Bending and Curls:

Investigating Some Classic Problems in General Relativity and Electromagnetism

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Wells College, October 6, 2017

Overview

- **Relativistic Hydrodynamics – a history**
- **Research project #1: Light bending by black holes**
  - Work w/Nate Barlow (SMS), Steve Weinstein (ChemE), Ryne Beachley (SMS Undergrad)
  - Barlow, Weinstein, JF CQG 34, 135017 (2017)
- **Research Project #2: Magnetic fields and Vector Potentials**
  - Work w/Zach Silberman (AST PhD grad student), Z Etienne & I. Ruchlin (WVU)
- **The future: Multimessenger Astrophysics**
- **Bonus: Promotional Material**

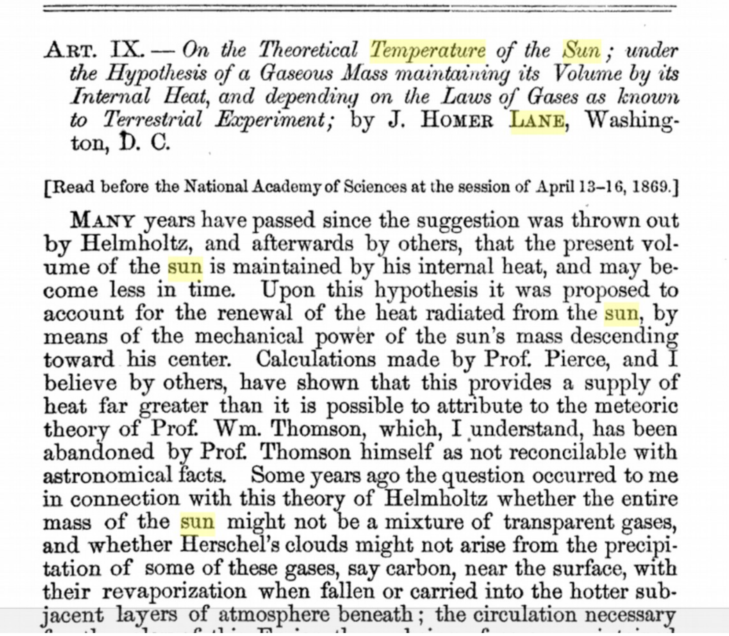
A brief history of numerical relativity

People have been simulating astrophysical objects for longer than there have been computers.

- e.g. The Lane-Emden equations describing stellar structure date back to 1870.

Computers opened up the world of more complex, multidimensional simulations.

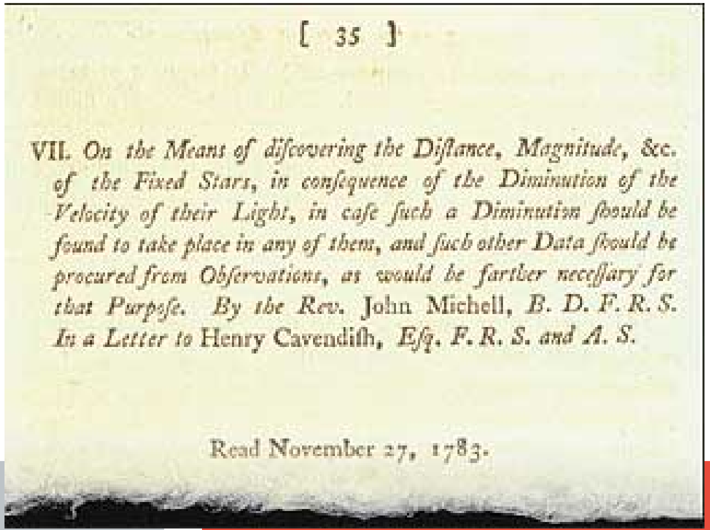
Up until the 1990s, even simulations of black holes and neutron stars were typically done in Newtonian, "quasi-relativistic", or background general relativistic (GR) gravity



ART. IX. — *On the Theoretical Temperature of the Sun; under the Hypothesis of a Gaseous Mass maintaining its Volume by its Internal Heat, and depending on the Laws of Gases as known to Terrestrial Experiment;* by J. HOMER LANE, Washington, D. C.

[Read before the National Academy of Sciences at the session of April 13–16, 1869.]

MANY years have passed since the suggestion was thrown out by Helmholtz, and afterwards by others, that the present volume of the sun is maintained by his internal heat, and may become less in time. Upon this hypothesis it was proposed to account for the renewal of the heat radiated from the sun, by means of the mechanical power of the sun's mass descending toward his center. Calculations made by Prof. Pierce, and I believe by others, have shown that this provides a supply of heat far greater than it is possible to attribute to the meteoric theory of Prof. Wm. Thomson, which, I understand, has been abandoned by Prof. Thomson himself as not reconcilable with astronomical facts. Some years ago the question occurred to me in connection with this theory of Helmholtz whether the entire mass of the sun might not be a mixture of transparent gases, and whether Herschel's clouds might not arise from the precipitation of some of these gases, say carbon, near the surface, with their revaporization when fallen or carried into the hotter subjacent layers of atmosphere beneath; the circulation necessary



[35]

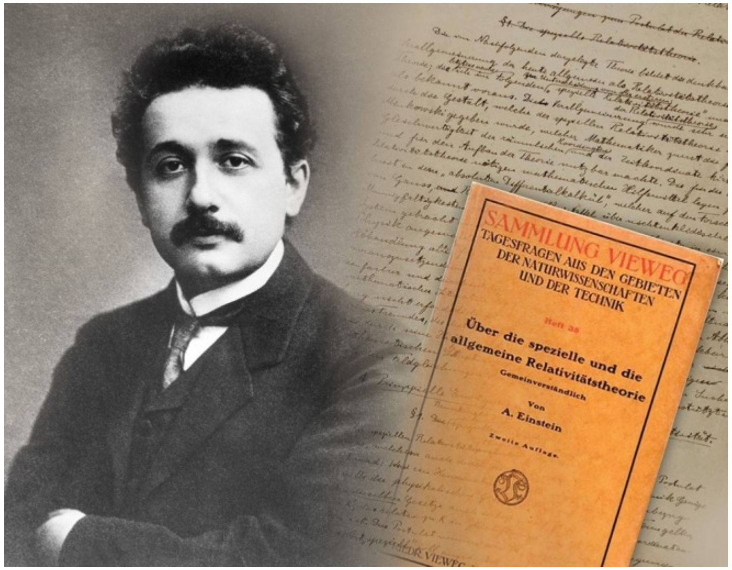
VII. *On the Means of diſcovering the Diſtance, Magnitude, &c. of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

Read November 27, 1783.

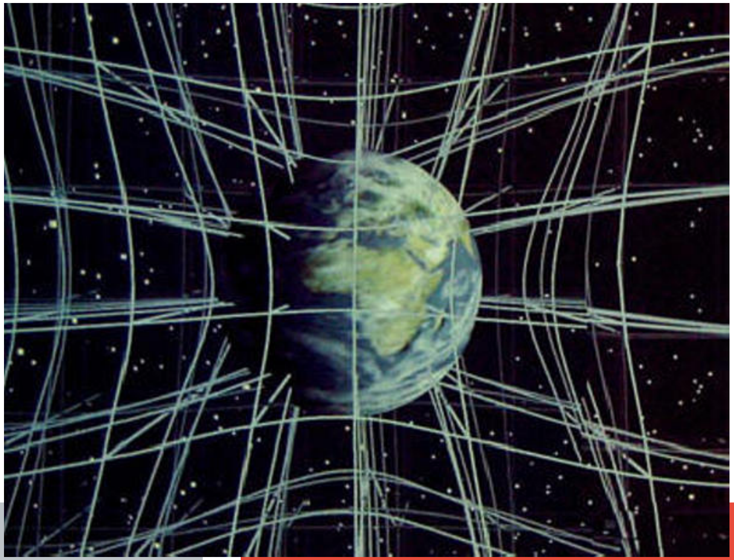
Top: Lane (1870); Bottom: Michell (1783)

General Relativity in one slide

Einstein's General theory of relativity was published in 1915. It is one of two fundamental scientific theorems which we currently use to understand the universe – QCD is the other.



Space is curved by matter, and objects trying to move in straight lines on the curved background seem to bend toward masses → gravity!

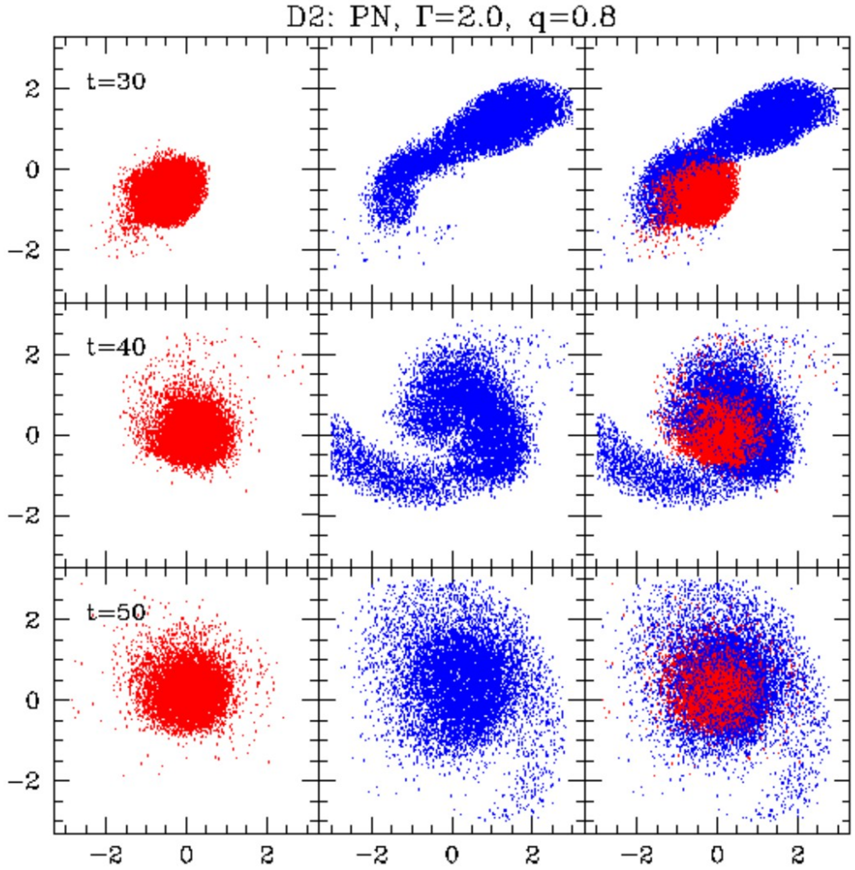


The "metric" is the 4x4 object describing how distances between points can be measured.

