Hořava–Lifshitz gravity: What's the matter?

based on arXiv:0904.0829 and arXiv:0905.3740

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1855

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Plan of the talk

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- Sketchy review of Hořava–Lifshitz gravity.

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- Cosmology.

The idea

Hořava, arXiv:0812.4287, arXiv:0901.3775

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- The critical exponent z determines the dimension D at which the field propagator becomes logarithmic, critical behaviour of correlation functions near a phase transition.

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- Dispersion relation $\omega^2 \sim |\mathbf{k}|^2 + \alpha|\mathbf{k}|^{2z}$, improves short-distance behaviour. Theory power-counting renormalizable if $z \geq D$.

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$$\boxed{[\kappa^2] = z - 3}$$

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$$\mathcal{H} = \frac{1}{\sqrt{g}}\left[\Pi^T\mathbb{G}\Pi - \left(\frac{\delta W}{\delta q}\right)^T \mathbb{G}\frac{\delta W}{\delta q}\right]$$

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Minimal coupling prescription: scalar and grav. sectors factorize:

$$\pi_{ij} = \frac{\delta W_g}{\delta g^{ij}}\,, \qquad \pi_\phi = \frac{\delta W_\phi}{\delta \phi}$$

Minimally coupled theory: Gravity sector

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Relevant deformations push the system towards the IR f.p.:

$$S_g \sim \frac{2}{\kappa^2}\int dt d^3x\,\sqrt{g}\,N\left[K_{ij}K^{ij} - \lambda K^2 + c^2(R-3\Lambda_W)\right]$$

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$$c \equiv \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G \equiv \frac{\kappa^2}{32\pi c}$$

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IR limit:

$$S_\phi \sim \frac{1}{2}\int dtd^3x\sqrt{g}N\left[\frac{\dot{\Phi}^2}{N^2} - |\beta_2|\partial_i\phi\partial^i\phi - m^2\phi^2\right]$$

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Friedmann equation:

$$H^2 = \frac{8\pi\tilde{G}c}{3}\rho - \frac{B^2}{a^4} - \frac{c^2\tilde{\mathsf{K}}}{a^2} - \frac{c^2|\tilde{\Lambda}|}{3}$$

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Possibility of a bounce at

$$\rho_* = \frac{c|\Lambda|}{8\pi G} + \frac{3c\mathsf{K}}{8\pi G a_*^2}\left(1 + \frac{B^2}{a_*^2 c^2\tilde{\mathsf{K}}}\right)$$

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Proposed in arXiv:0904.0829, later considered by Brandenberger and others.

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$$\delta^{(2)}S_g = -\frac{1}{2\kappa^2}\int d\tau d^3x\, a^2\left[h^{ij}h''_{ij} - \left(\frac{\kappa^2}{2\nu^2}\right)^2 a^2 h_{ij}\Delta^3 h^{ij}\right]$$

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In momentum space, $v_k = ah_k$,

$$v''_k + \left[+\left(\frac{\kappa^2}{2\nu^2}\right)^2 \frac{k^6}{a^4} - \frac{a''}{a}\right] v_k = 0$$

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Corley–Jacobson dispersion relation as in trans-Planckian cosmology, strongly scale-dependent scalar spectrum!

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$\Rightarrow$ Abandoning detailed balance, signs get fixed.

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UV marginal kinetic terms:

$$\frac{\kappa^2}{2}\left(s_0^2\phi^2 - \frac{4}{\nu^4}\right)h_{ij}\Delta^3 h^{ij}\,, \qquad \left[\frac{s_3^2}{2} - s_0^2\kappa^2(2\lambda-1)\right]\delta\phi\Delta^3\delta\phi$$

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$$W = \frac{1}{\nu^2}\int \omega_3(\Gamma) + \int d^3x\sqrt{g}\left[\mu R + s_0 g^{ij}\phi\Delta^{1/2}R_{ij} - 2L(\phi)\right]$$

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$4D$ action very complicated but its properties can be inferred by looking only at a few terms.
UV marginal kinetic terms:

$$\frac{\kappa^2}{2}\left(s_0^2\phi^2 - \frac{4}{\nu^4}\right)h_{ij}\Delta^3 h^{ij}\,, \qquad \left[\frac{s_3^2}{2} - s_0^2\kappa^2(2\lambda-1)\right]\delta\phi\Delta^3\delta\phi$$

UV stability if

$$|\phi| > \frac{2}{\nu^2|s_0|}\,, \qquad s_3^2 > 2s_0^2\kappa^2(2\lambda-1)$$

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At the IR point one may define the conformal transformation $f(\partial_i\Omega, \Omega)\Omega^{-2} + c^2(\phi) = \text{const}$ but only on inhomogeneous backgrounds.

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- Detailed balance should be abandoned. This is clear only in the theory with matter.

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- **Fractal structure**: Spectral dimension flows from $1+D$ in the infrared to $1+D/z=2$ in the ultraviolet, like CDT, QEG and spinfoams. Newton potential $G(|\mathbf{x}|) \sim |\mathbf{x}|^{-1}$ at large scales, $G(|\mathbf{x}|) \sim |\mathbf{x}|^{2z-D}$ in general, typical of fractal manifolds. Since integrals on net fractals can be approximated by fractional integrals, it is natural to consider fractional integrals over a space with fractional dimension.