Constraining a Relativistic Mean-Field Theory with Pure Neutron Matter Calculations

(A Microscopic EoS for Neutron Star Merger Simulations)

Liam Brodie

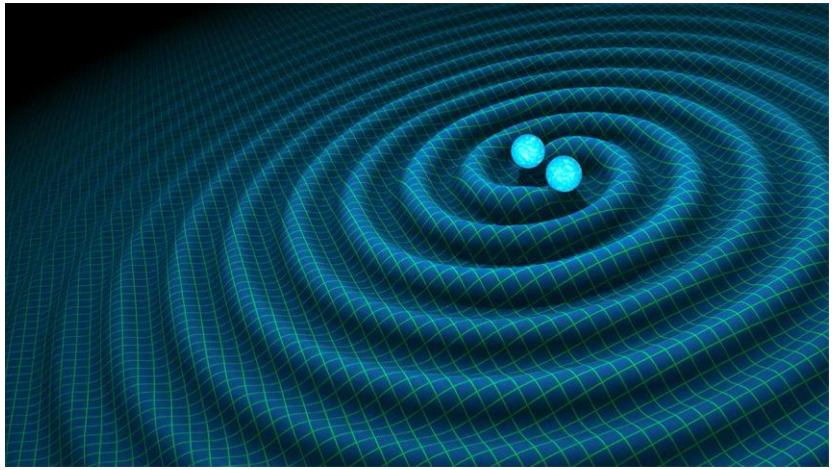
Washington University in St. Louis

Collaboration with: Alex Haber, Mark Alford, Ingo Tews

arXiv: 2205.10283

Why should I pay attention?

- We use a **microscopic model** to study neutron stars
  - Why?
- Model requirements for simulations of **neutron star mergers**:
  - Valid at temperatures between 1 MeV and 100 MeV
  - Valid for a range of proton fractions
  - Provides an equation of state
  - Provides spectrum of low-energy excitations



R. HURT/CALTECH-JPL

Conclusions first!

A **better** microscopic model of nuclear matter that is constrained by

- Pure neutron matter binding energy
- Observations of neutron stars

Ready for use in simulations of neutron star mergers

Microscopic Model Choice

- Quantum Chromodynamics (QCD)?
  - Fundamental theory of the strong interaction ☺
  - Not yet solvable at relevant densities ☹

- Settle for **approximate** methods
  - Chiral Effective Field Theory (ChiEFT)?
    - Controlled approximation of QCD ☺
    - Only valid at low densities ☹
  - Relativistic mean-field theory (RMFT)
    - Protons and neutrons interact through background meson fields

Relativistic Mean-Field Theory (RMFT)

Advantages	**Limitations**
Relativistic theory (e.g., $v_{sound} < c$) ☺	Coupling constants need to be fit to something ☺☹
Tractable calculations ☺	Not a controlled approximation ☹
Useable in the relevant range of temperatures and proton fractions ☺	Valid to about 6 times nuclear saturation density ☺ ☹
Low-energy excitations ☺	No phase transition to deconfined quarks ☹

How to Constrain this Model?

- **Coupling constants** not derived from QCD
  - Need to constrain six parameters in the model

- Constraints:
  - Data derived from nuclear physics around 0.5-2 times nuclear saturation density

  - Mass and radius measurements of neutron stars

How to Constrain this Model?

- **Coupling constants** not derived from QCD
  - Need to constrain six parameters in the model

- Constraints:
  - Data derived from nuclear physics around 0.5-2 times nuclear saturation density
  - Mass and radius measurements of neutron stars

Nuclear Physics Data to Constrain Our Model

- Typically, microscopic models fit to nearly **symmetric** (50% protons 50% neutrons) nuclear matter properties
- **But** neutron stars are $\sim\mathbf{90}\%$ neutrons
- Cannot probe **neutron rich** matter experimentally
  - <u>Chiral effective field theory</u>: binding energy for pure (**100%**) neutron matter
- Fit relativistic mean-field theory to **symmetric** <u>and</u> **pure neutron** matter

Chiral Effective Field Theory

- Based on the symmetries of QCD with nucleon and pion degrees of freedom

- Controlled approximation to QCD valid at low densities

- Theory fitted to data from scattering experiments

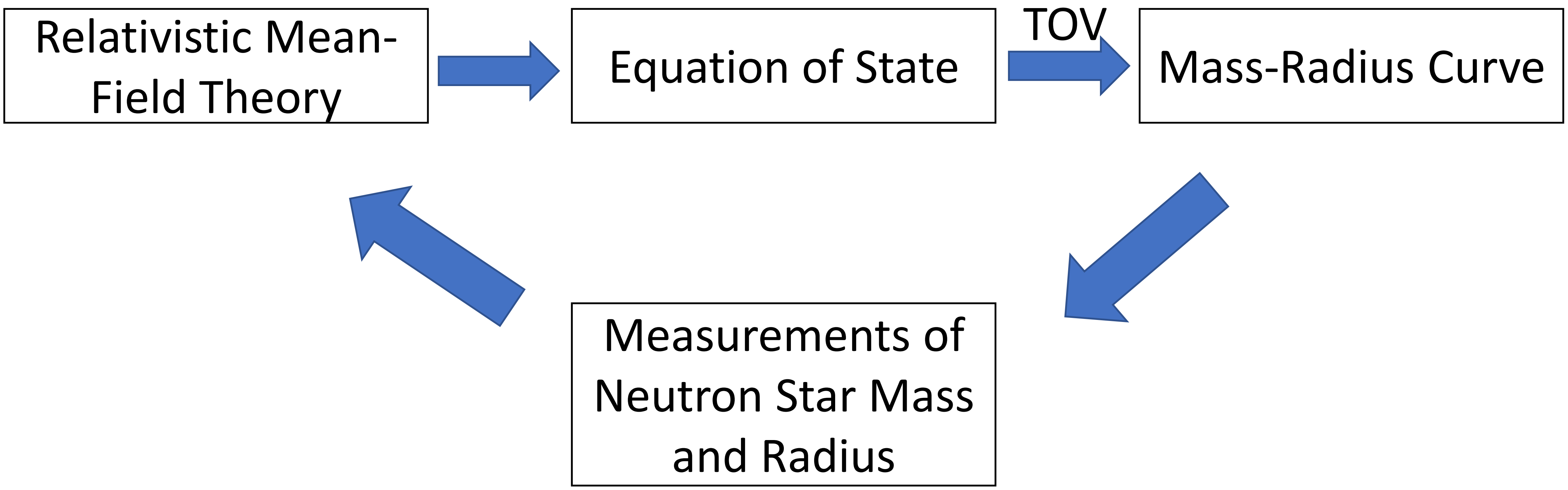
- Able to calculate the low-density binding energy for pure neutron matter. Use that data to **constrain** the relativistic mean-field theory

How to Constrain this Model?