"Matter tells space how to curve; curvature tells matter how to move." – John Wheeler

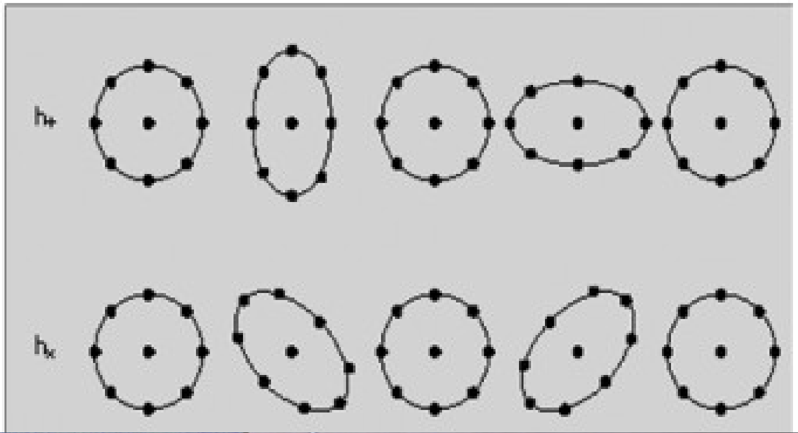
Relativistic hydrodynamics – early days

- **Beginning in the 1990s, simulations of neutron stars using post-Newtonian and approximate relativistic metrics began to appear more widely**
  - The gravitational background evolves following the fluid, but not self-consistently
- **Increasing interest in gravitational waves (GW) due to LIGO motivated work into the nature of GW signatures and what they could tell us about nuclear physics**
- **Rapidly increasing computer power played an important role in driving results forward**



Faber & Rasio 2002

Gravitational Waves & LIGO in one slide



Gravitational waves are predicted by GR

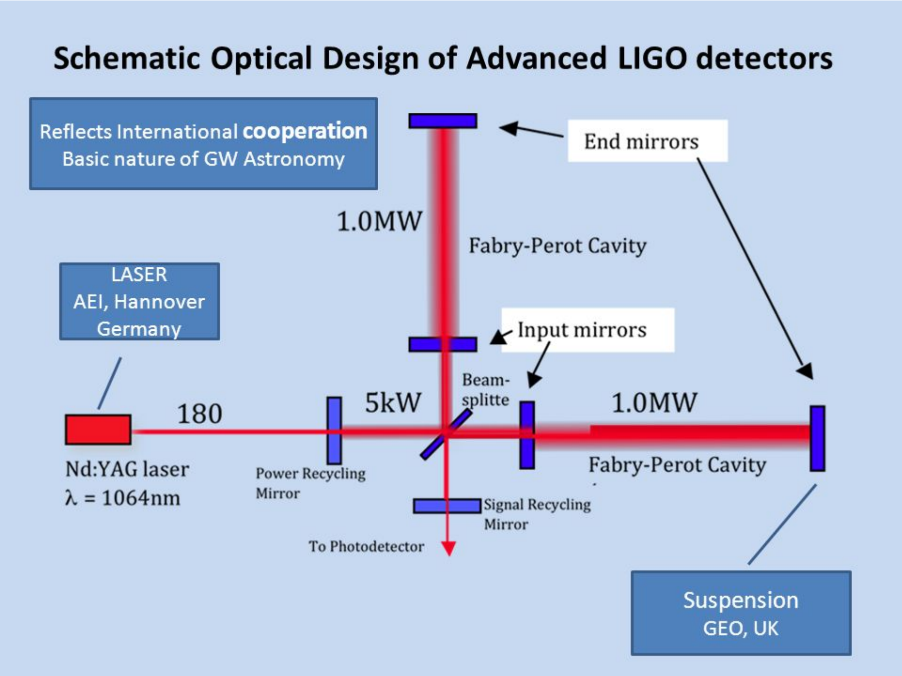
Accelerating matter produces “ripples” in spacetime – distances between points oscillate

- Accelerating charges give of cyclotron/synchrotron radiation, by a similar process

A stretch in one direction leads to a compression perpendicularly

Interferometers are a natural choice to look for these

Distance change is <1/100 times the radius of a proton over 4km.



Schematic Optical Design of Advanced LIGO detectors

Reflects International **cooperation**
Basic nature of GW Astronomy

End mirrors

1.0MW

Fabry-Perot Cavity

LASER
AEI, Hannover
Germany

Input mirrors

Beam-
splitte

180

5kW

1.0MW

Nd:YAG laser
λ = 1064nm

Power Recycling
Mirror

Fabry-Perot Cavity

Signal Recycling
Mirror

To Photodetector

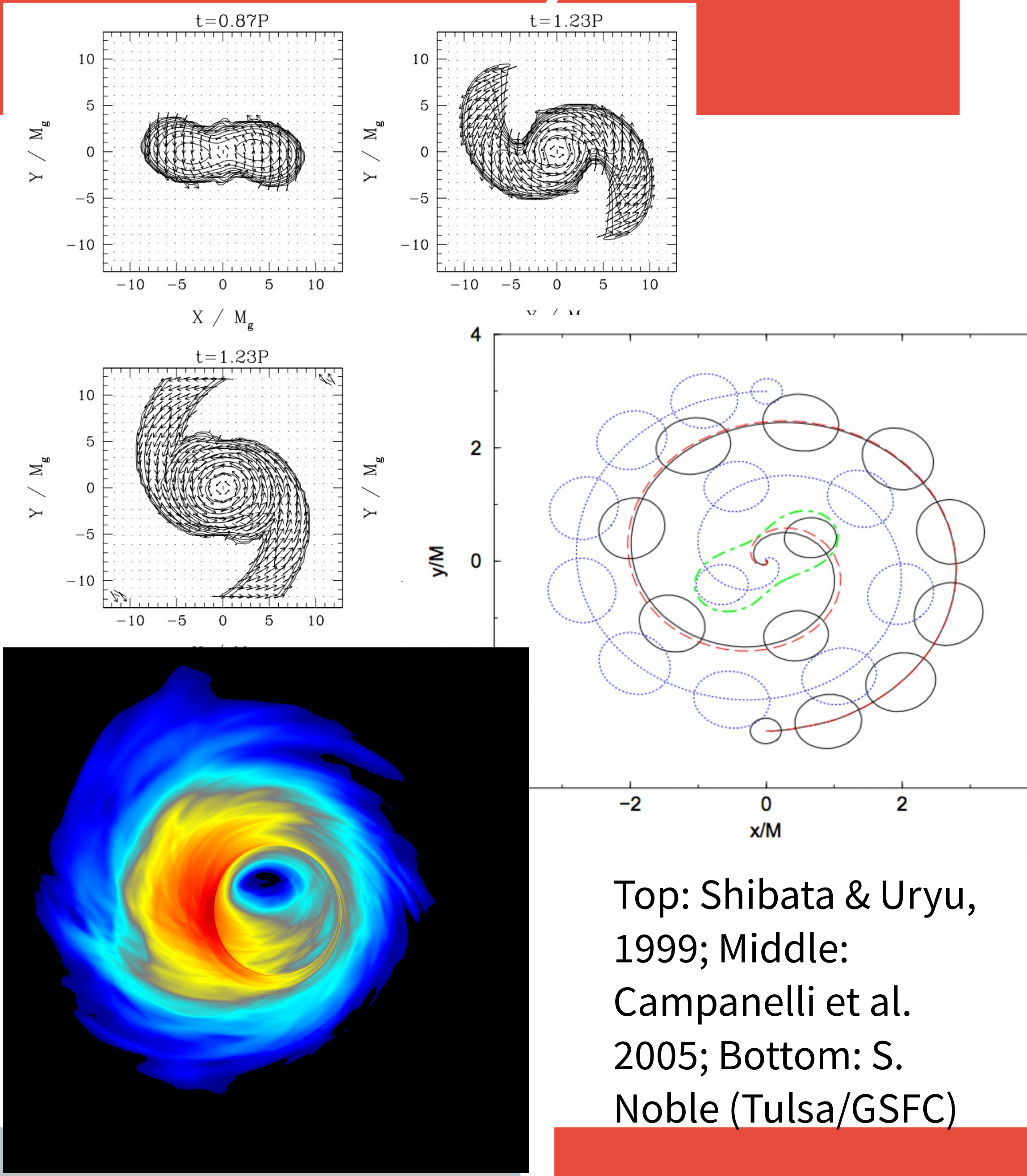
Suspension
GEO, UK

Relativistic Hydrodynamics – recent years

In the 2000s, we figured out how to evolve the spacetime metric around gravitating objects stably in full General Relativity

