Simulations of the Gravity Dual in an AdS/CFT Correspondence

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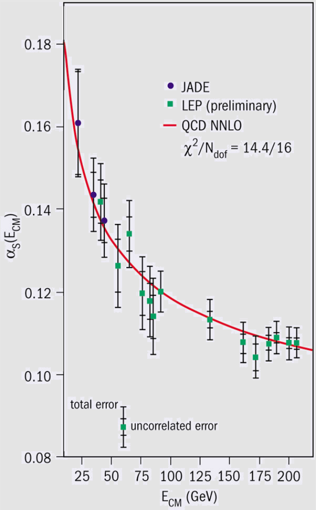
June 15, 2009

Outline

- Motivations
  - heavy ion collision
  - dual system on the gravity side
- AdS/CFT
  - approximation to QCD
  - dictionary to translate between a CFT and gravity
- Preliminary simulations
- Initial data
  - evolution “forward in time”
  - constraints for initial spatial hypersurface
- Summary

Motivations



- Many problems in QCD are non-perturbative
  - not amenable to usual methods based on Feynman graphs
  - hard to solve
- Perturbative methods questionable because the QCD running coupling α_s can be as large as $\frac{1}{2}$
  - seen from RG equation for α_s $p\frac{d\alpha_s}{dp} \sim \beta(\alpha_s) < 0$
  - instead, gravity description via AdS/CFT

Figure: Running coupling α_s as a function of CM energy E_{CM} [1]

[1] *CERN Courier*

Motivations

- Hope is that we could use the AdS/CFT duality to make experimentally testable predictions about QCD processes
  - work by Steve Gubser's group in linearized gravity to describe a QGP in terms of a BH's quasinormal modes
- Heavy ion collision at RHIC
  - Au-Au collision with center-of-mass energy $\sqrt{s} = 200$ [MeV]
  - Forms deconfined fluid of quarks and gluons called a quark-gluon plasma (QGP)
  - QGP thermalizes, contracts, then expands under its own pressure, with lifetime of ~ 10 [fm/c]
- Gravity dual
  - BH-BH collision in $d = 5$ anti-de Sitter space (AdS_5)
  - merger of compact massive binaries is the subject of much work in numerical relativity

Approximation to QCD

- Major obstacle is the current lack of a gravity dual for QCD ($N = 3$, so AdS/CFT not directly applicable)
  - so try approximating QCD with a CFT toy model
  - replace QCD by $\mathcal{N} = 4$ super-Yang-Mills, with symmetry group $SU(N)$ in the limit as $N \to \infty$

QCD vs SYM (zero temperature $T = 0$)

Quantum Chromodynamics	$\mathcal{N} = 4$ super-Yang-Mills
confining	not confining
not conformal	conformal
quarks	adjoint matter
not supersymmetric	supersymmetric
$N = 3$	$N \to \infty$

QCD vs SYM (finite temperature $T > T_c$)[2]

Quantum Chromodynamics	$\mathcal{N} = 4$ super-Yang-Mills
not confining	*not confining*
not conformal	*not conformal*
quarks	adjoint matter
not supersymmetric	*not supersymmetric*
$N = 3$	$N \to \infty$

[2] $T_c \sim 190$ *[MeV] critical temperature above which QCD deconfines*

AdS/CFT Duality

- The idea behind the duality: describe a limiting case of string theory in two alternative ways
  - two alternative descriptions of string theory in the low-energy limit

AdS/CFT Duality

Gauge Theory Description	Supergravity Description
$\mathcal{N} = 4$ super-Yang-Mills ($N \to \infty$ for $SU(N)$ symmetry group)	extremal supergravity solution ($r \to 0$ for radial coordinate r)

AdS/CFT Duality

- Extremal $d = 10$ supergravity solution

$$g = \left(1 + \frac{L^4}{r^4}\right)^{-\frac{1}{2}} (-dt^2 + d\vec{x}_3^2) + \left(1 + \frac{L^4}{r^4}\right)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$$

