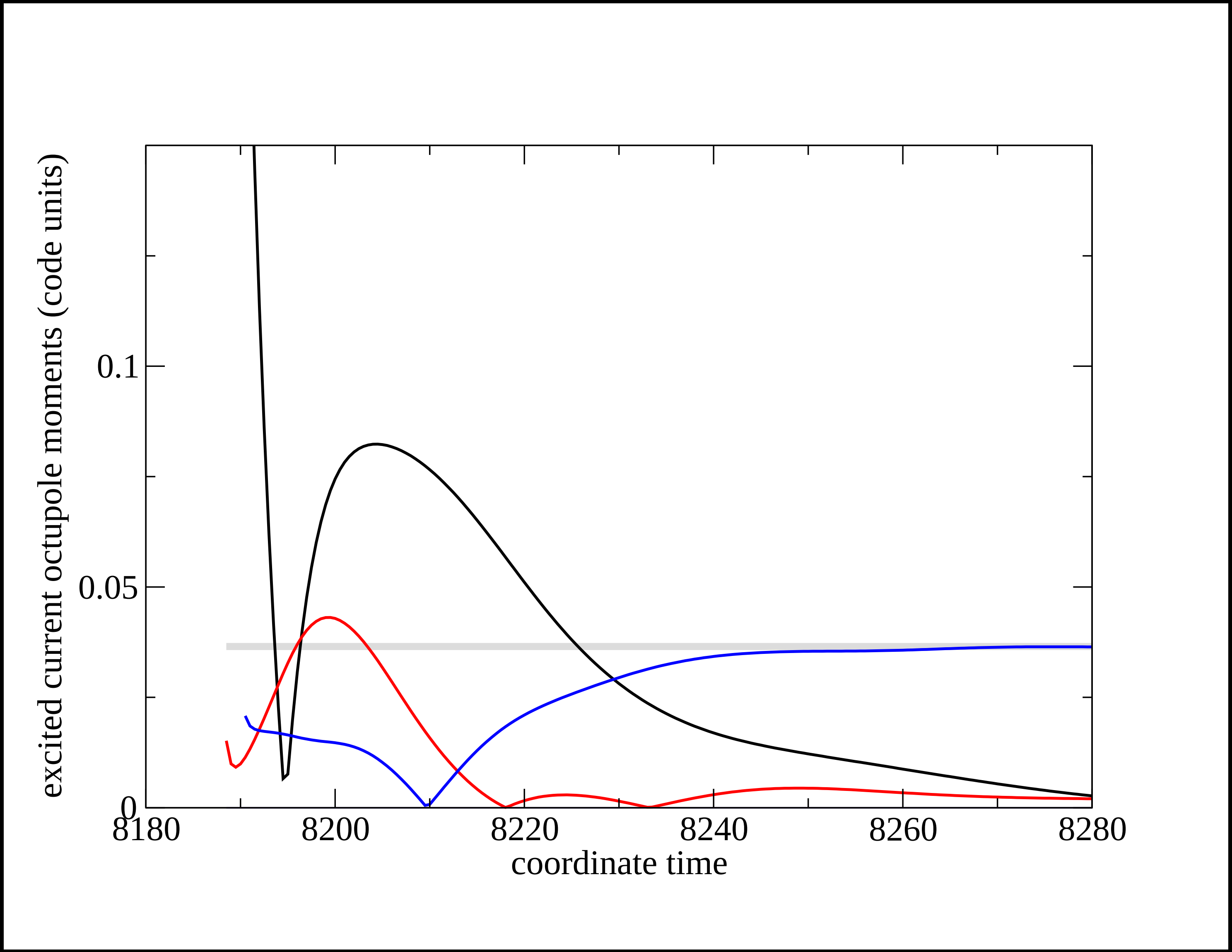


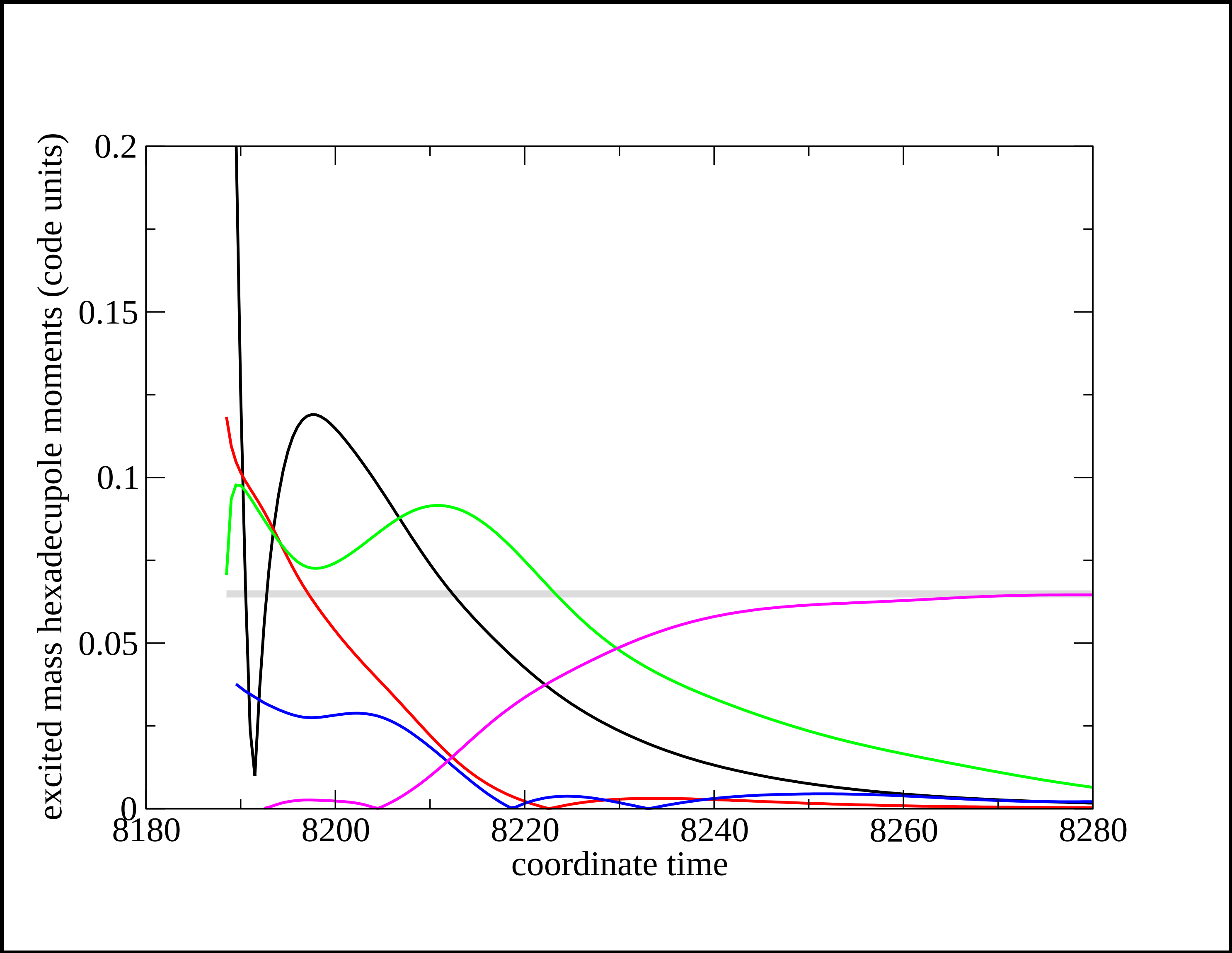
Multipoles:

Excited current octupole moments:

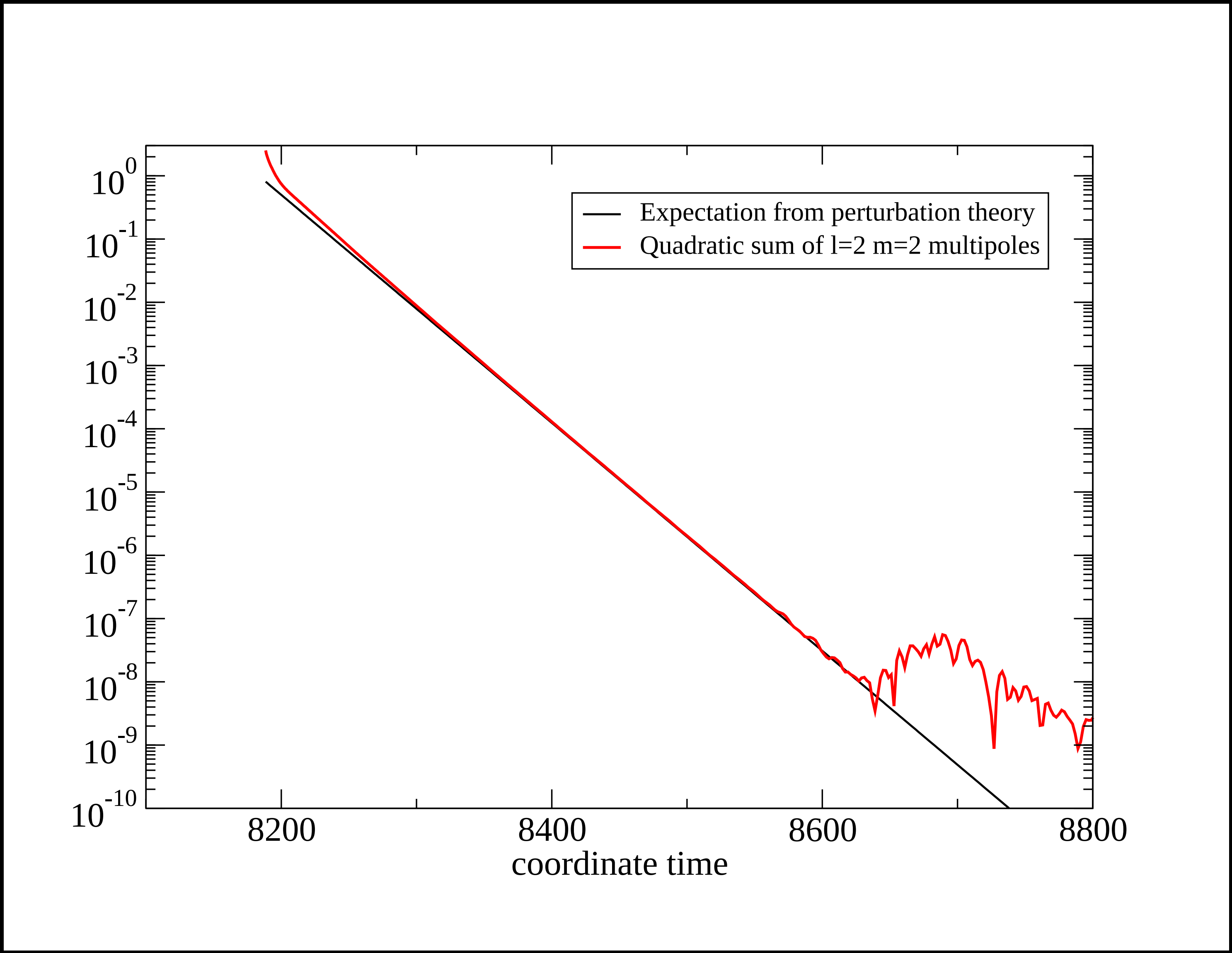


Multipoles:

Excited mass hexadecupole moments:



Another interesting result: quasinormal ringing.



Another interesting result: quasinormal ringing.

Next session, Geoffrey Lovelace will discuss a merger that settles down to a (kicked) Schwarzschild black hole. There is enough symmetry in this situation that the harmonics can't "rotate with the bulge," so this allows a better picture of the quasinormal oscillations.