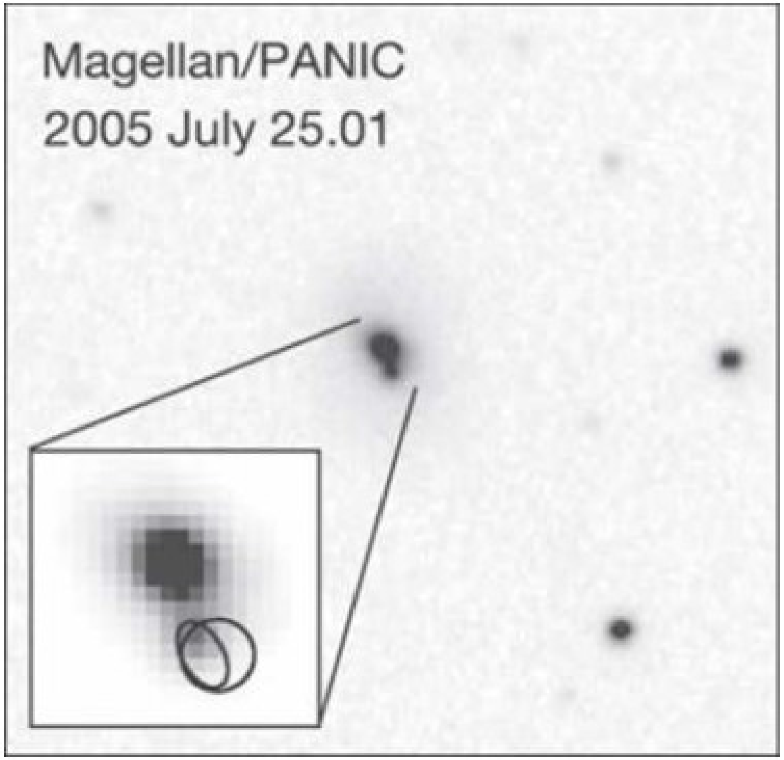
Much of the discussion focuses on black holes, but the same techniques cure the same problems for neutron stars

Today, we can evolve neutron stars, black holes, accretion disks, etc.



Top: Shibata & Uryu, 1999; Middle: Campanelli et al. 2005; Bottom: S. Noble (Tulsa/GSFC)

Current challenges



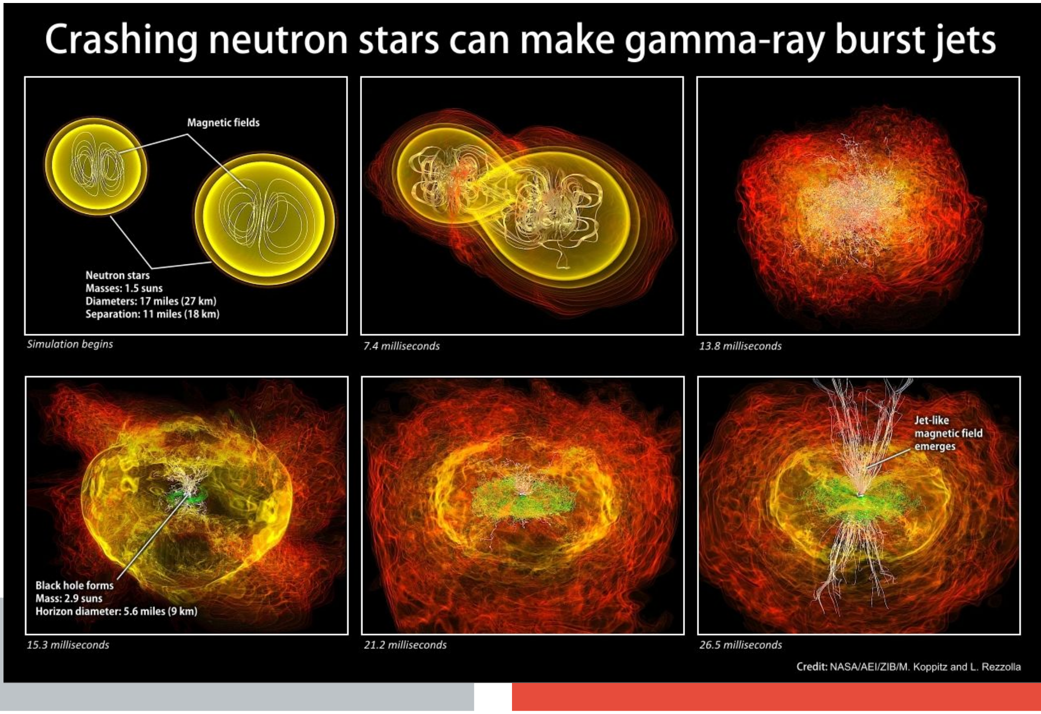
Multimessenger astronomy – GWs predictions are "easy" to predict, as they depend only on the large-scale "bulk" motion of a fluid object.

Electromagnetic (EM) predictions are difficult – they depend on nuclear physics, thermodynamics, magnetic fields, etc. operating on many different scales

- We know astrophysical jets exist, and have a pretty good idea why they do, but no first principles numerical model for them.

Multiscale/multidimensional physics – Some physical problems are difficult to handle numerically, and problems with multiple scales even more so.

- e.g. radiative transfer – photons have intensities per frequency interval in different directions at different positions in time



Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

Top: Berger: 2005; Bottom: Caltech

Overview: Light bending by black holes

Black holes are responsible for the vast majority of the high-energy EM signals (x-rays and gamma rays) that astronomers observe.

We don’t see the black holes themselves, or the matter inside. Instead, we see matter about to fall into black holes, as it heats up and starts moving at speeds approaching the speed of light.

Gravity bends the light, delays it, redshifts it, etc.

Same process as gravitational lensing, but potentially much stronger effects.

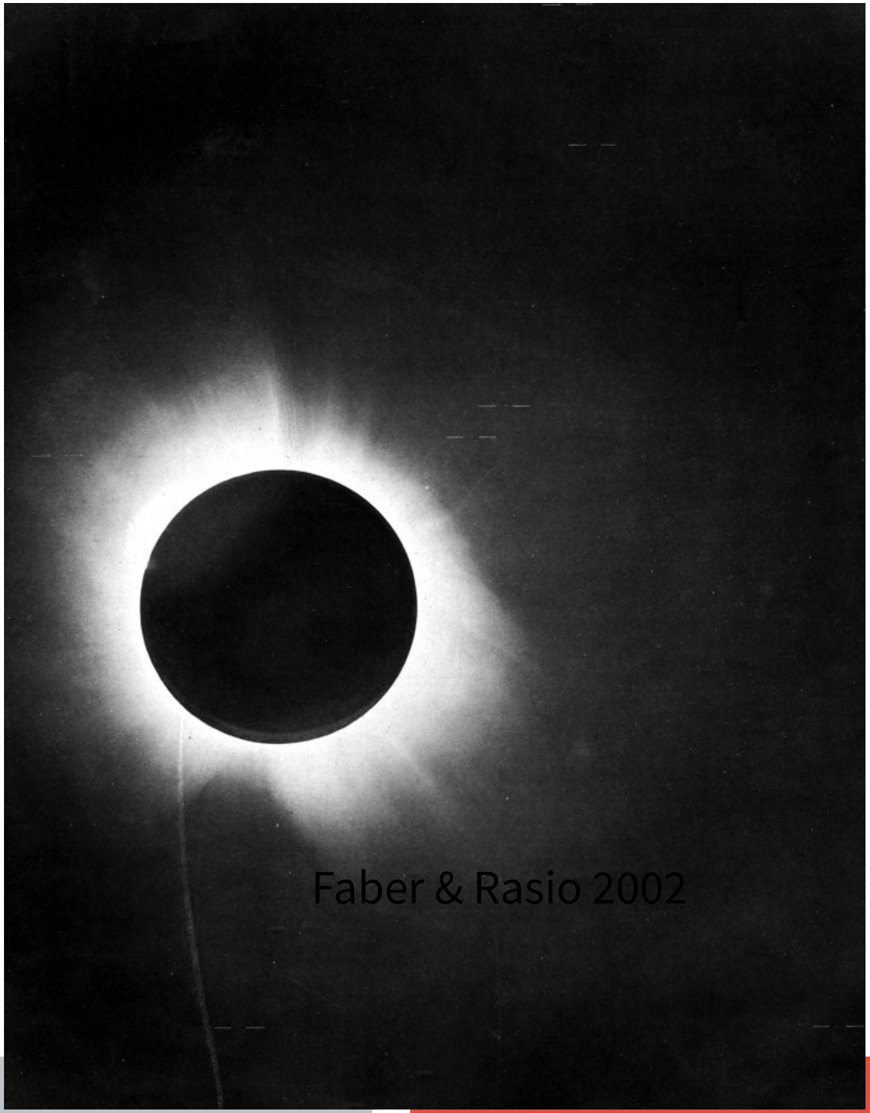
Bottom: HST

Overview: Light bending by black holes

In order to work out the EM emission from an accretion disk around a black hole, we need to work out which directions emitted photons end up going

One of the first tests of GR was that gravity bends light, as measured for stars near the sun during the 1919 eclipse

This was a weak-field test. The sun is NOT very relativistic. Bending angles are small.