- **Coupling constants** not derived from QCD
  - Need to constrain six parameters in the model

- Constraints:
  - Data derived from nuclear physics around 0.5-2 times nuclear saturation density
  - Mass and radius measurements of neutron stars

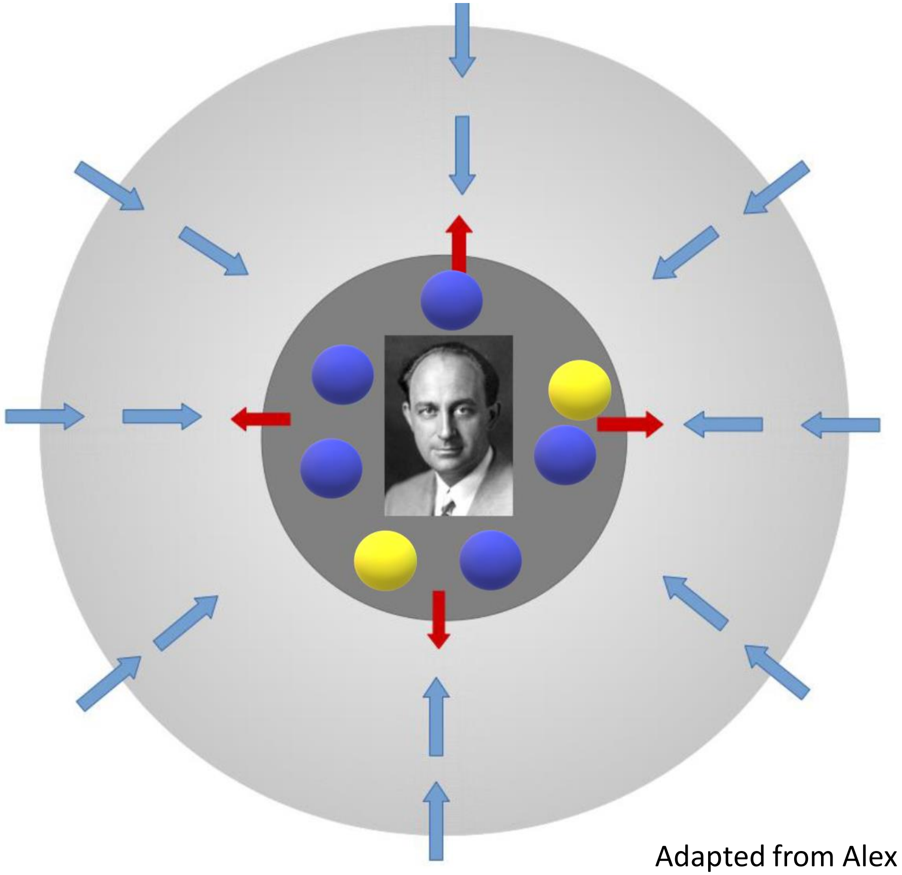
How can neutron stars constrain our model?



Equation of State

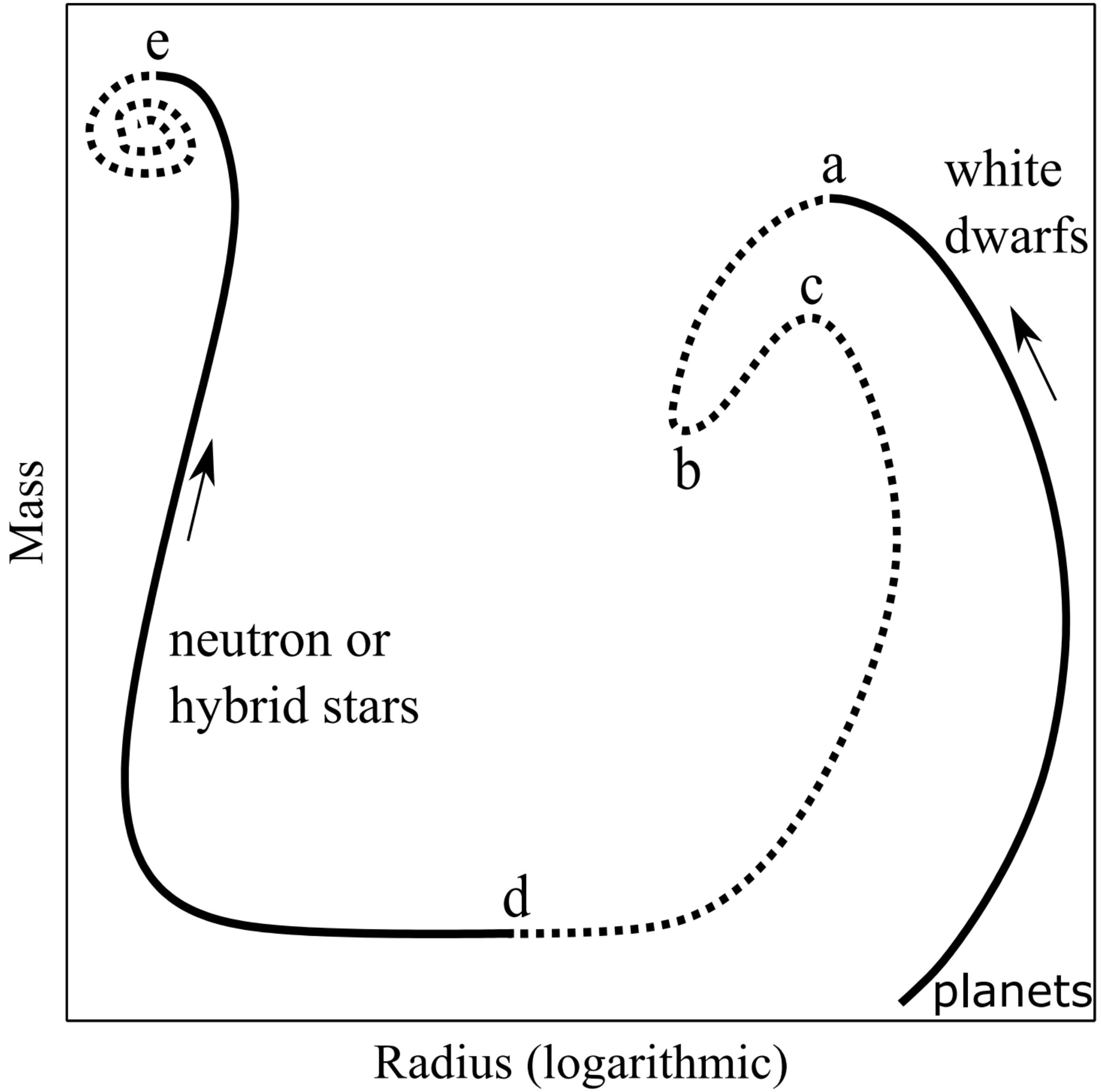
- Amount of pressure needed to keep matter at a fixed density
  - How "squishy" matter is

- Neutron stars
  - Natural sample of neutron-rich matter
  - Gravity acts as a **piston**



