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Cauchy-Characteristic Patching with Improved Accuracy

Rochester, NY, June 15-16, 2009

Introduction

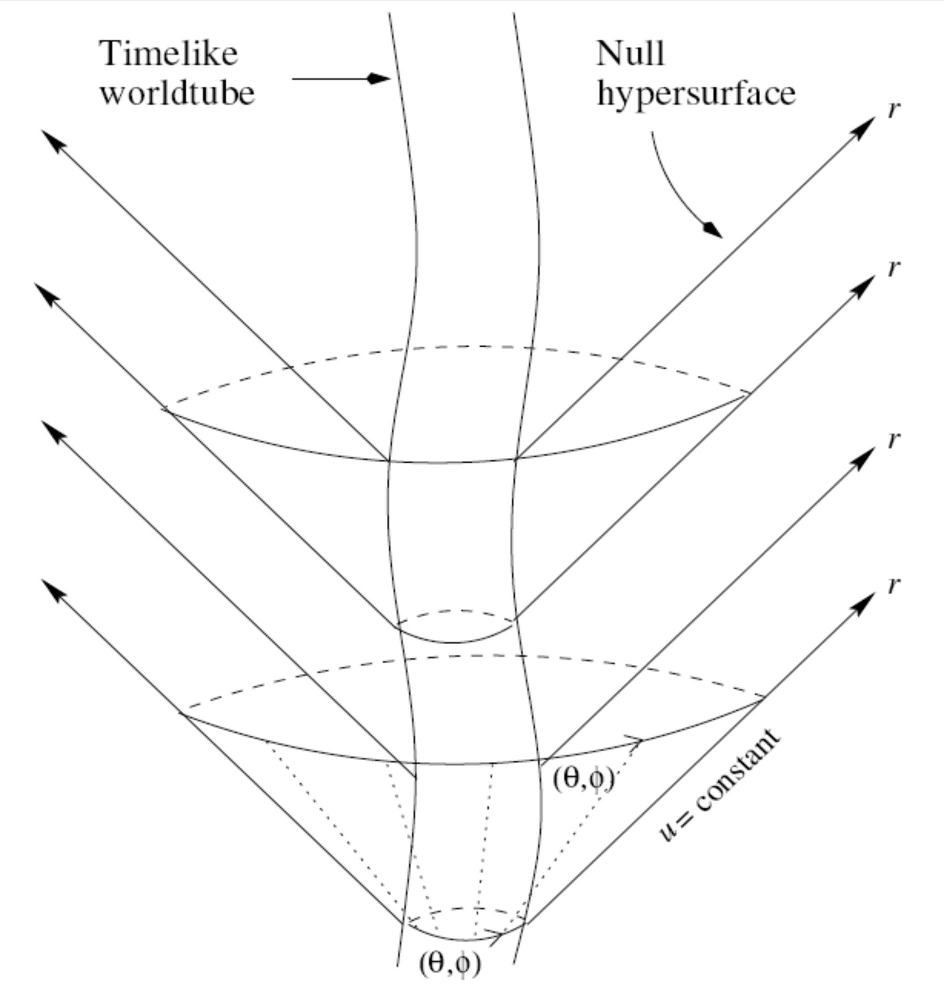
- Cauchy-characteristic extractions (CCE) avoids the errors due to extraction at finite worldtube
- The Cauchy and the characteristic approaches have complementary strengths and weaknesses.
- Unification of the two methods is a promising way of combining the strengths of both formalisms.

Advantages

- Avoids the errors due to gravitational waveform extraction at finite worldtube.
- The grid domain is exactly the region of the waves propagation (no artificial boundary).
- Gives the waveform and polarization state at infinity (no ongoing radiation).
- Offers flexibility and control in prescribing initial data (very little gauge freedom).
- Combine Cauchy & Characteristic methods.

The Characteristic Method

- Extract characteristic data at inner worldtube from Cauchy evolution,
- Propagate the characteristic solution along the outgoing light cones,
- Extracts the waveform at infinity for each retarded time.