Faber & Rasio 2002

Eddington 1919

Strong-field limit

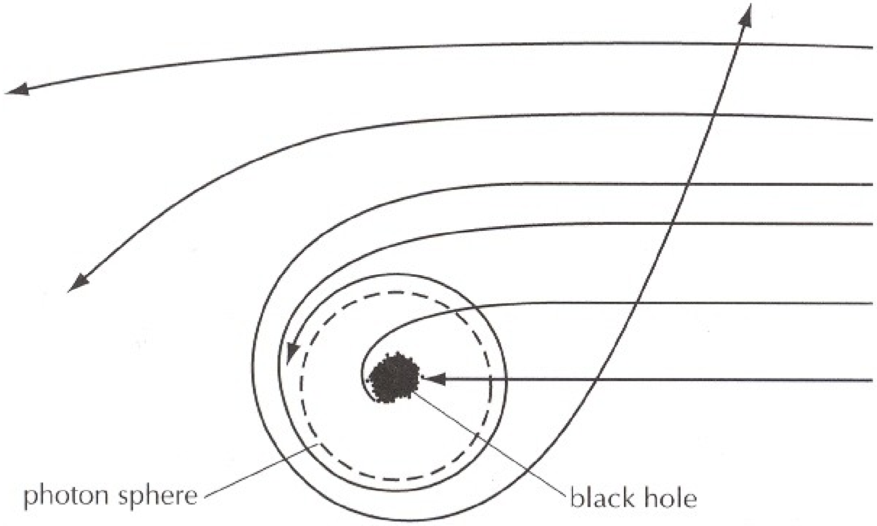
The strong-field limit involves light passing very near a black hole.

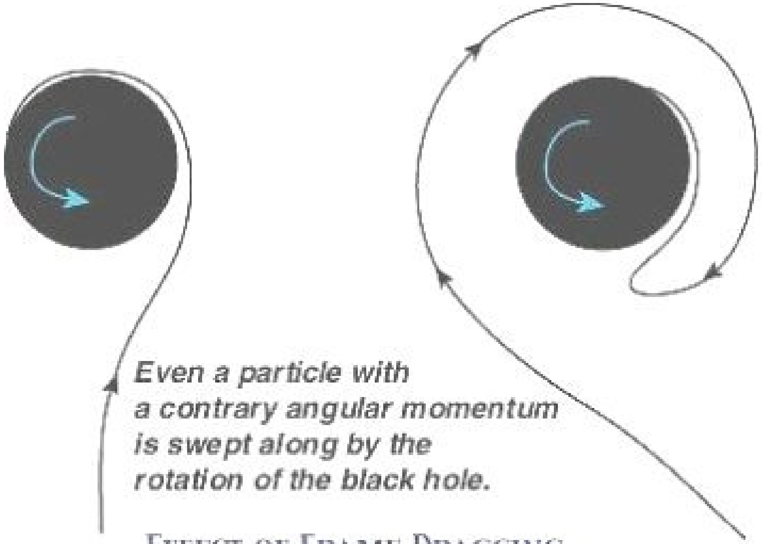
Too close, and the BH absorbs the photon.

There is an “Innermost circular orbit” (ICO) – as light approaches this distance, it ends up in “orbit” around the BH.

These effects depend strongly on the BH’s spin (angular momentum), not just its mass.

More complicated than the weak-field, but still fairly easily described.




Even a particle with a contrary angular momentum is swept along by the rotation of the black hole.

EFFECT OF FRAME DRAGGING

Connecting Limits

We know:

- How light bends when passing far from the BH (r → infty)
- How light bends when passing close to the BH (r → r_ICO)
- Light bends more when the BH spins more in the prograde direction (BH spin and photon angular momentum aligned)
- Light bends more the closer it gets to the BH

What we want is a simple, easy-to-use formula to describe light bending as it passes by a BH. We begin with photons in the equatorial plane of spinning black holes.

The exact solution

The formula for this case is known in terms of a reasonably awful elliptic integral.

We want a simpler approximate form that could be used repeatedly, taking less time, to track many photons originating in a disk or similar configuration.

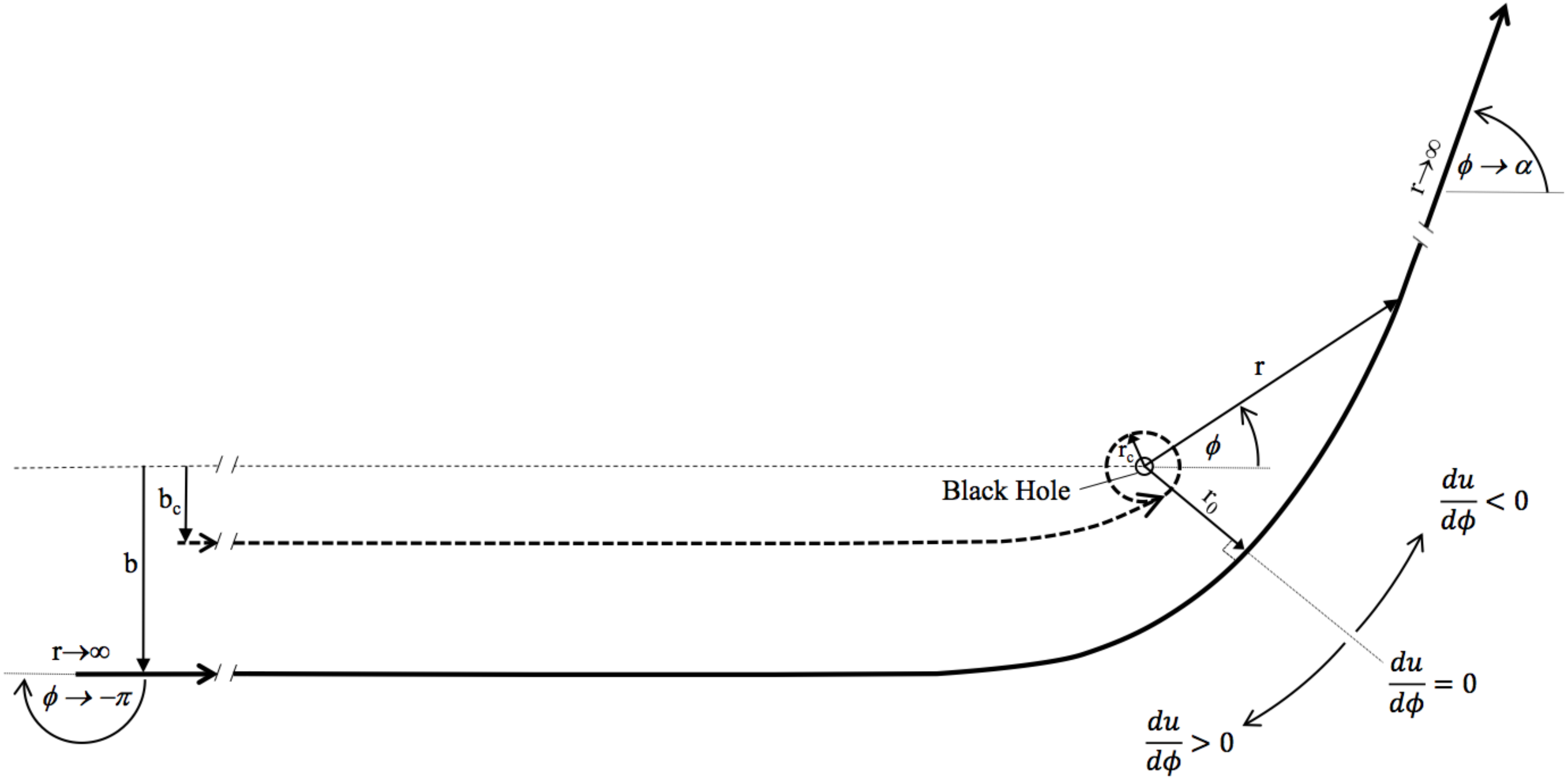
$$\alpha = -\pi + 2\int_0^{u_0} \frac{1 - 2u\left(1 - \frac{a}{b}\right)}{\left[1 - 2u + a^2u^2\right]\sqrt{2\left(1 - \frac{a}{b}\right)^2 u^3 - \left(1 - \frac{a^2}{b^2}\right)u^2 + \frac{1}{b^2}}}du.$$

$$\begin{aligned}
\hat{\alpha} &= -\pi + \sqrt{\frac{2}{\mathsf{m}_\bullet}}\ \frac{C_+}{1-\omega_s}\int_0^{1/r_0} \frac{du}{(u_+ - u)\sqrt{(u-u_1)(u-u_2)(u-u_3)}} \\
&\qquad + \sqrt{\frac{2}{\mathsf{m}_\bullet}}\ \frac{C_-}{1-\omega_s}\int_0^{1/r_0} \frac{du}{(u_- - u)\sqrt{(u-u_1)(u-u_2)(u-u_3)}} \\
&= -\pi + \sqrt{\frac{2}{\mathsf{m}_\bullet}}\,\frac{C_+}{1-\omega_s}\left[\int_{u_1}^{u_2} \frac{du}{(u_+ - u)\sqrt{(u-u_1)(u_2-u)(u_3-u)}}\right. \\
&\qquad\qquad \left. - \int_{u_1}^{0} \frac{du}{(u_+ - u)\sqrt{(u-u_1)(u_2-u)(u_3-u)}}\right] \\
&\qquad + \sqrt{\frac{2}{\mathsf{m}_\bullet}}\,\frac{C_-}{1-\omega_s}\left[\int_{u_1}^{u_2} \frac{du}{(u_- - u)\sqrt{(u-u_1)(u_2-u)(u_3-u)}}\right. \\
&\qquad\qquad \left. - \int_{u_1}^{0} \frac{du}{(u_- - u)\sqrt{(u-u_1)(u_2-u)(u_3-u)}}\right]
\end{aligned}$$

$$\hat{\alpha} = -\pi + \frac{4}{1-\omega_s}\sqrt{\frac{r_0}{Q}}\left\{\Omega_+\Big[\Pi(n_+, k) - \Pi(n_+, \psi, k)\Big] + \Omega_-\Big[\Pi(n_-, k) - \Pi(n_-, \psi, k)\Big]\right\}, \quad (34)$$