Adapted from Alex Haber

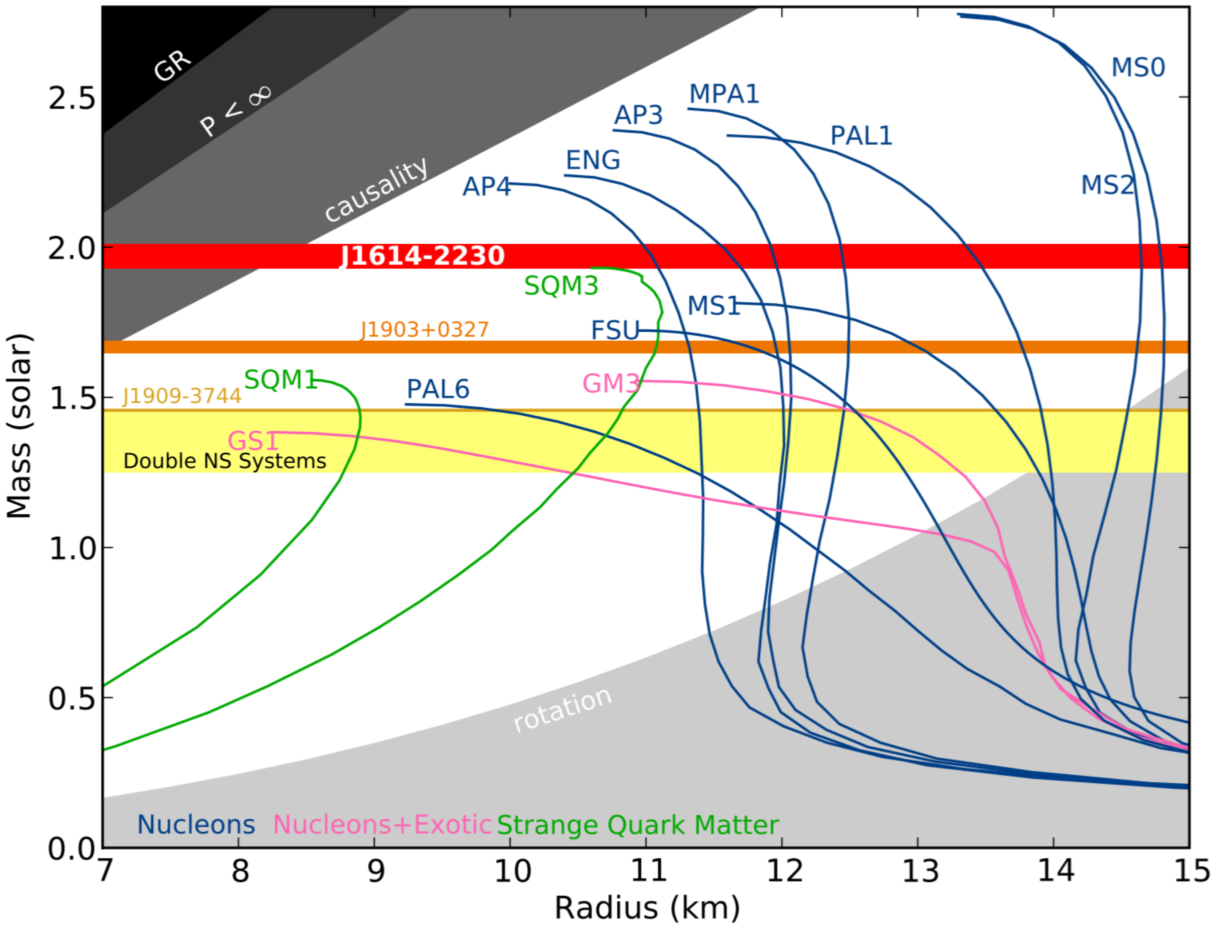
Mass Radius Curves



Alford et al. (arXiv:1705.09880)

Neutron Star Observations as Constraints

- Many models **discarded** after recent neutron star mass measurement

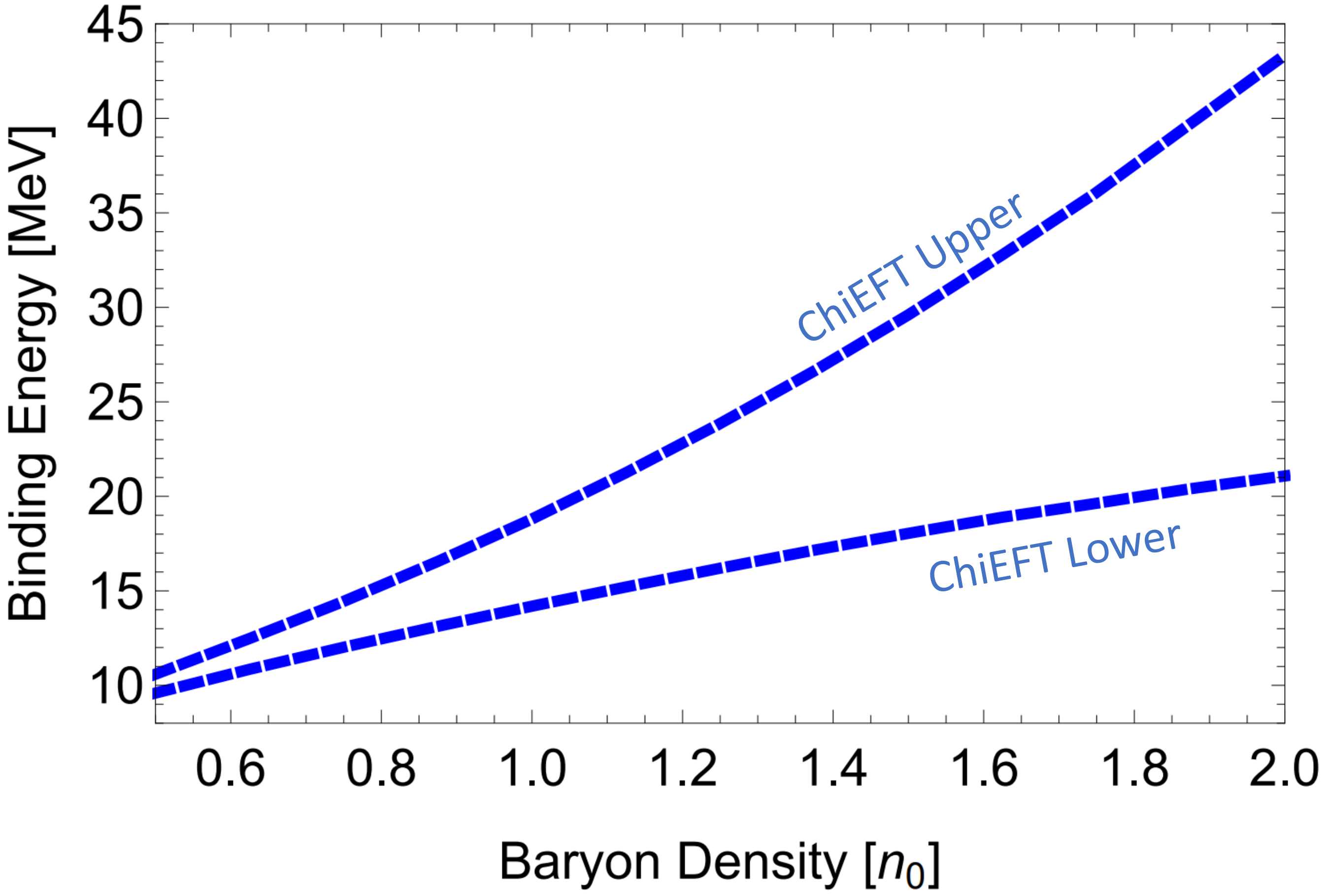


arXiv:1010.5788

How do we actually constrain our model?