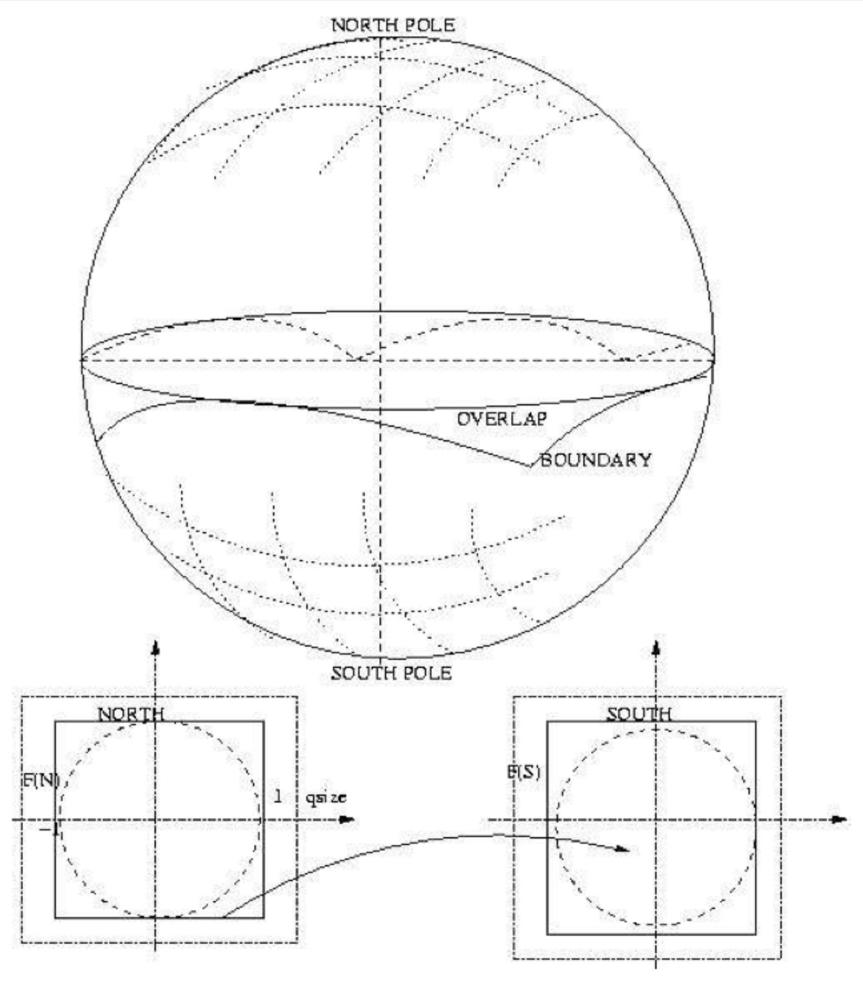


The Stereographic Module

- Newman-Penrose eth-formalism on the sphere

  $$q_A q_B D^A U^B = \eth U$$

- Numerical noise introduced by inter-patch interpolation.
- Two improvements:
  - Circular boundary,
  - 4^{th} order derivatives.



Higher Order Approximations

- 4th order approximations in finite differences.
- 1st and 2nd derivatives of the 5th order, 3rd derivative of the 7th order Lagrange polynomial.

$$D_1F(x_i) = \frac{F(x_{i-2}) - 8F(x_{i-1}) + 8F(x_{i+1}) - F(x_{i+2})}{12\Delta x}$$

$$D_2F(x_i) = -\frac{F(x_{i-2}) - 16F(x_{i-1}) + 30F(x_i) - 16F(x_{i+1}) + F(x_{i+2})}{12\Delta x^2}$$

$$D_3F(x_i) = \frac{F(x_{i-3}) - 8F(x_{i-2}) + 13F(x_{i-1}) - 13F(x_{i+1}) + 8F(x_{i+2}) - F(x_{i+3})}{8\Delta x^3}$$