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$A\mathrm{dS}_5 \times \mathrm{S}^5$ metric! (in Poincaré coordinates)

AdS/CFT Dictionary

Entry: AdS_5 metric perturbations δg_{ij} near the boundary / CFT energy-momentum tensor

$$\langle T_{ij} \rangle_{CFT} = \frac{L^3}{4\pi G_5} \lim_{z\to 0} \frac{1}{z^4} \delta g_{ij}$$

Preliminary Simulations

- Dimensionally reduce a $4+1$ simulation in $(t, \rho, \chi, \theta, \phi)$ to an effective $2+1$ simulation in (t, ρ, χ)
  - each point in the effective $2+1$ spacetime is a 2-sphere parametrized by θ, ϕ
  - θ, ϕ only appear in the full metric as the static 2-sphere metric $d{\Omega_2}^2$, multiplied by an overall conformal factor
  - left with radial $\rho \in [0, 1]$ and angular $\chi \in [0, \pi]$

Preliminary Simulations

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Preliminary Simulations

- Solve generalized-harmonic form of the Einstein equations $R_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right)$
  - introduce source functions H^μ such that the coordinates satisfy $\Box x^\mu = H^\mu$
  - reexpress $R = d\Gamma + \Gamma \wedge \Gamma$ in terms of H^μ
  - the principal part of Einstein equations then becomes the wave operator $g^{\mu\nu}\partial_\mu\partial_\nu$ (hyperbolic), giving stable evolution
- The coordinate degrees of freedom are encoded in the source functions H^μ
  - gauge choice amounts to specifying the evolution equations satisfied by H^μ (eg: harmonic gauge $H^\mu = 0$)

Initial Data

- Foliate an $(n + 1)$-dimensional manifold into a family of n-dimensional hypersurfaces $\{\Sigma_t\}_{t \in \mathbb{R}}$
  - amounts to a "space + time" splitting that arbitrarily singles out a timelike coordinate t
  - used to generate the time evolution of objects on the Σ_t spatial hypersurfaces
  - decomposes Einstein equations into evolution equations and constraint equations

Initial Data

- The constraint equations are conditions within each Σ_t
  - ensures that Σ_t hypersurfaces can be embedded in AdS_5

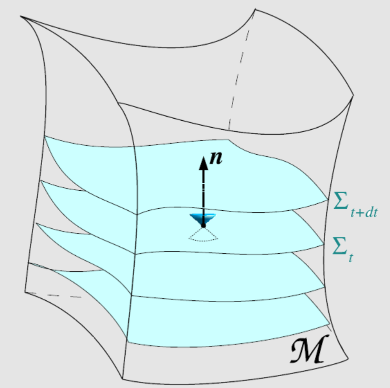

Figure: dim-$n+1$ manifold $\mathcal{M}$ foliated by dim-n hypersurfaces Σ_t [3]

[3] *Bases of Numerical Relativity*, E. Gourgoulhon

Initial Data

- Start from the Einstein equations with energy-momentum source $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda_5 g_{\mu\nu} = 8\pi T_{\mu\nu}$
  - contract with two copies of a time-like one-form $n = -\alpha(x)dt$ normal to the initial data hypersurface Σ_{t_0}
  - reexpress time-time contraction $R(n,n)$ of the Ricci tensor in terms of hypersurface objects

- Result is the Hamiltonian constraint equation
  - solved to reconstruct consistent initial data

Summary

- What physics can we hope to extract from these simulations?
  - full numerical analysis of colliding black holes in an asymptotically AdS_5 spacetime
  - extract $\langle T_{ij} \rangle_{CFT}$ from the AdS_5 metric perturbations δg_{ij} near the boundary
  - compare extracted parameters (eg: thermalization time) to known parameters of QGPs produced at RHIC

- Working towards a generic code
  - can be applied to a wide variety of systems on AdS_5
  - mapped to a variety of dual problems relevant to RHIC