Our method

We want a formula that reproduces the weak and strong-field limits, but works for intermediate values, regardless of BH spin:



Results

We constructed an asymptotic approximant that matches the series expansion in the weak-field limit and the known terms in the strong field limit, which converges toward the true function:

$$\alpha_{\mathrm{AN}} = -\pi + \beta + \gamma \ln \zeta + \delta_{a,1} \frac{\sqrt{3}}{b'} - \gamma \ln b' + \sum_{n=1}^{N+1} B_n b'^{\frac{n}{2}} \left(\Delta_{n+1} \sqrt{b'} \ln b' + \Delta_n \right) \quad (13)$$

$$\begin{aligned} \alpha_{\mathrm{A1}} = {} & \left(-\pi + \beta + \gamma \ln \zeta\right) \left(1 - b' + b' \ln b'\right) - 4 \left(b' \ln b'\right) / b_c \\ & + \gamma \left(b' - 1\right) \ln b' + \delta_{a,1} \sqrt{3} \left(1/b' - b' + 2b' \ln b'\right) \end{aligned}$$

Coefficients may be pre-computed, to any order desired.

Simple? Definitely faster than numerical integrals.

Problem 2: Magnetized disks

Accretion disks are ubiquitous around black holes:

- SMBH in the centers of galaxies
- Stellar-mass BH in binary systems

The most popular theoretical models invoke "viscosity" to explain angular momentum transport outward and the generation of heat/light, but the gas in these systems isn't actually viscous.

Magnetic field effects drive the time evolution of the disk.

Magnetic effects also closely related to the formation of jets along the polar axes.

Numerical Codes

Disks are long-lived phenomena, so typical 3-dimensional Cartesian codes can’t always be run long enough to examine phenomena of interest

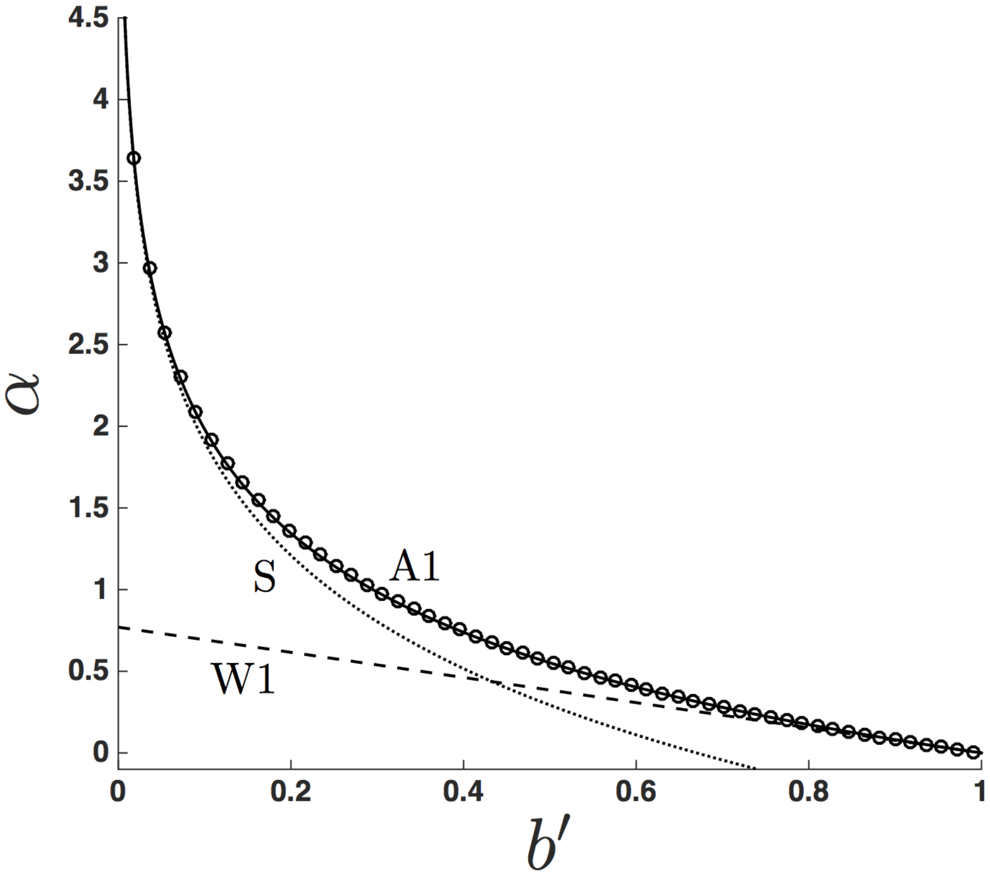
Numerical codes can be much more efficient when tuned to the symmetries of a problem.

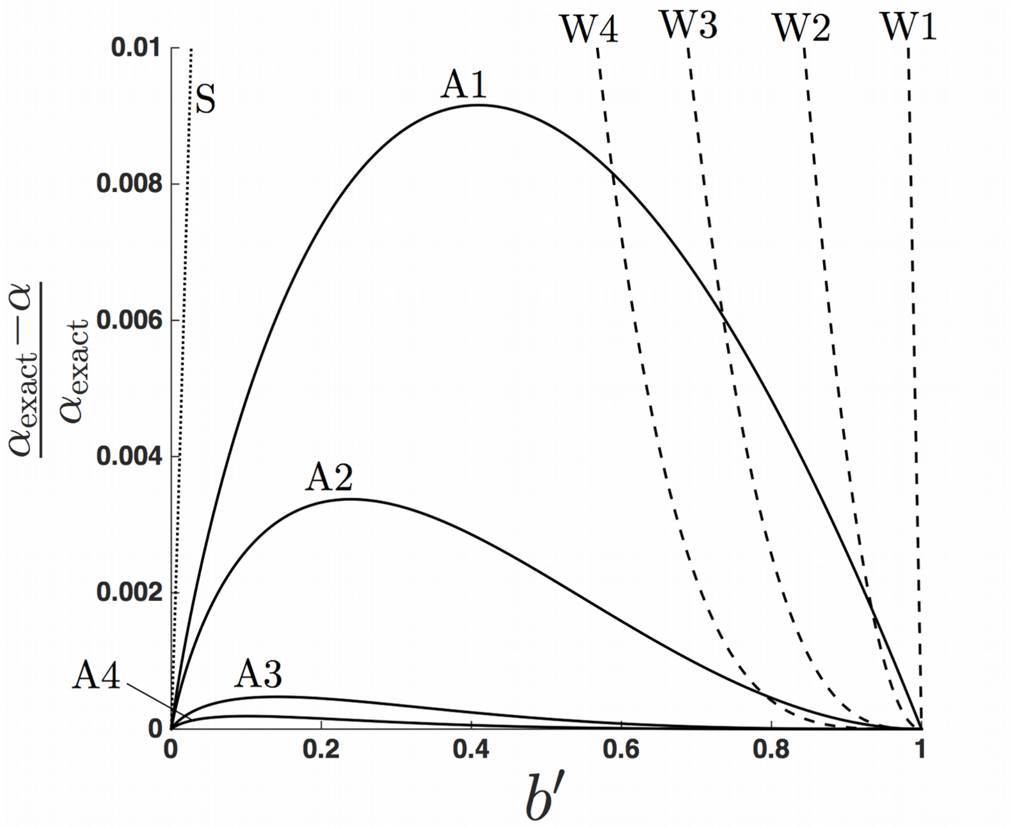
Here: spherical coordinates, high degree of axisymmetry.

Spacetime is largely determined by the black hole itself – doesn’t evolve much in time if at all (disks are typically NOT self-gravitating).