Errors in Angular Derivatives

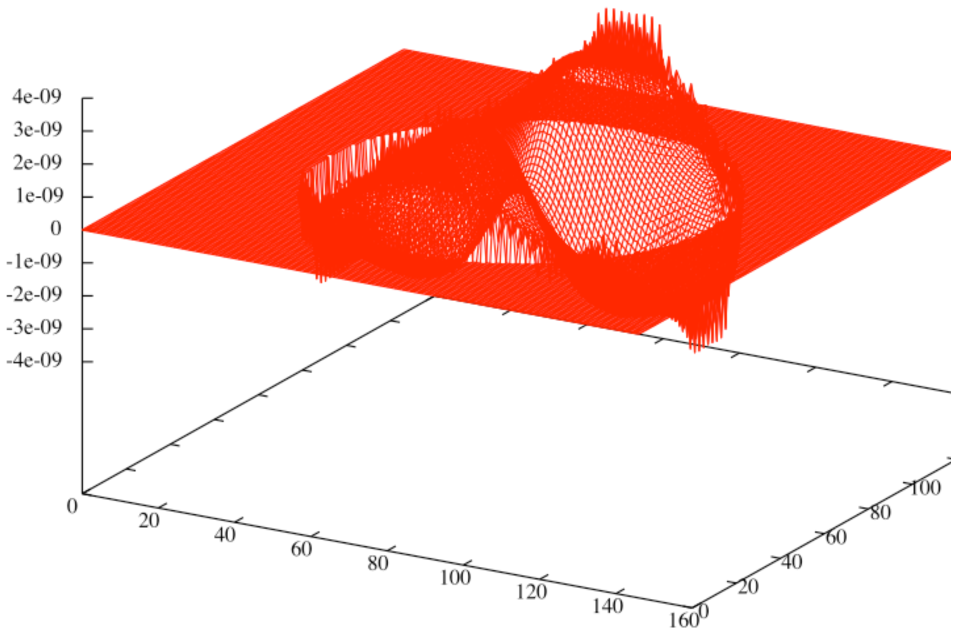
- 2D scalar wave propagation on the sphere:
  $$-\partial_t^2\Phi + \eth\bar{\eth}\Phi = 0, \Phi = \cos(\omega t)Y_{lm}, \omega = \sqrt{l(l+1)}$$
- Convergence rates for errors in $\partial_1\Phi, \partial_2\Phi$ *and* $\partial_3\Phi$.

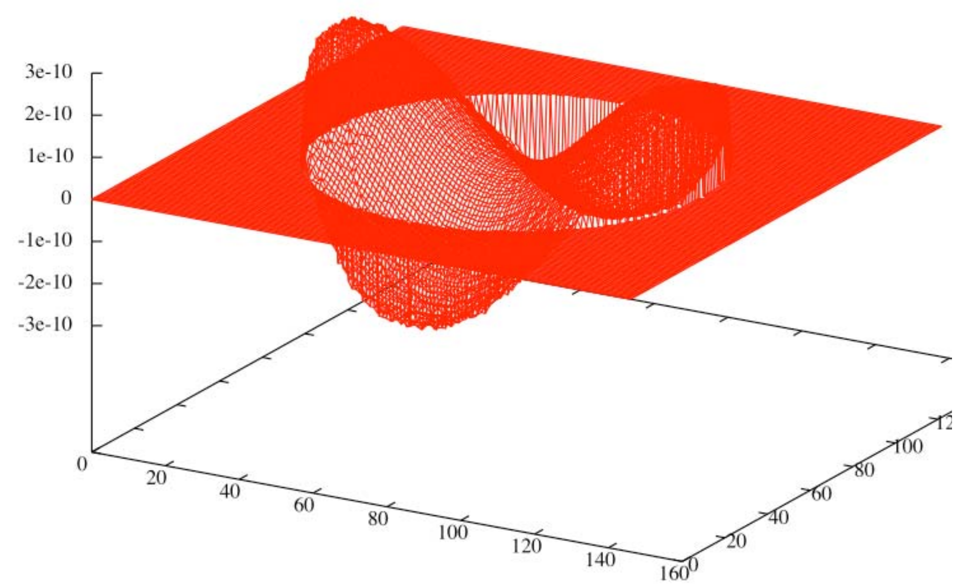
Error	N=80	N=120	N=160	N=200	N=240
$\xi(\partial_1\Phi)$	3.99	4.04	4.11	4.35	4.85
$\xi(\partial_2\Phi)$	3.99	4.07	4.24	4.80	3.95
$\xi(\partial_3\Phi)$	3.94	3.98	3.95	3.92	3.86