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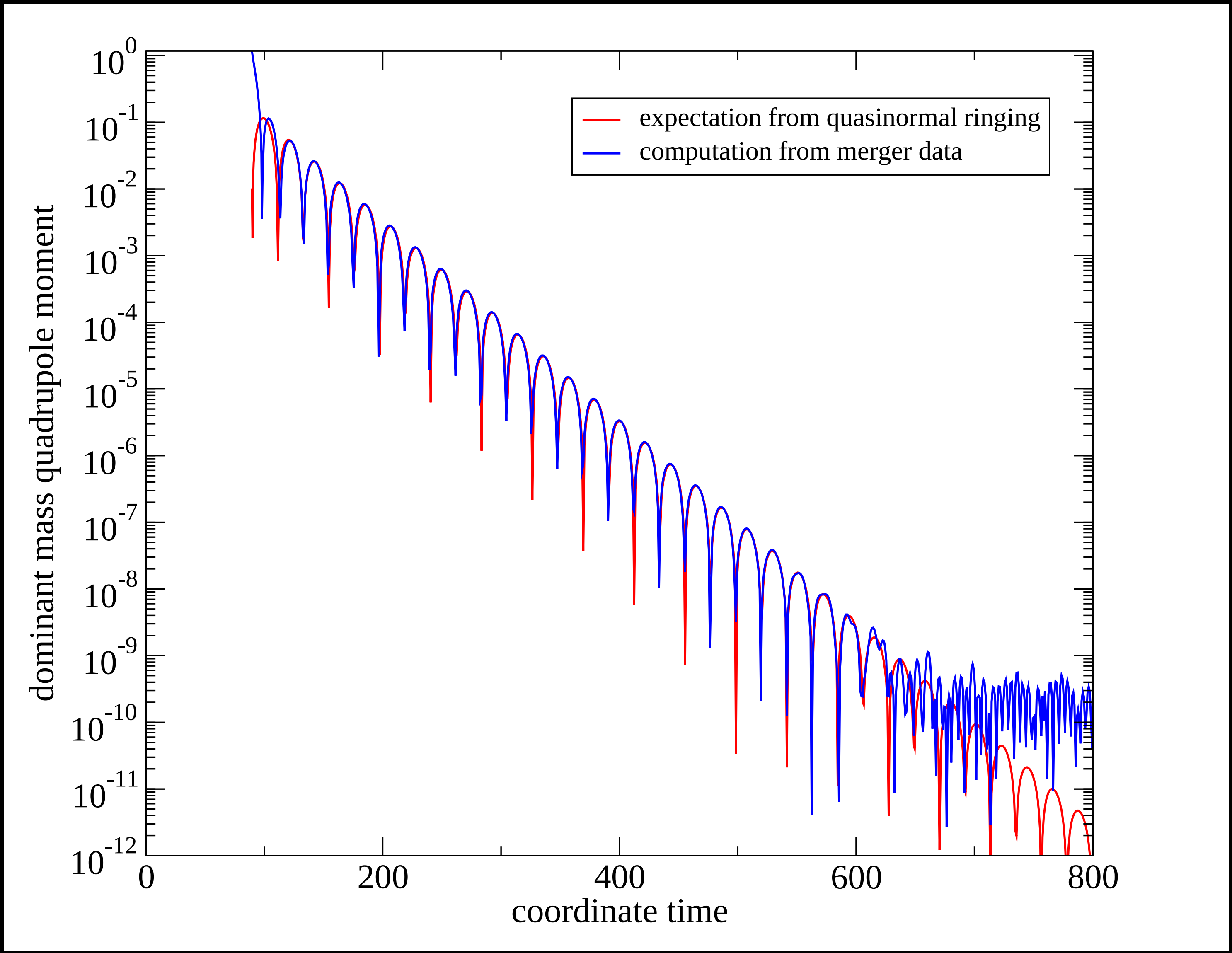