Accuracy of the approximant

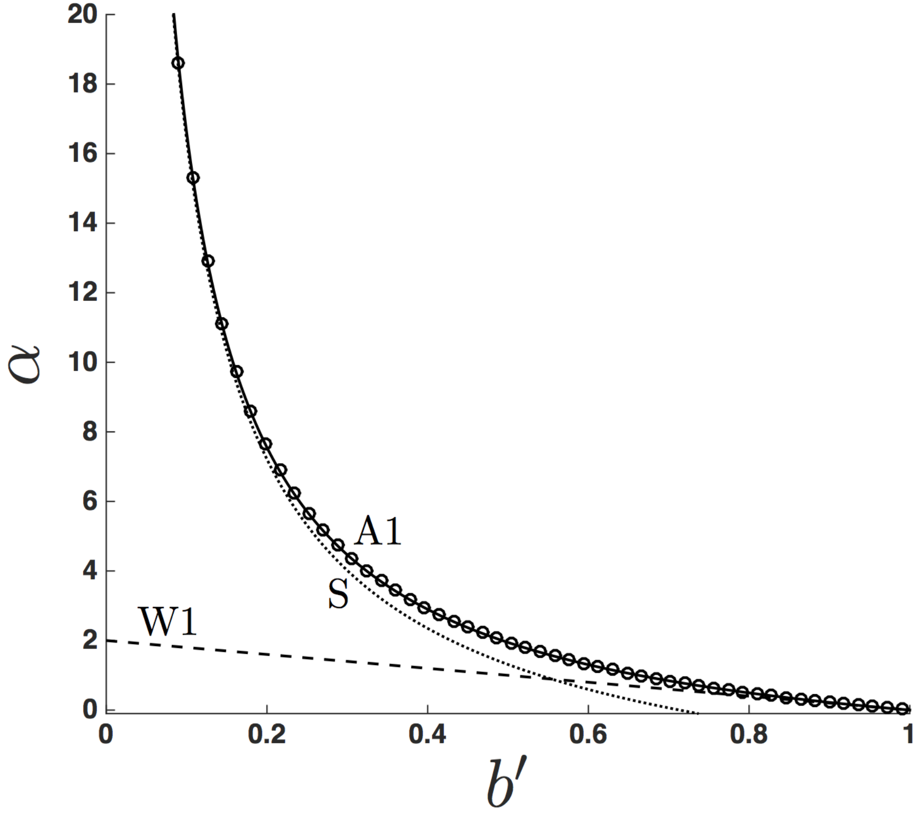
Schwarzschild (a=0) Black hole

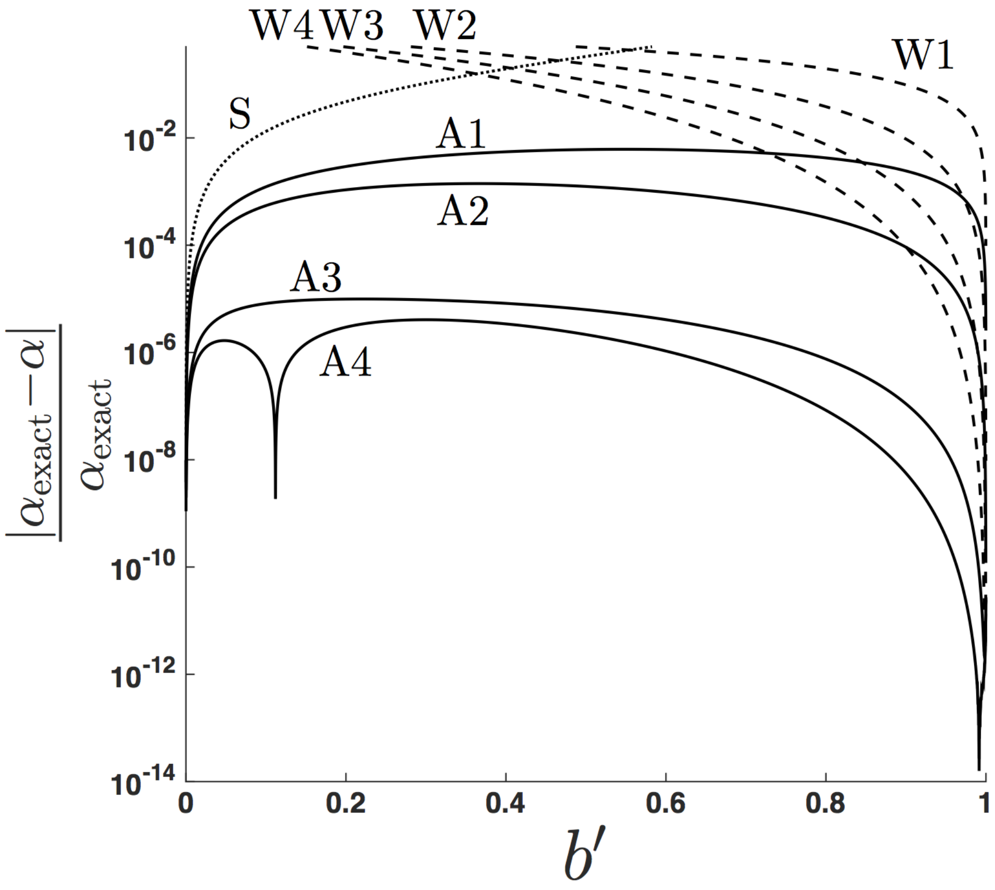




Accuracy of the approximant

Maximal Kerr (a=1) Black hole

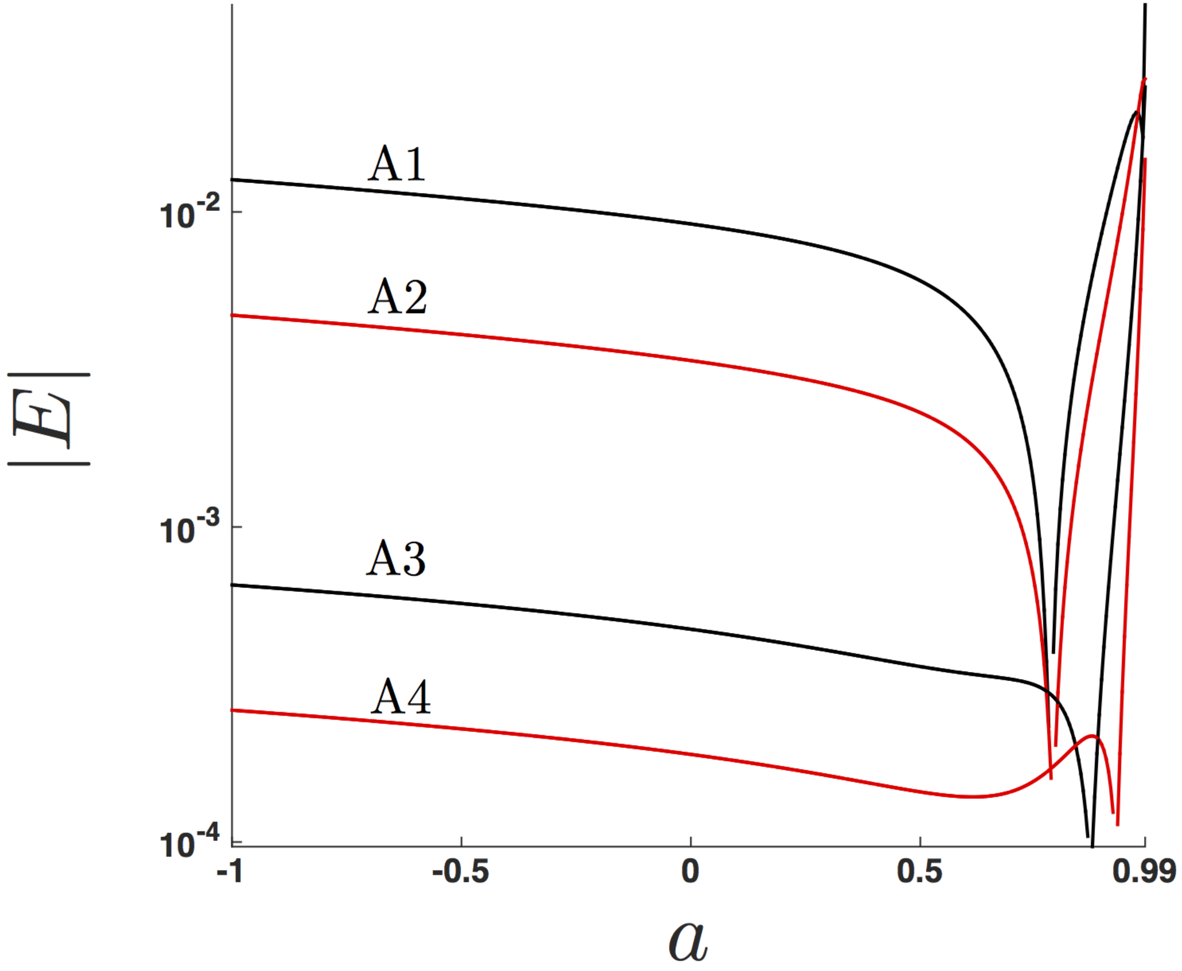




Accuracy of the approximant: max error vs. spin

Approximant is extremely accurate across wide range of spins

Errors in a simulation would be dominated by other physical effects



Future work

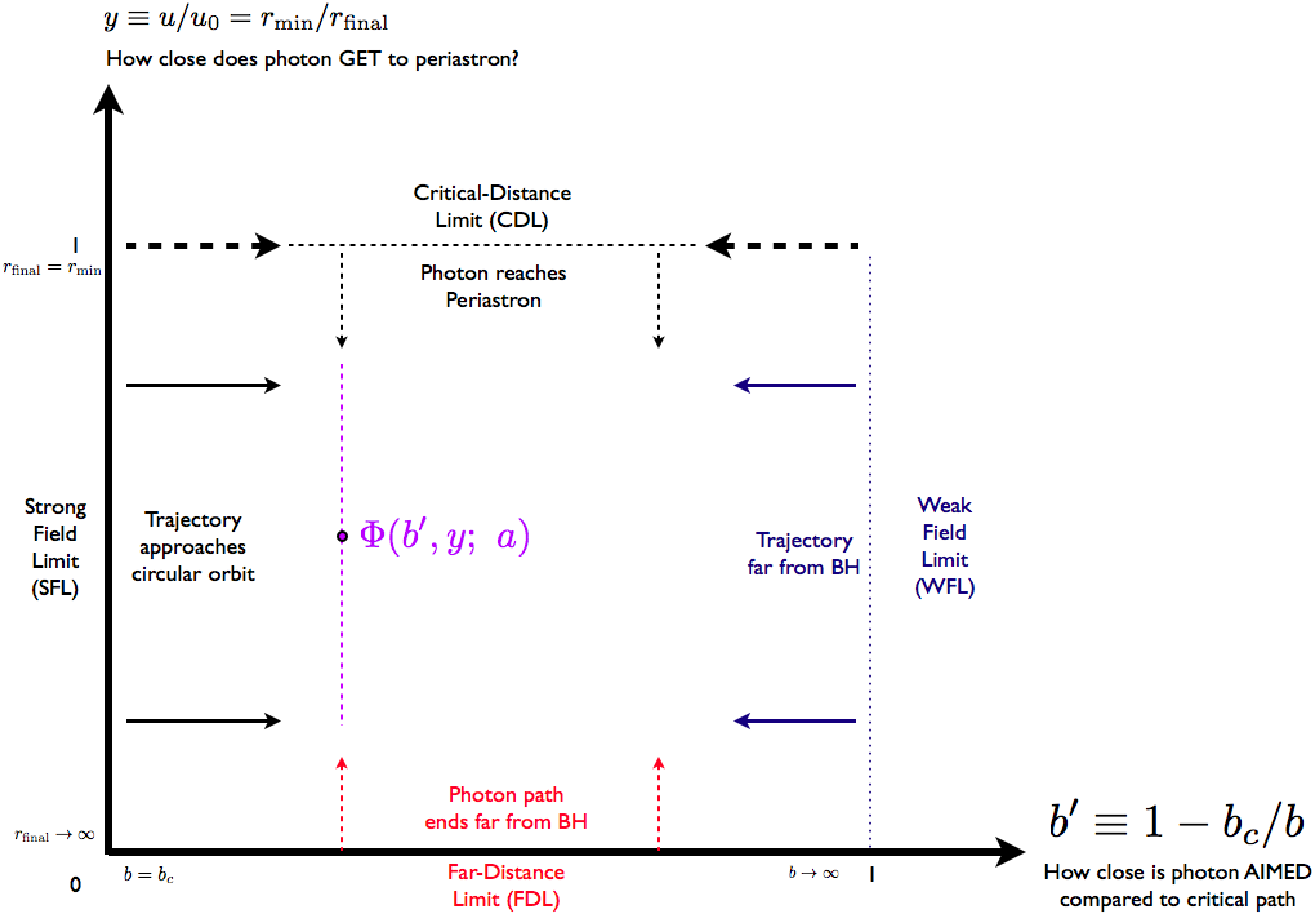
These results were for complete passage – photon started far away, went by BH, finished far away again

Currently focusing on case where starting/ending point is at arbitrary distance

Allows for generation of full approximate trajectories

Position vs. time can be added to evolution codes to track photon emission and absorption.

Future work



$y \equiv u/u_0 = r_{\rm min}/r_{\rm final}$

How close does photon GET to periastron?

Critical-Distance Limit (CDL)

1

$r_{\rm final} = r_{\rm min}$

Photon reaches Periastron

Strong Field Limit (SFL)

Trajectory approaches circular orbit

$\Phi(b', y;\ a)$

Trajectory far from BH

Weak Field Limit (WFL)