News for a Linearized Test

- Bondi News function: $N = N_+ + iN_\times = \partial_t h_+ + \partial_t h_\times$
- Linearized vacuum Bondi-Sachs solutions to Einstein equations on Minkowski background

$$h = \sqrt{(l-1)l(l+1)(l+2)}\,{}_2Y_{lm}\,\mathrm{Re}(h_l(r)e^{i\nu u})$$

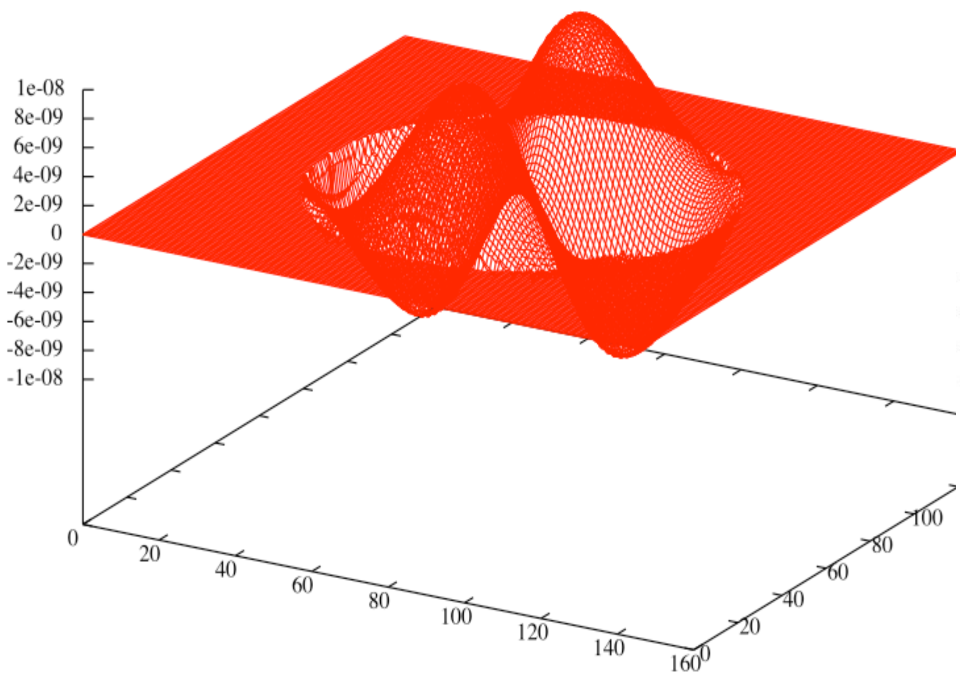


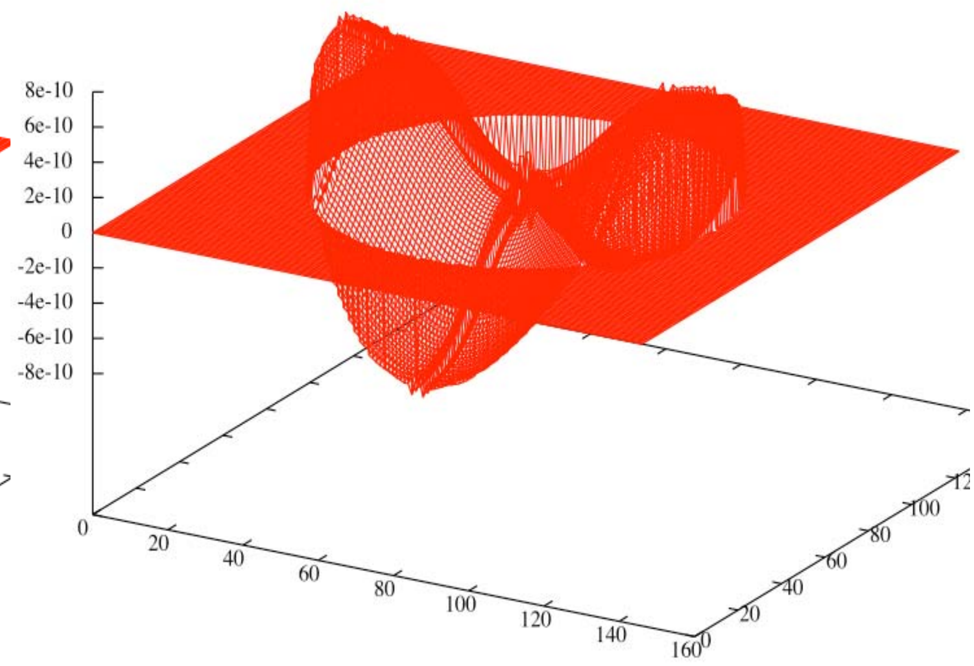


Alternate Method: Ψ_4

- The Newman-Penrose Weyl component ψ_4

$$\Psi_4 = l\frac{\partial N}{\partial u} + O(l^2)$$

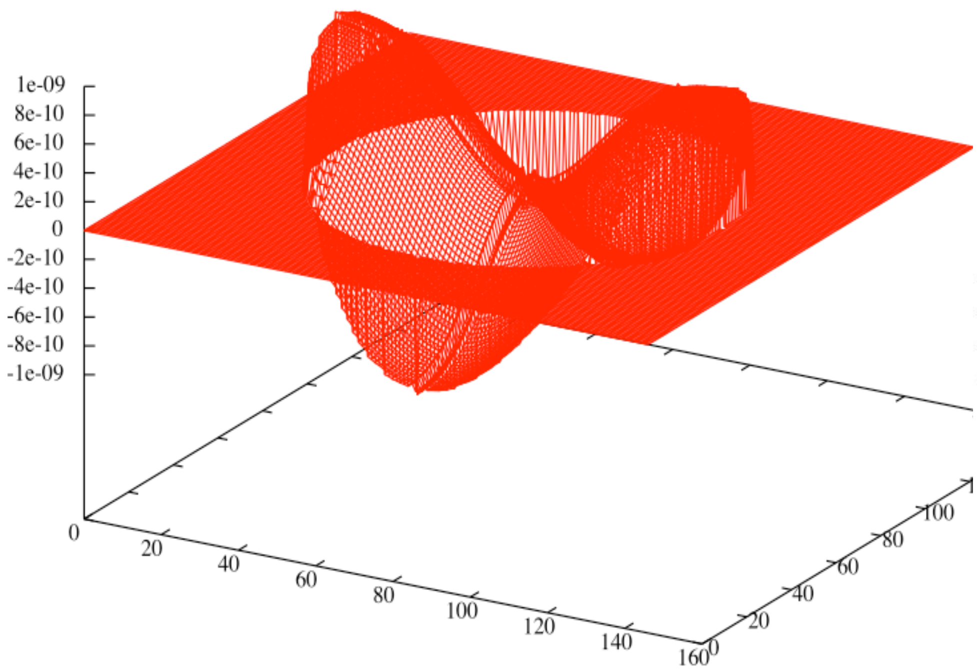