Uncertainty in Chiral Effective Field Theory

- No specific curve to fit ➔Parameterize the binding energy per nucleon uncertainty



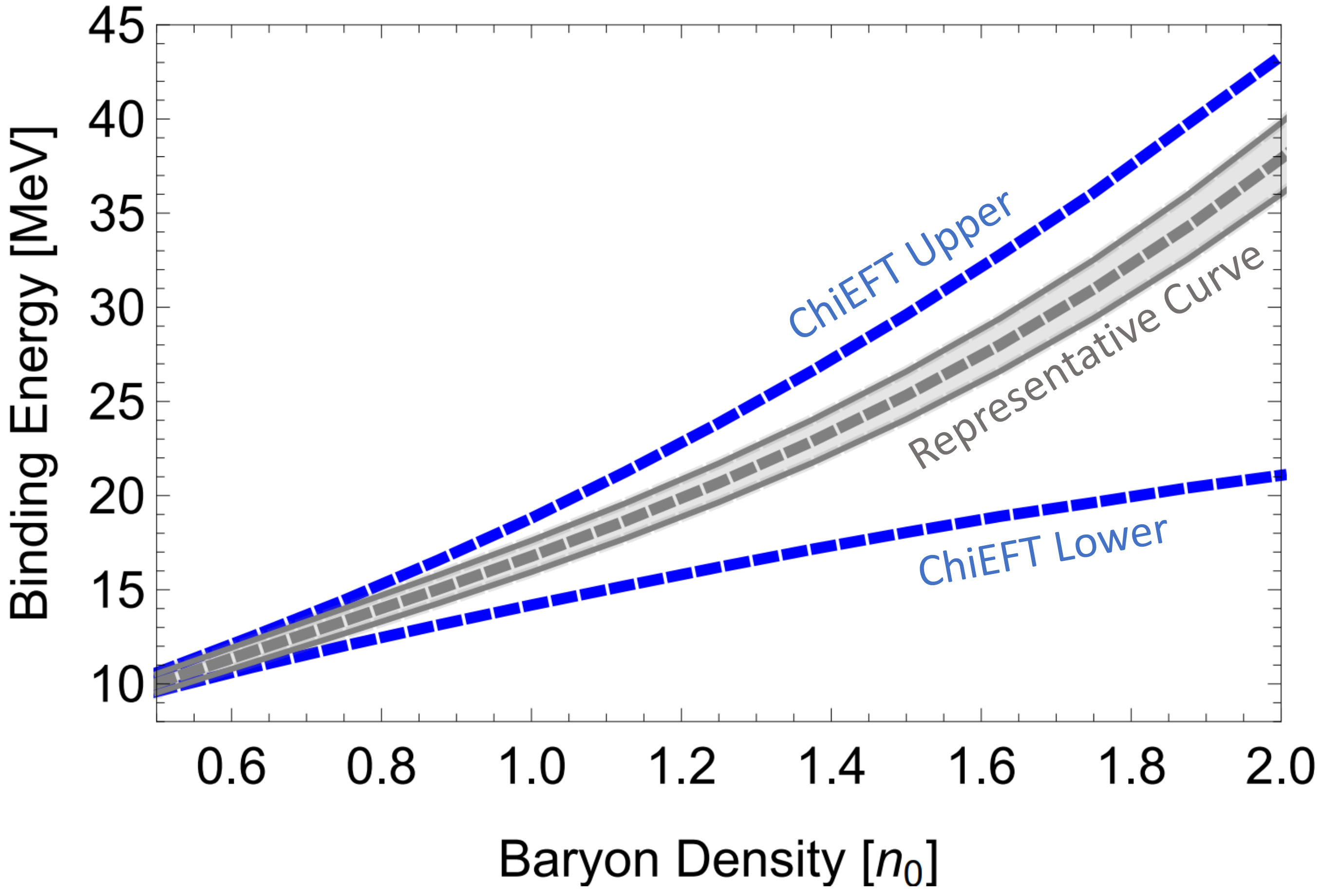
ChiEFT data from Tews et al. (arXiv:1801.01923)

Uncertainty in Chiral Effective Field Theory

- No specific curve to fit ➔ Parameterize the binding energy per nucleon uncertainty

$$\varepsilon(n_B) = a\, n_B{}^{\alpha} + b\, n_B{}^{\beta}$$

Gandolfi et al. (arXiv:1101.1921)