Photon path ends far from BH

$r_{\rm final} \to \infty$

0

$b = b_c$

Far-Distance Limit (FDL)

$b \to \infty$

1

$b' \equiv 1 - b_c/b$

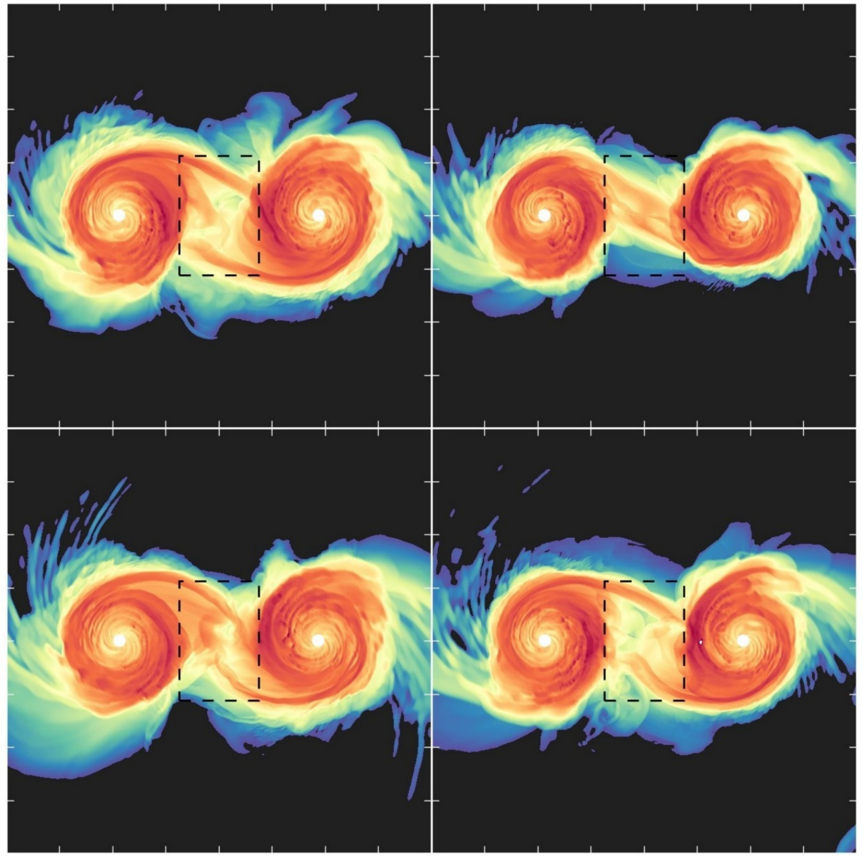
How close is photon AIMED compared to critical path

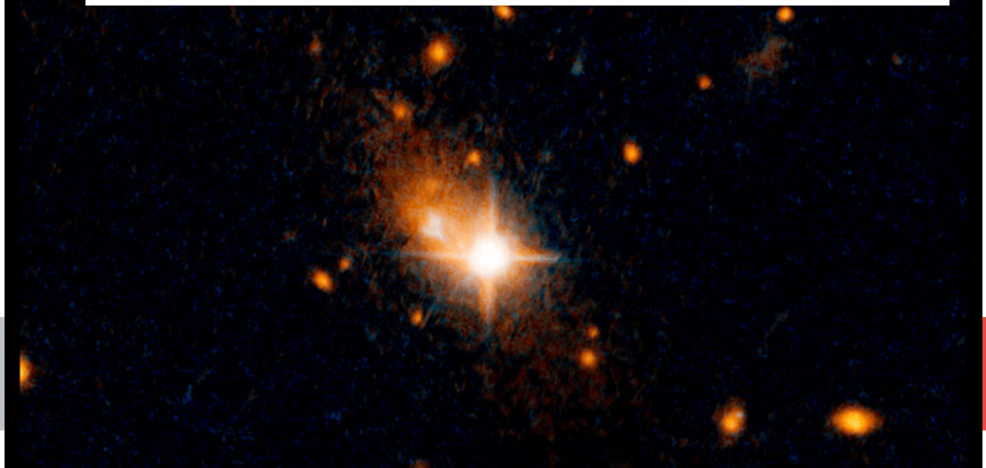
Research Project 2: Magnetized disk simulations and the B-to-A problem

Disks around binary black holes are a fascinating astrophysical source:

- Multi-disk structure with circumbinary disk + 2 BH disks
- Binary BH systems have been observed by LIGO
- Merger may produce an EM transient during merger
- We may have observed post-merger kicked disks already

These are long-lived systems until merger, requiring codes optimized for disks – background metrics, large physical scales





Top: Bowen et al. 2017; Bottom: HST

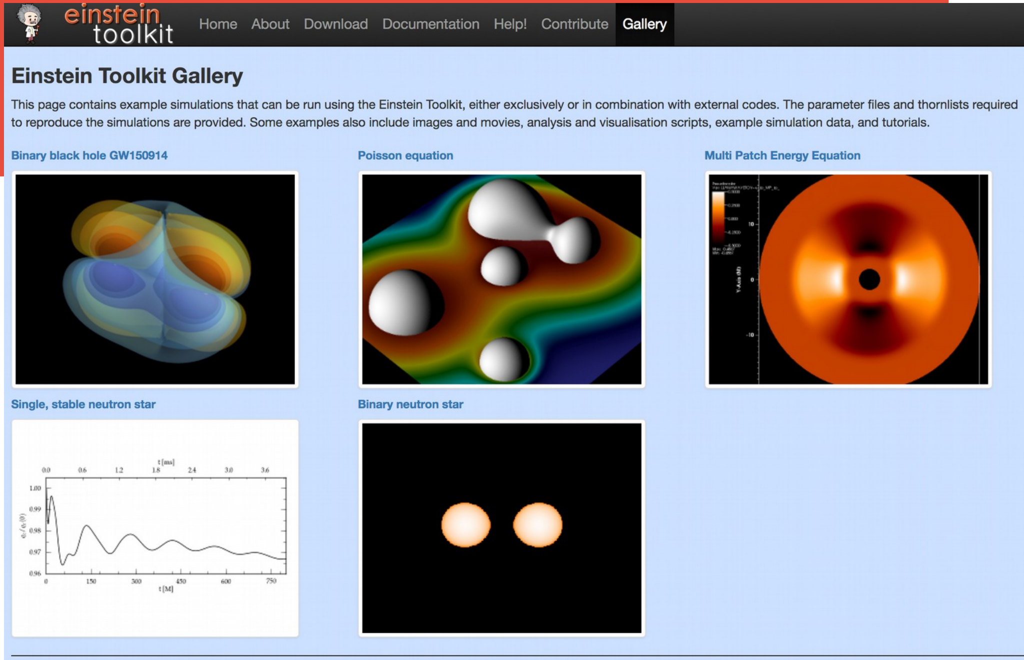
Our problem: B to A

Evolving the black holes through a merger requires full numerical GR

The code we wish to use evolves the magnetic vector potential, not the magnetic field vector

$$\vec{B} = \nabla \times \vec{A}$$

Our problem: how do we invert the curl operator to find the vector potential?