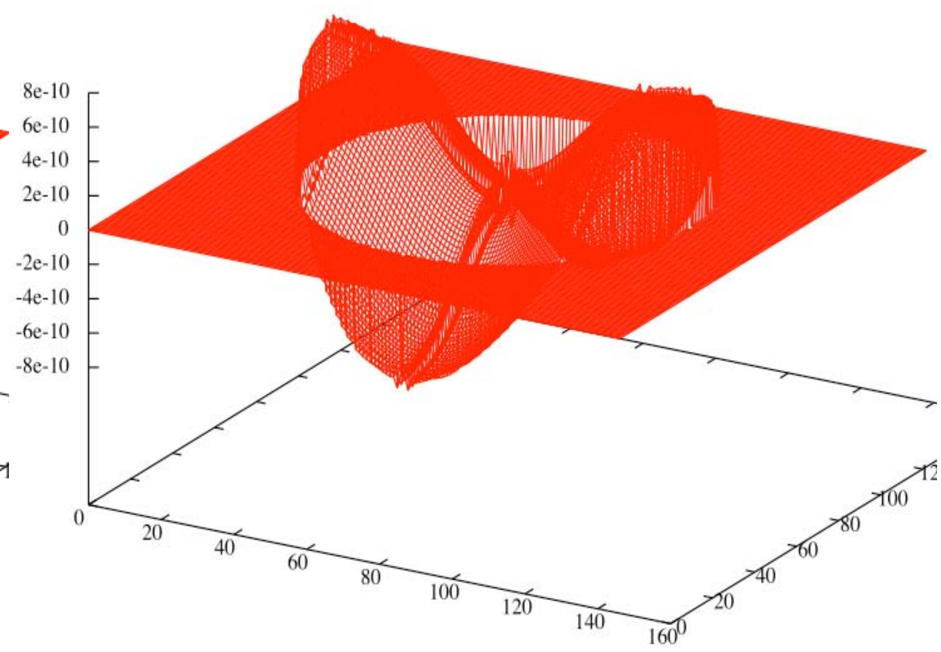


News and Comparison

- Weyl tensor extraction N_ψ is slightly more accurate than the News function extraction *N*.

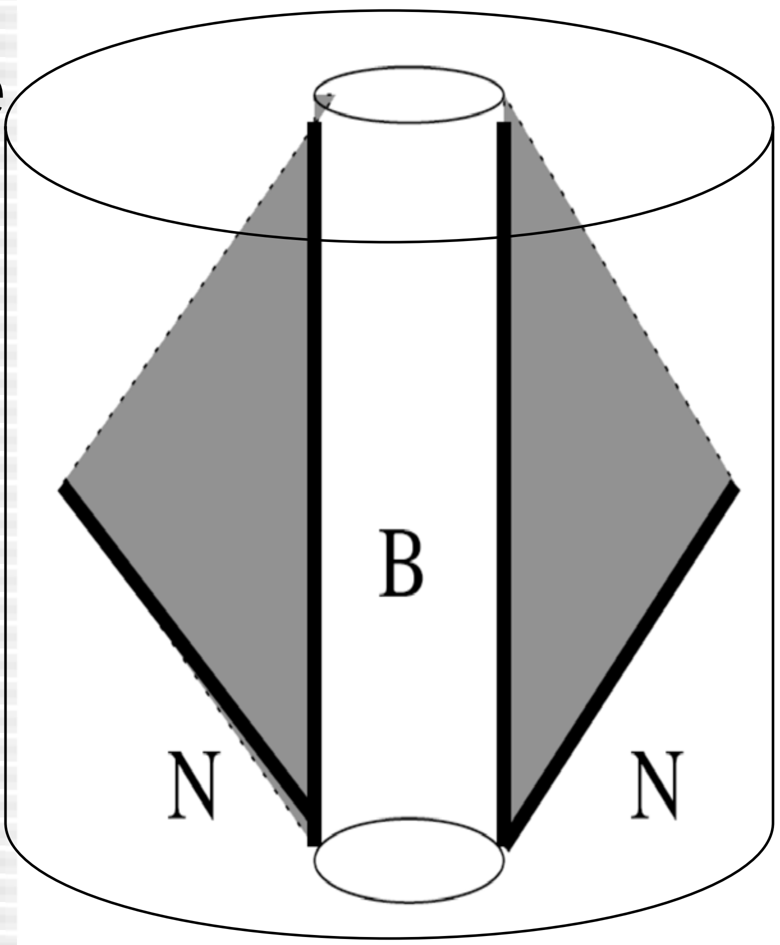
$$\Psi = N_{,u}, \quad N_\Psi = N\big|_{u=0} + \int_0^u \Psi du$$





The Patching Scheme

- Cauchy and characteristic evolution are patched in the vicinity of a worldtube,
- Characteristic data is provided by Cauchy evolution at worldtube B,
- Free initial characteristic data is given on the initial null hypersurface N,
- All is embedded in the Cauchy evolution.



Online Extraction

- Sets the Cartesian coordinates on the sphere on which to extract the Cauchy data,
- Interpolates the Cauchy metric, lapse, shift, and spatial derivatives of the metric on the sphere,
- Calculates the Jacobeans from Cartesian to affine null metric, and from the affine to Bondi metric,
- Calculates the boundary data and puts it on the worldtube with a Taylor expansion to be evolved,
- Advances to the next time level and repeat.