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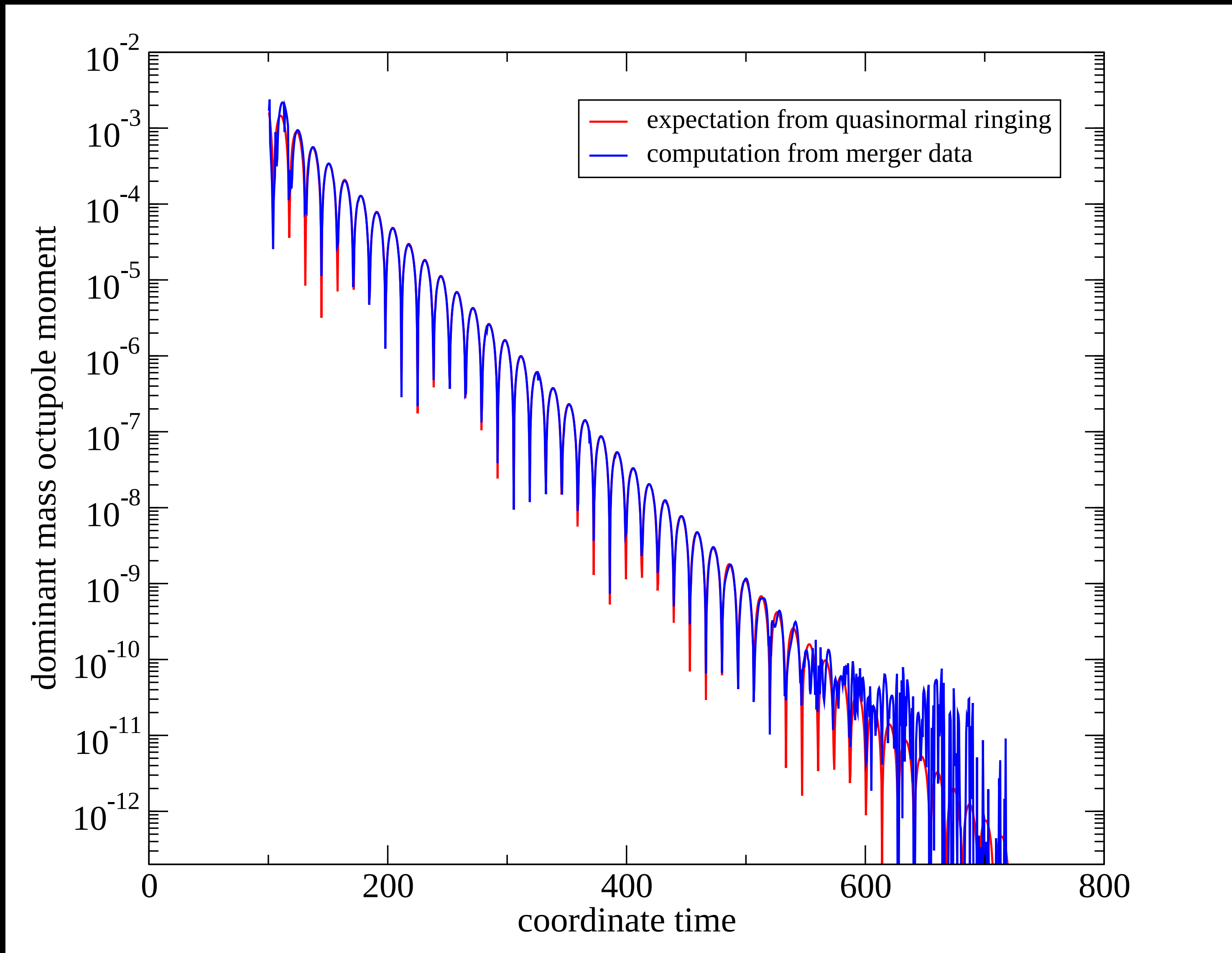