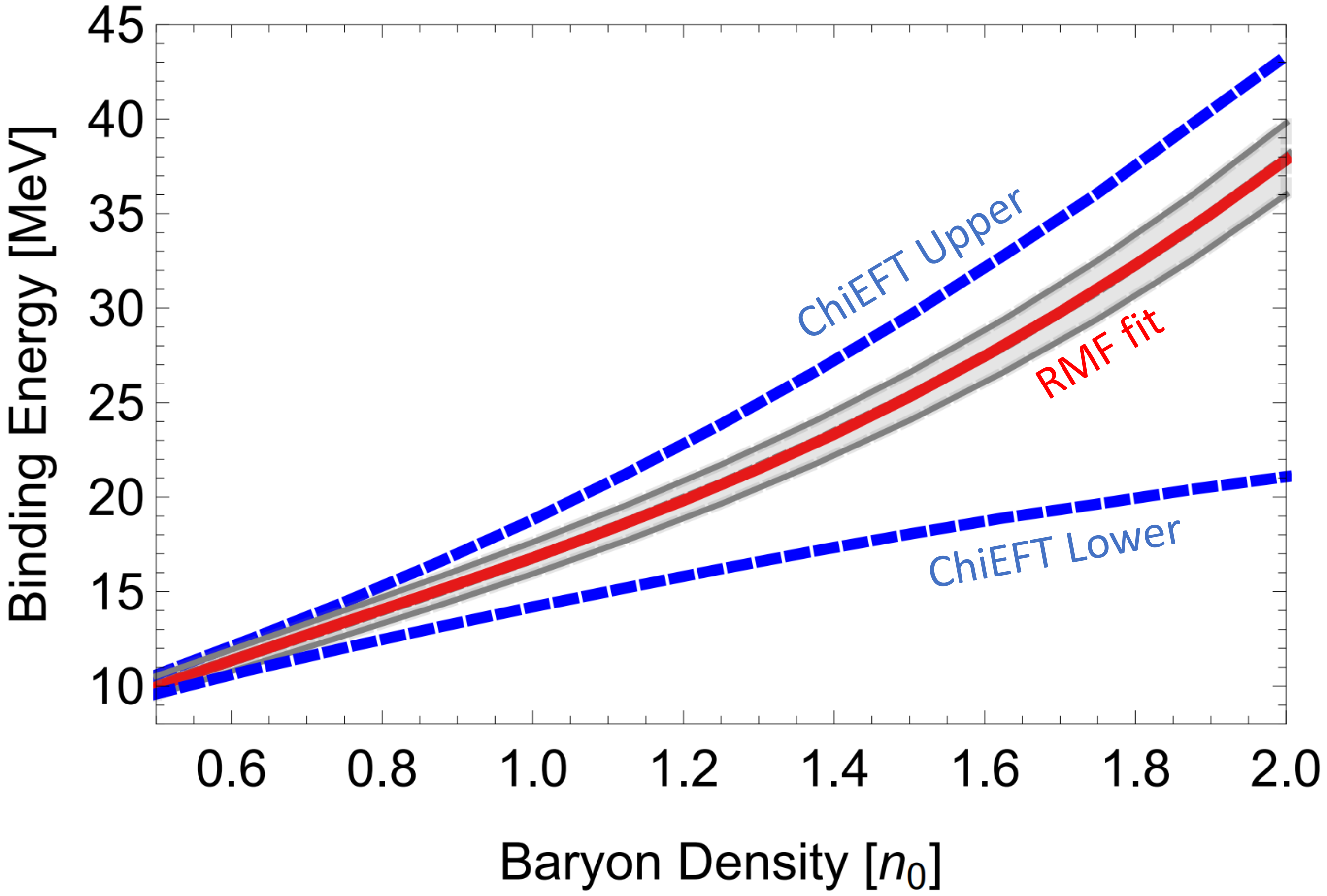
ChiEFT data from Tews et al. (arXiv:1801.01923)

Uncertainty in Chiral Effective Field Theory

- No specific curve to fit ➔Parameterize the binding energy per nucleon uncertainty

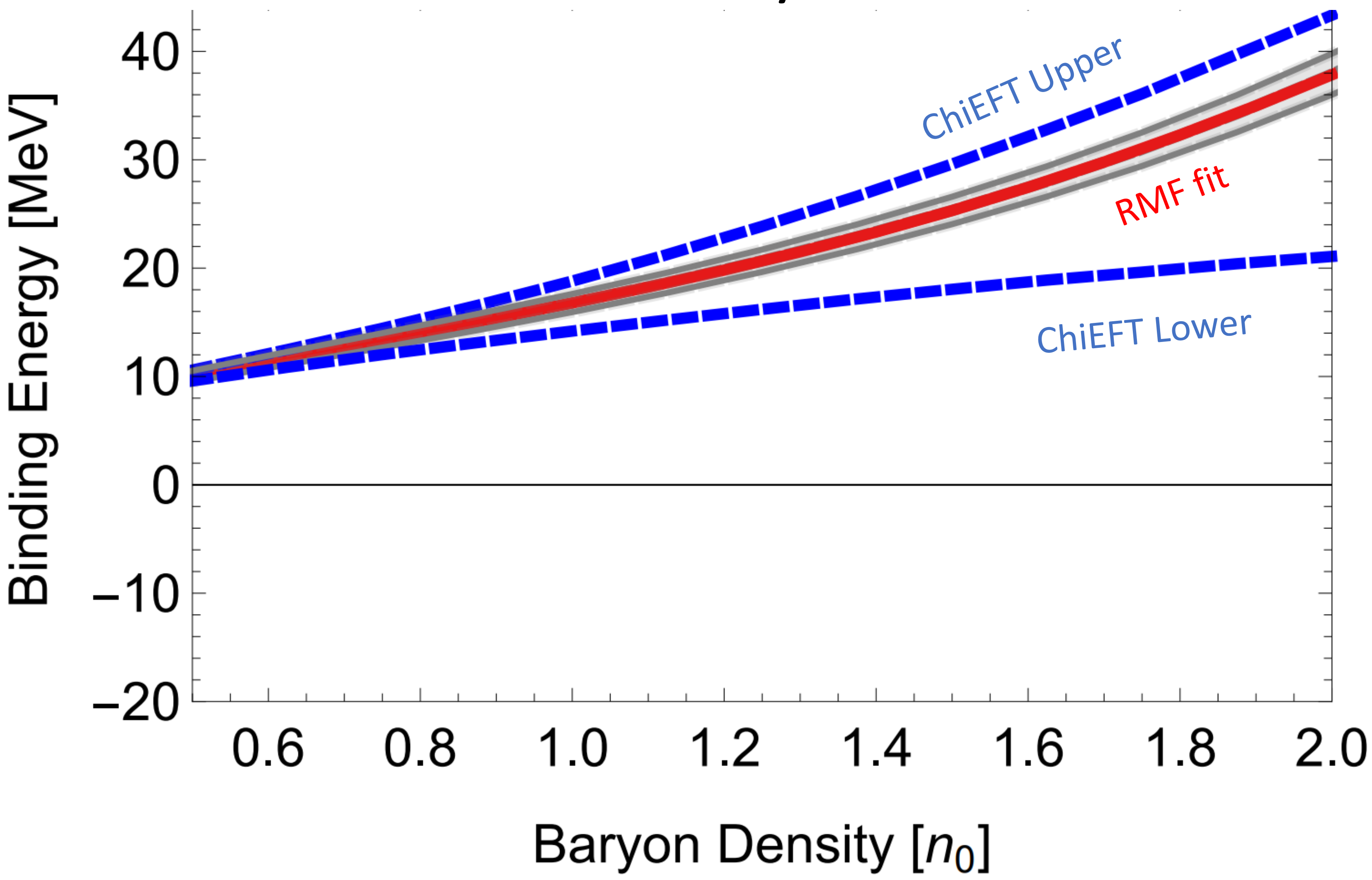
$$\varepsilon(n_B) = a\, n_B{}^{\alpha} + b\, n_B{}^{\beta}$$

Gandolfi et al. (arXiv:1101.1921)