Towards a Versatile Extraction

- CCE does not have to run simultaneously, if the data is given on the world tube, before extracting.
- Steps towards a new IO interface for Extraction:
  - Read from file Cauchy data in Cartesian coordinates between two determined radii, at a chosen resolution,
  - Convert data into a set of analytic functions using a Chebyshev and spherical harmonic decomposition,
  - Take the analytic functions and feed them to the extraction module (no interpolation necessary),
  - Evolve the data and compute the waveform at infinity.

Chebyshev Spherical Harmonic Decomposition

- Normalization for Chebyshev polynomials:

$$\int_{-1}^{+1} wU_m U_n dt = \delta_{mn}\frac{\pi}{2} \Rightarrow f_n = \frac{2}{\pi}\int_{-1}^{+1} FwU_n dt, w = \sqrt{1-t^2}$$

- Change the integration limits, add Y_{lm} and do the coordinate transformation (r,θ,ϕ)->(x,y,z):

$$f_{lmn} = \frac{2}{\pi}\int_{R_1}^{R_2} \frac{dt}{dr}\frac{w}{r^2} Y_{lm}^* U_n F dxdydz$$

- Recover the analytic function:

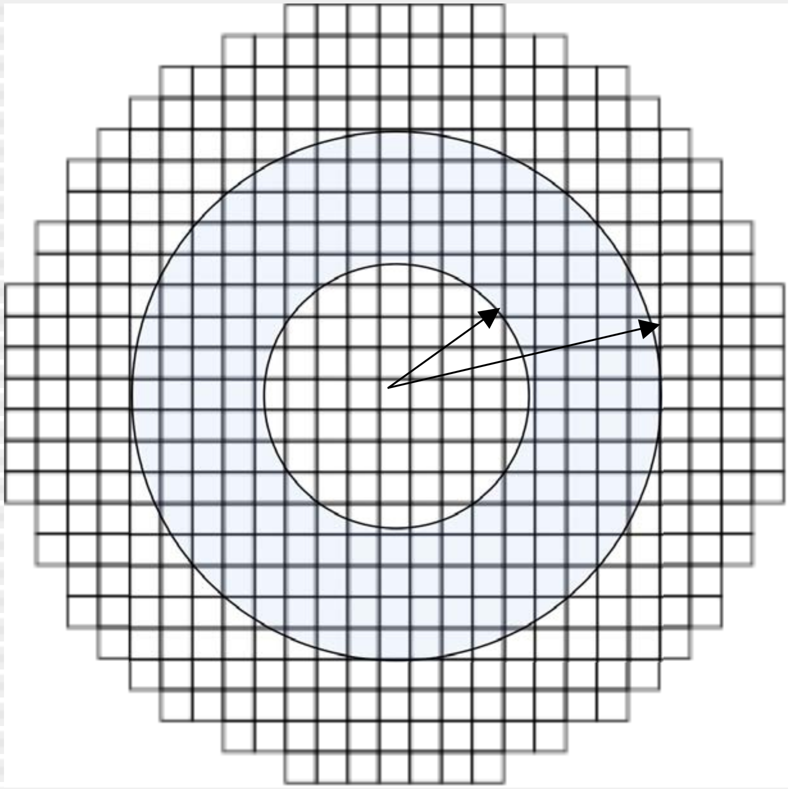
$$F = \sum_l \sum_m \sum_n f_{lmn} Y_{lm} U_n$$

Implementation

- Define a mask between two relevant radii (R_1, R_2)
- Read the functions in (x,y,z) coordinates,
- Compute the expansion coefficients f_{lmn} as a sum over the masked points,

$$f_{lmn} = mask \sum_{i,j,k} F_{lmn}^{i,j,k} \Delta x \Delta y \Delta z$$

- Reconstruct the function,
- Populate the worldtube.



Further Improvements

- Include higher order approximations in the post-processing of the news function
- Improve the characteristic boundary by changing the data on the inner worldtube from Dirichlet to Sommerfeld
- Produce a Cactus CCE module for wave extraction that will be freely available to the numerical relativity community