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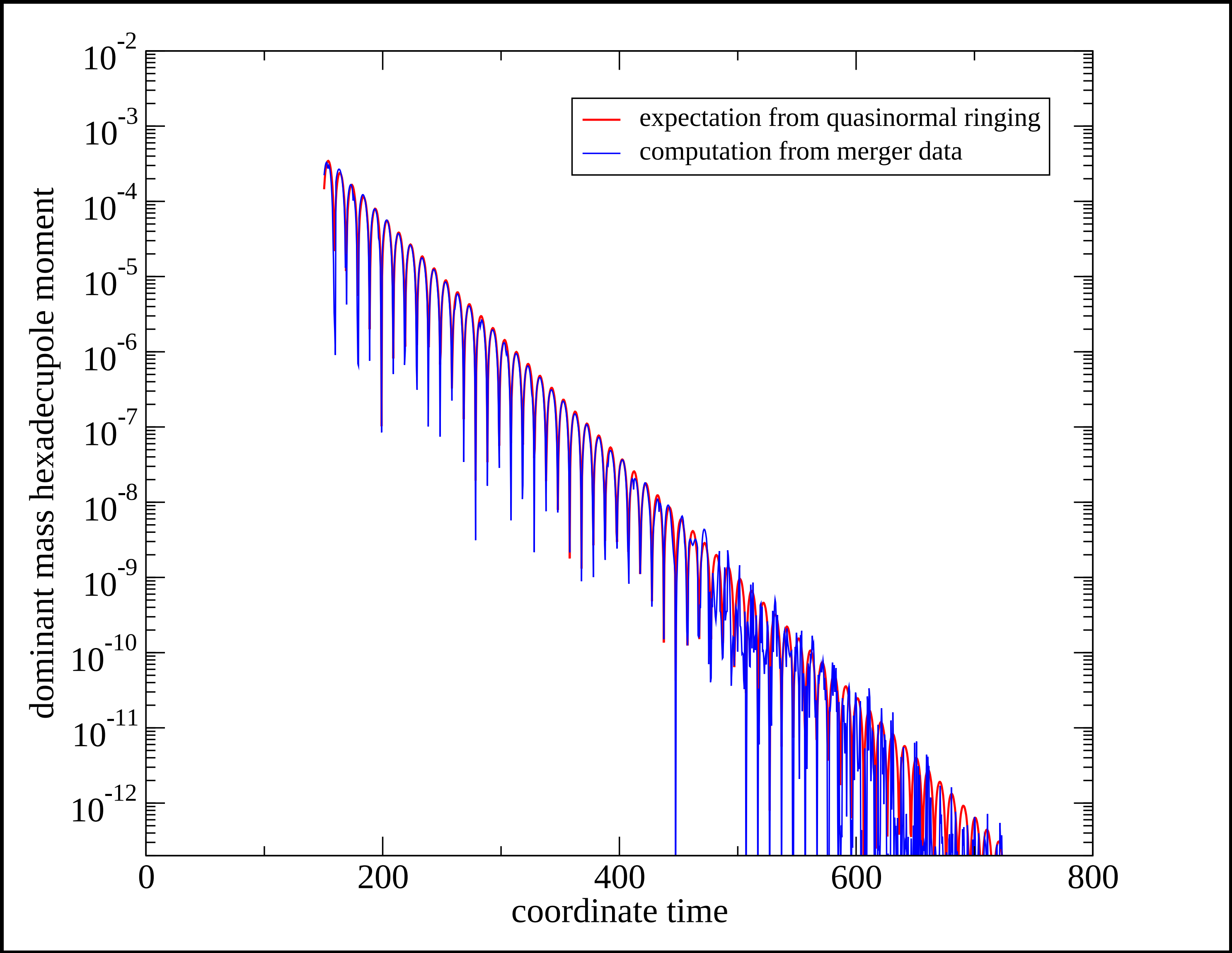