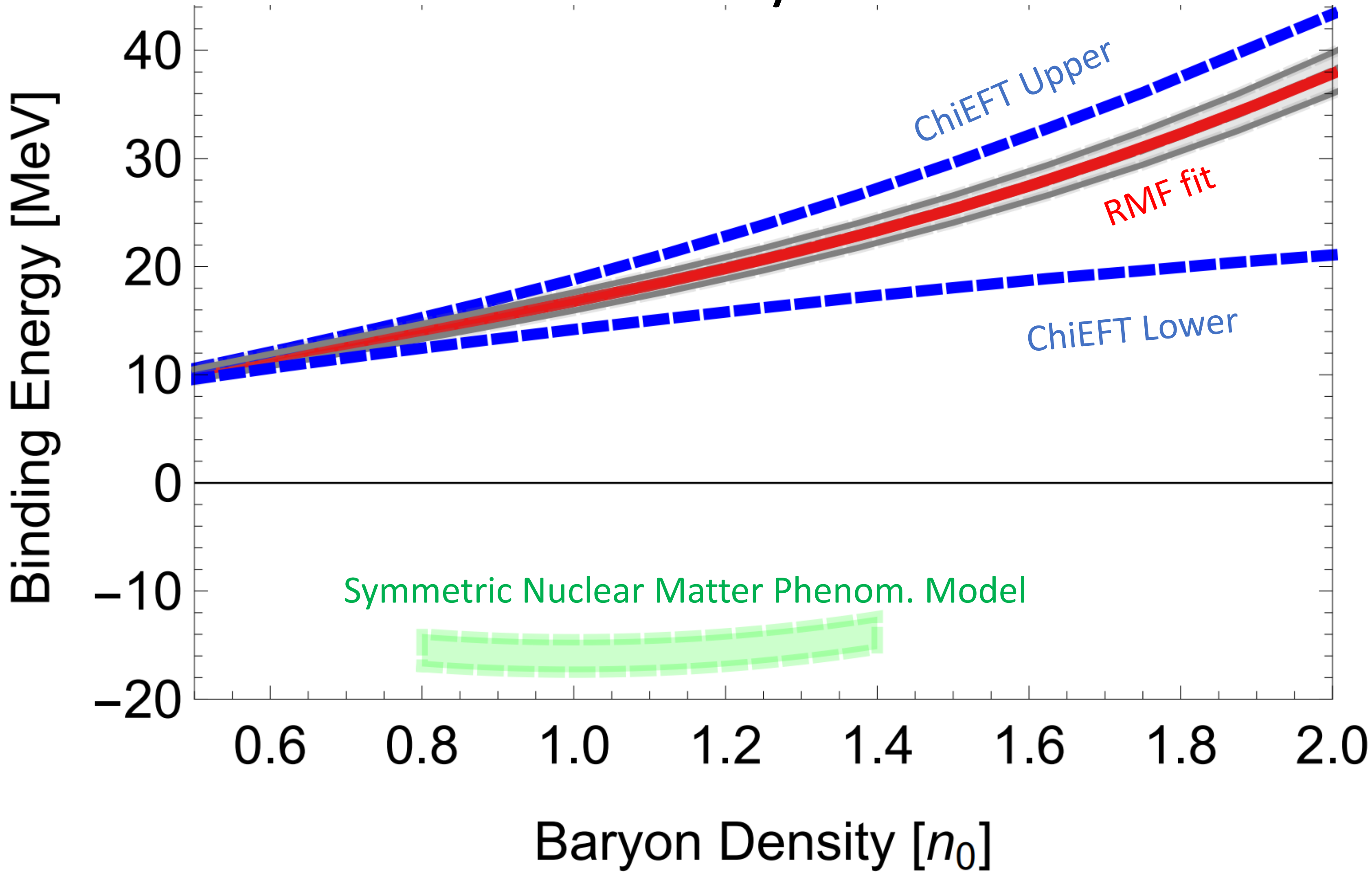


ChiEFT data from Tews et al. (arXiv:1801.01923)

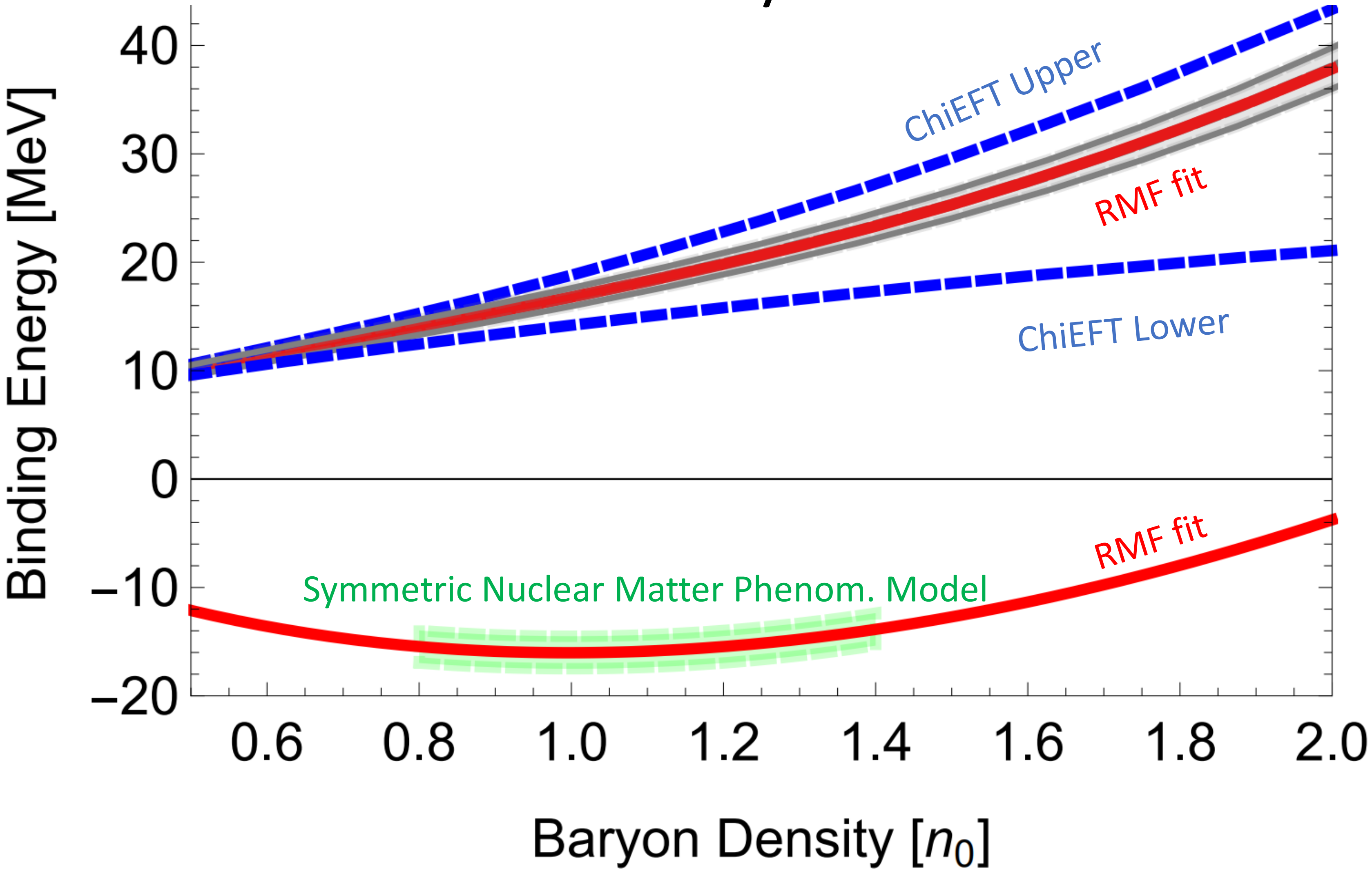
Simultaneous Nuclear Physics Constraints



