Effective Four-Dimensional Actions in Braneworld Scenarios

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Motivations for looking at extra dimensions

- String Theory
- Dark Matter
- Dark Energy
- Particle Physics

Features of Randall-Sundrum Models

Two 4-dimensional “branes” floating in a 5-dimensional “bulk”, compactified (orbifolded).

$$S = \int d^4x \int dy \sqrt{-g^{(5)}} \left\{ \frac{R^{(5)}}{2\kappa^2} - \Lambda \right\} + S_{\text{branes}}$$

- Separation of matter fields (“sequestering”)
- Natural hierarchy
- Radion mode: scalar field
  - Fixed
  - Dynamical
  - Free?

Randall-Sundrum: Compactified

$$S = \int d^4x \int dy \sqrt{-g^{(5)}} \left\{ \frac{R^{(5)}}{2\kappa^2} - \Lambda \right\} + S_{\mathrm{branes}} + S_{\mathrm{matter}}$$

$$ds^2 = e^{-2k\Phi(x^\lambda)|y|} \gamma_{\mu\nu}(x^\lambda) dx^\mu dx^\nu + \Phi(x^\lambda)^2 dy^2$$

$$\begin{aligned} S^{(4)} &= \int d^4\xi \sqrt{-g} \left[\frac{1}{2\kappa_4^2} R[g] - \frac{1}{2} (\nabla_a \psi)(\nabla^a \psi) \right] \\ &\quad + S_+[g_{+\ \mu\nu}, \chi_+] + S_-[g_{-\ \mu\nu}, \chi_-] \end{aligned}$$

$$g_{\mu\nu} = \left[\mathrm{e}^{2k\Phi} - 1 \right] \gamma_{\mu\nu}$$

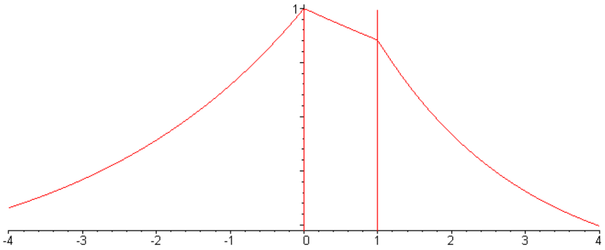
$$g_{-\ \mu\nu} = \sinh^2 \left(\frac{\kappa_4 \psi}{\sqrt{6}} \right) g_{\mu\nu}, \quad g_{+\ \mu\nu} = \cosh^2 \left(\frac{\kappa_4 \psi}{\sqrt{6}} \right) g_{\mu\nu}$$

Uncompactified Case

Two branes without compactification.

$$S = \int d^4x \int dy \sqrt{-g^{(5)}} \left\{ \frac{R^{(5)}}{2\kappa^2} - \Lambda \right\} + S_{\text{branes}} + S_{\text{matter}}$$

$$ds^2 = e^{2\Phi(x^\lambda)Q(y)} \gamma_{\mu\nu}(x^\lambda) dx^\mu dx^\nu + \Phi(x^\lambda)^2 dy^2$$



Plot of the warp factor $Q(y)$

Uncompactified Case

$$S^{(4)} = \int d^4\xi\sqrt{-g}\left\{\frac{1}{2\kappa_4^2}R^{(4)}[g] - \frac{1}{2}(\nabla_a\psi)(\nabla^a\psi)\right\} + S_1[g_{1\mu\nu}, \chi_1] + S_0[g_{0\mu\nu}, \chi_0]$$

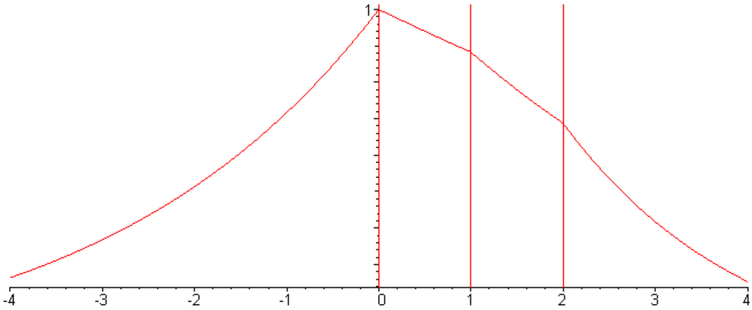
$$g_{\mu\nu} = \left[1 - \mathrm{e}^{-2k_2\Phi}\right]\gamma_{\mu\nu}$$

$$g_{0\ \mu\nu} = \hat{\kappa}\sinh^2\left(\frac{\kappa_4\psi}{\sqrt{6}}\right)g_{\mu\nu}, \quad g_{1\ \mu\nu} = \cosh^2\left(\frac{\kappa_4\psi}{\sqrt{6}}\right)g_{\mu\nu}$$

Goal

To look for a consistent dynamical model with a scalar field which models the dynamics of dark energy and also addresses dark matter and/or the hierarchy problem

New Models



A class of models to investigate

Techniques to find 4-dimensional theories

- Classical field theory reduction from 5D $\rightarrow$ 4D
- Identify a regime in which a 4-dimensional effective theory is a reasonable approximation to the full 5-dimensional dynamics
- Consider the radius of curvature in our 4-dimensional world, compare it to the radius of curvature in the bulk (dominated by the cosmological constant in the bulk)
- Expand in ratio of lengthscales

$$\varepsilon = \frac{\mathrm{R}_{\mathrm{Bulk}}}{\mathrm{R}_{\mathrm{4D}}}$$

Techniques to find 4-dimensional theories

- Full 5D equations $\rightarrow$ linear/quadratic perturbations
- Full 5D equations $\rightarrow$ projected onto 4D brane
- Direct computation of 4D action from 5D action by integrating out 5th dimension
  - General procedure
  - Requires full 5-D metric

Deriving the 4D Action

- Construct generalised coordinates in each region between branes
- Construct general metric based upon such coordinates (without any gauge assumptions)
- Scale brane coordinates x^α by ε (coordinates become $(\varepsilon x^\alpha, y)$)
- Calculate 5-D action to lowest order in ε
- Minimise action to obtain metric ansatz in limit $\varepsilon \longrightarrow 0$

Deriving the 4D Action

- Use this metric ansatz to calculate 5-D action to second order in ε
- Minimise second order action (zeroth order action is already minimised by ansatz, first order action is vanishing)
- Second order action contains Ricci scalar and radion fields
- Integrate over 5th dimension in action to obtain 4D effective theory
- Recast fields in Einstein conformal frame with canonical normalisation

$$\begin{aligned} S^{(4)} &= \int d^4\xi\sqrt{-g}\left\{\frac{1}{2\kappa_4^2}R^{(4)}[g] - \frac{1}{2}(\nabla_a\psi)(\nabla^a\psi)\right\} \\ &\quad + S_1[g_{1\mu\nu},\chi_1] + S_0[g_{0\mu\nu},\chi_0] \end{aligned}$$

Where to now?

- Calculate low-dimensional effective action in n brane case
- Understand how 5-D dynamics in $\varepsilon \longrightarrow 1$ regime of uncompactified models affect 4-D effective theory
- Investigate the situation of a black hole on a brane in this framework ($\varepsilon \longrightarrow 1$ near the brane)
- Search for n-brane configurations with a region of parameter space useful for dark energy/dark matter

Summary

- Have classes of models which have features which can relate to dark matter, dark energy and the hierarchy problem
- Have a general method to calculate the physics of these models
- Have models to go and explore!