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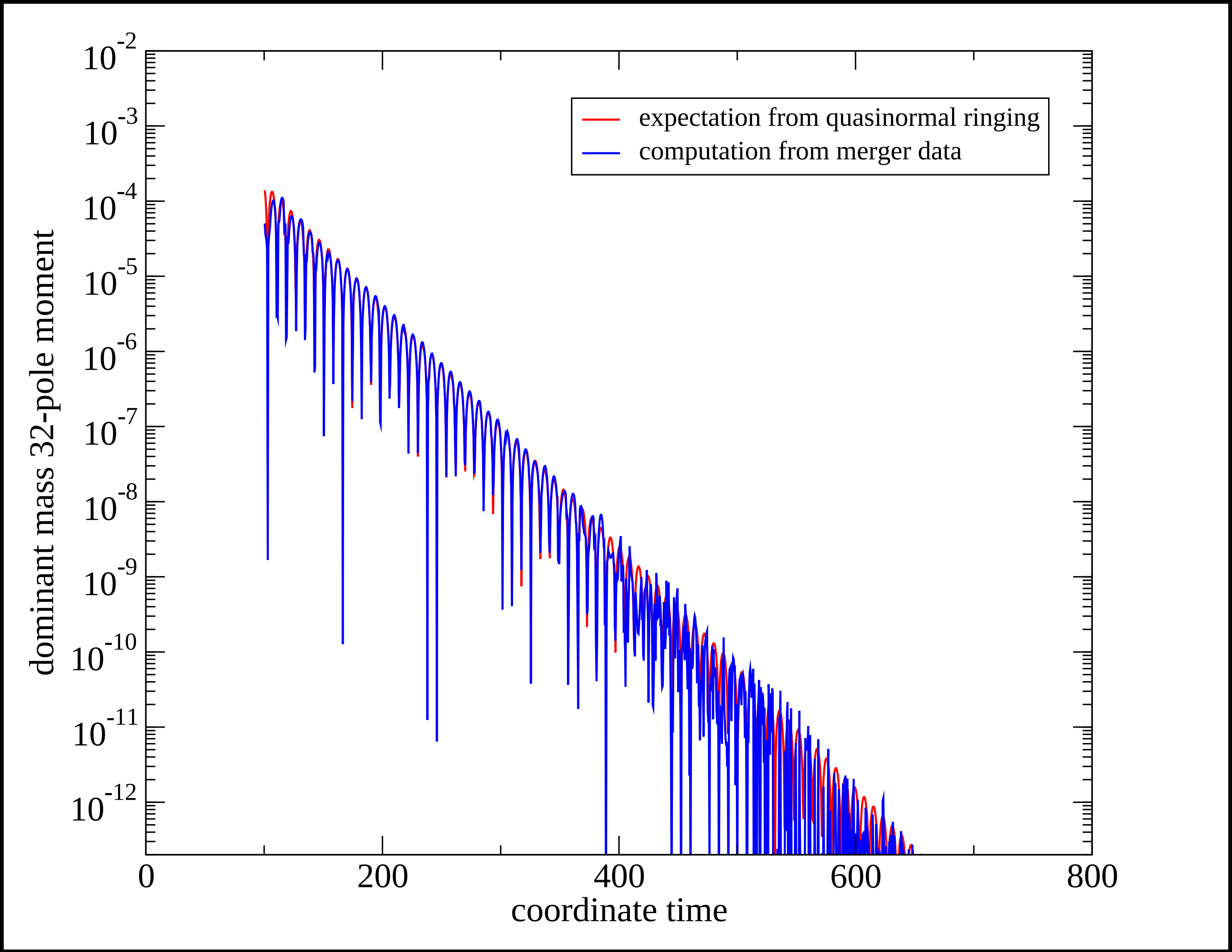

Conclusions:

- The source multipole formalism of Ashtekar et al. (2004) can be extended to nonaxisymmetric cases.
- The key to this generalization is a *spectral* definition of the spherical harmonics.
- The black hole merger described in Scheel et al. (2008) indeed settles down to a Kerr black hole, at least near the horizon.
- The quasinormal ringing of this dataset, as well as Lovelace's kick dataset, show remarkably fine agreement with results from perturbation theory, despite the general arbitrariness of the time slicing.