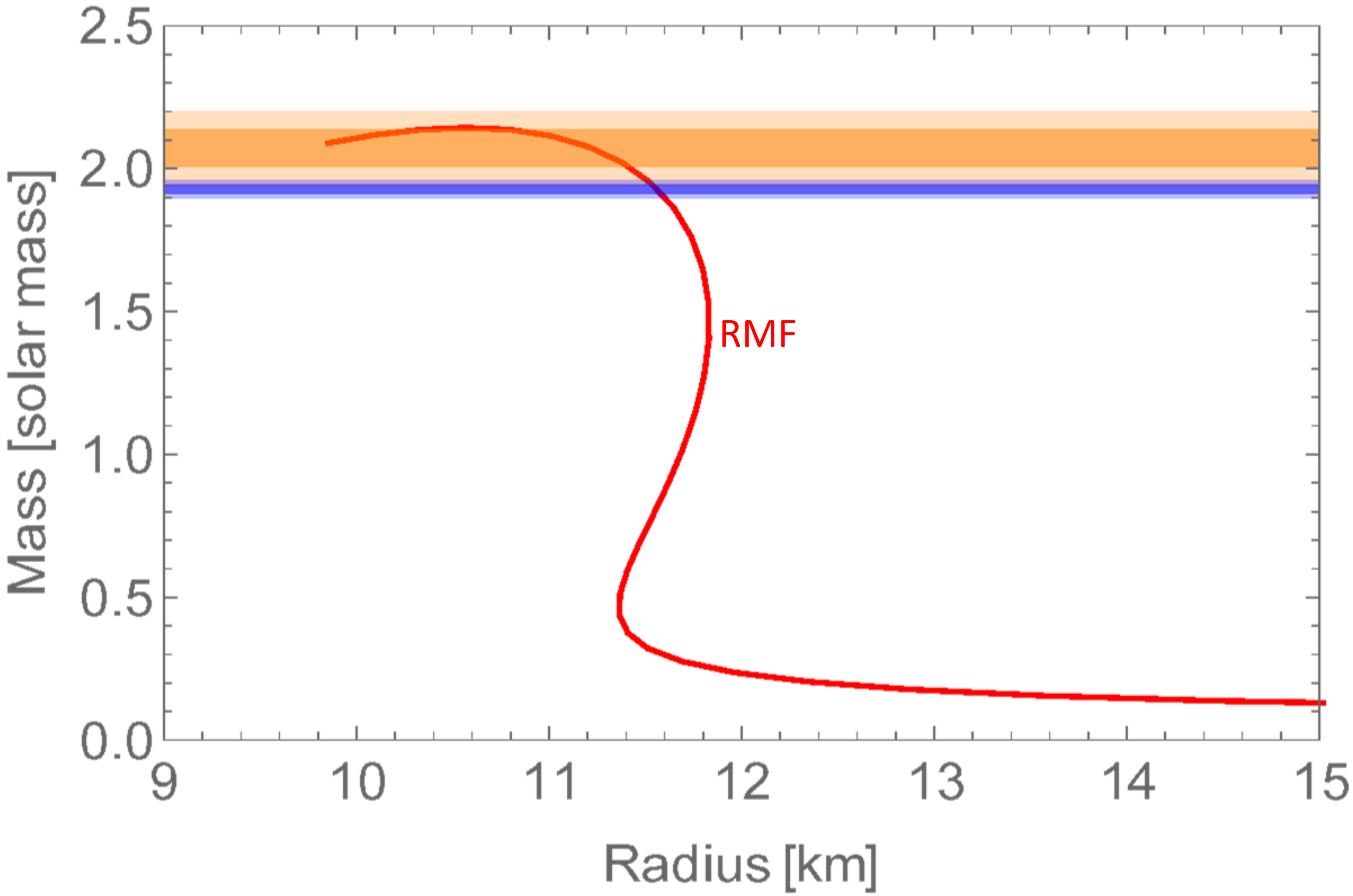
Simultaneous Nuclear Physics Constraints



Simultaneous Nuclear Physics Constraints

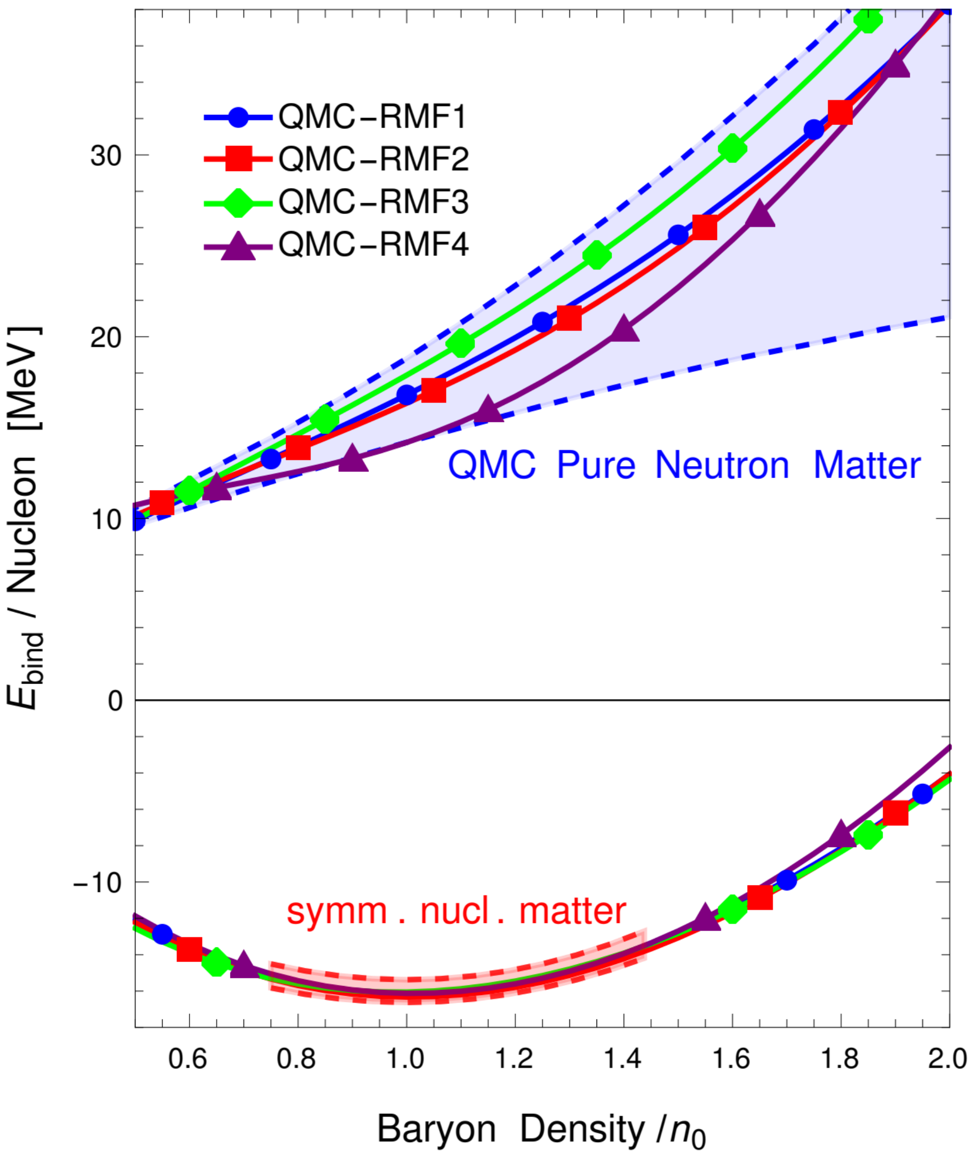


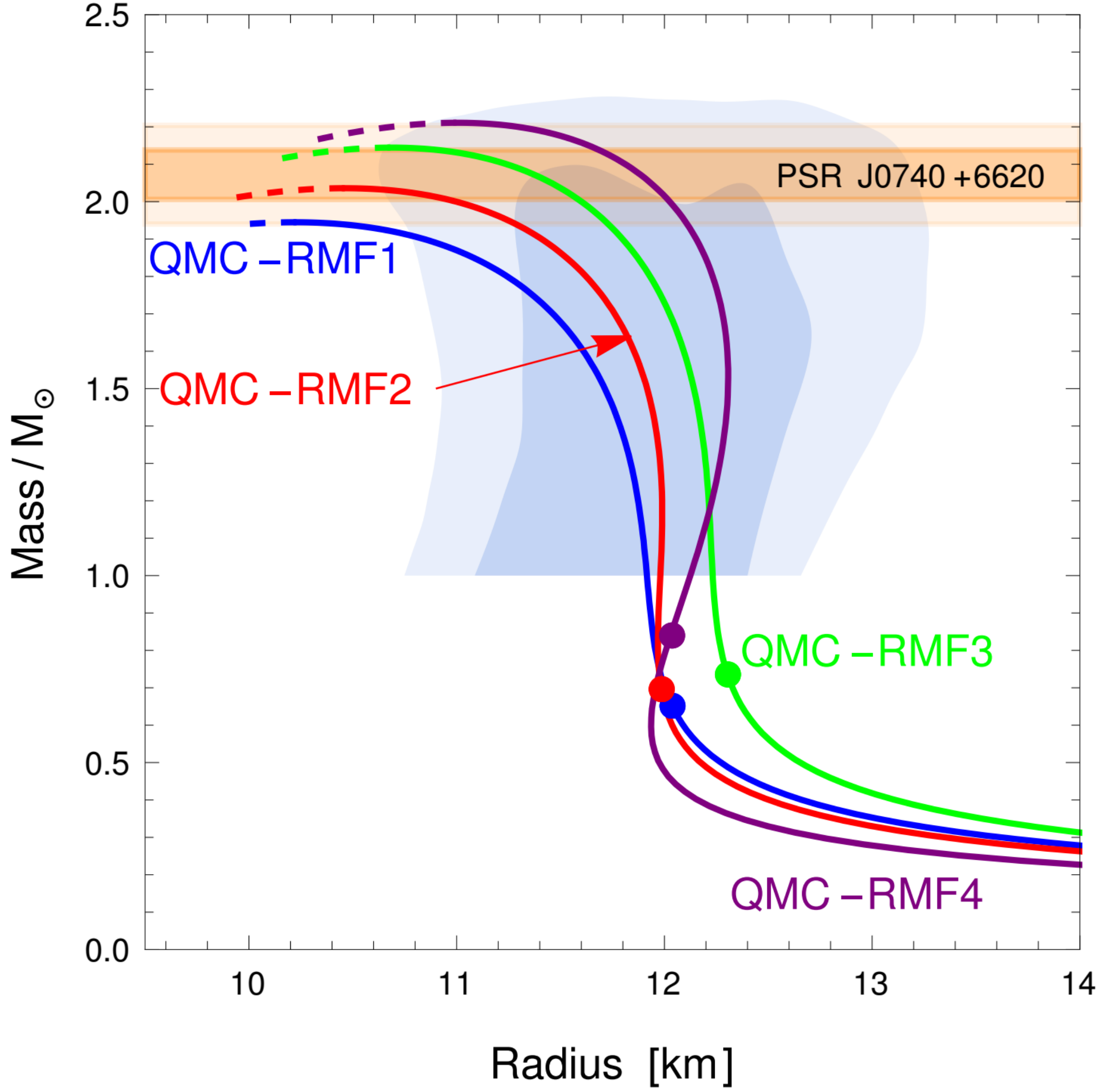
Resulting Mass-Radius Curve From Fit



PSR J0740+6620 (NICER XMM) [Orange]

PSR J1614-2230 (Nanograv) [Blue]





Max. mass constraint: arXiv 2105.06980

Radius constraint: arXiv 2105.06979

Conclusion

Developed a microscopic model of nuclear matter that is constrained by low-density symmetric **and** pure neutron matter and by observations of neutron stars

- Applicable at a range of **temperatures** and **proton fractions**
- Provides spectrum of **low-energy excitations**
- Provides an **equation of state**
- Ready for use in **neutron star merger simulations**

Extra: RMF Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{N}} + \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{l} \,,$$

$$\mathcal{L}_N = \bar{\psi}(i\gamma^\mu \partial_\mu - m_N + g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - \frac{g_\rho}{2} \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu \gamma^\mu)\psi \,,$$

$$\begin{aligned}\mathcal{L}_M =& \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{bM}{3}(g_\sigma \sigma)^3 - \frac{c}{4}(g_\sigma \sigma)^4 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{\zeta}{24} g_\omega^4 (\omega_\mu \omega^\mu)^2 \\ & - \frac{1}{4}\boldsymbol{B}_{\mu\nu} \cdot \boldsymbol{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \frac{\xi}{24} g_\rho^4 (\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu)^2 + g_\rho^2 \Big[\sum_{i=1}^{6} a_i \sigma^i + \sum_{j=1}^{3} b_j (\omega_\mu \omega^\mu)^j \Big] \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \,,\end{aligned}$$

$$\mathcal{L}_l = \bar{\psi}_e \left(i\gamma^\mu \partial_\mu - m_e \right) \psi_e \,,$$

Why Study Neutron Stars?