Einstein Toolkit Gallery

This page contains example simulations that can be run using the Einstein Toolkit, either exclusively or in combination with external codes. The parameter files and thornlists required to reproduce the simulations are provided. Some examples also include images and movies, analysis and visualisation scripts, example simulation data, and tutorials.

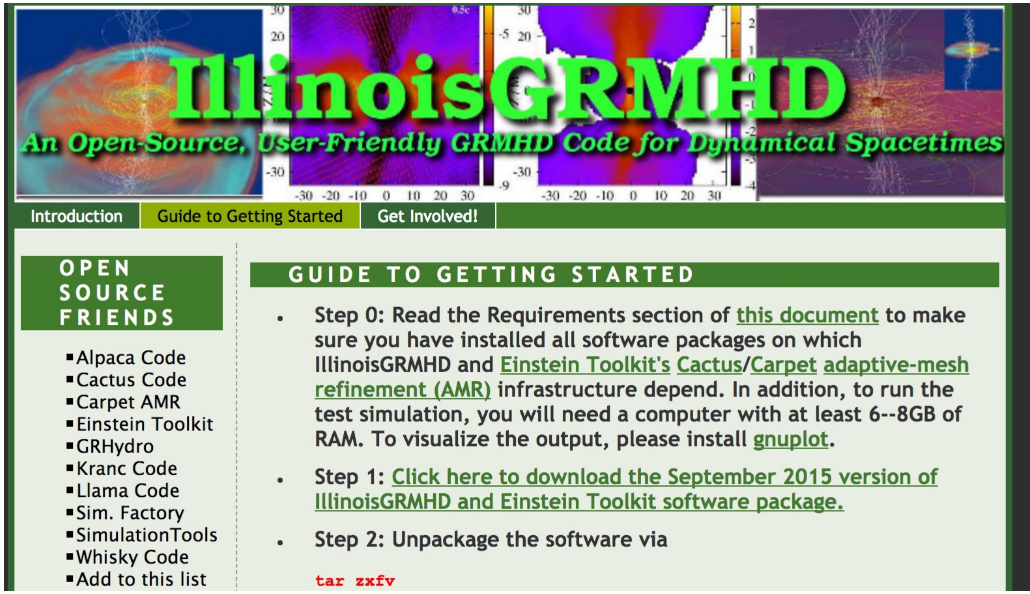
Binary black hole GW150914

Poisson equation

Multi Patch Energy Equation

Single, stable neutron star

Binary neutron star



OPEN SOURCE FRIENDS

- Alpaca Code
- Cactus Code
- Carpet AMR
- Einstein Toolkit
- GRHydro
- Kranc Code
- Llama Code
- Sim. Factory
- SimulationTools
- Whisky Code
- Add to this list

GUIDE TO GETTING STARTED

- Step 0: Read the Requirements section of this document to make sure you have installed all software packages on which IllinoisGRMHD and Einstein Toolkit's Cactus/Carpet adaptive-mesh refinement (AMR) infrastructure depend. In addition, to run the test simulation, you will need a computer with at least 6--8GB of RAM. To visualize the output, please install gnuplot.
- Step 1: Click here to download the September 2015 version of IllinoisGRMHD and Einstein Toolkit software package.
- Step 2: Unpackage the software via

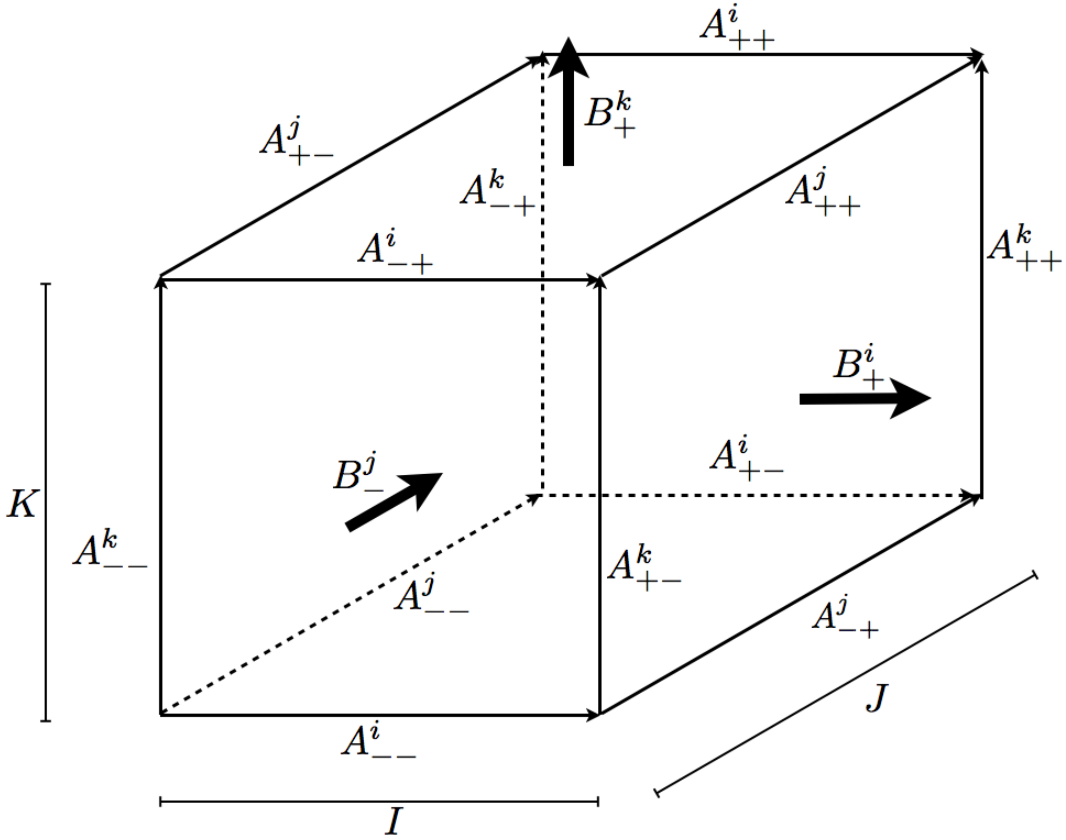
```
tar zxfv
```

A note on coordinates

We use a staggered approach to numerical grids. B-fields live on faces so that the divergence constraint $\nabla \cdot \vec{B} = 0$ is automatically centered for each cell.

Vector potentials (A-fields) live on grid edges, so that $\vec{B} = \nabla \times \vec{A}$ may be calculated by differencing around the edges of a face.

This approach automatically enforces the divergence constraint $\nabla \cdot \vec{B} = 0$



Potential Approaches

1. Direct cell-by-cell

You can symmetrically set 12 A-field values to reproduce 6 B-field values (only 5 are independent) for a single cell.

Progressing through the grid, there is always enough freedom to continue setting A-fields to reproduce a given B-field

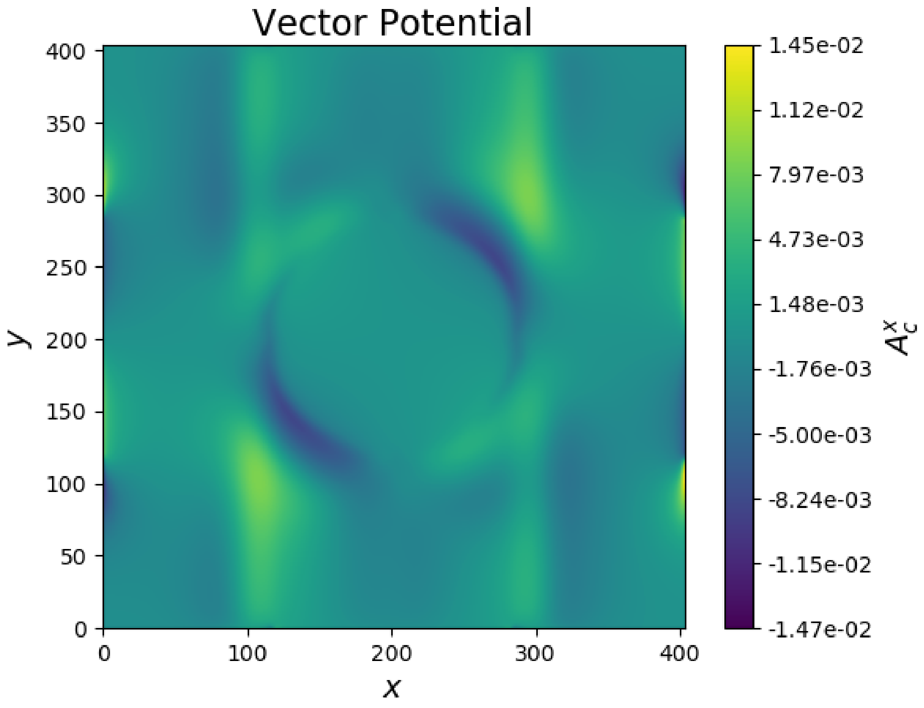
The method is highly efficient – scales linearly with the number of grid cells! – but lacks overall symmetry, since order matters.

2. Global linear algebra – Solve a VERY big linear algebra problem – about 3 equations per grid cell

Parallel linear algebra solvers for sparse matrices exist

For grids of (100)^3, we need up to 12-16 processors to be able to guarantee a solution will be found

Test 1: The rotor

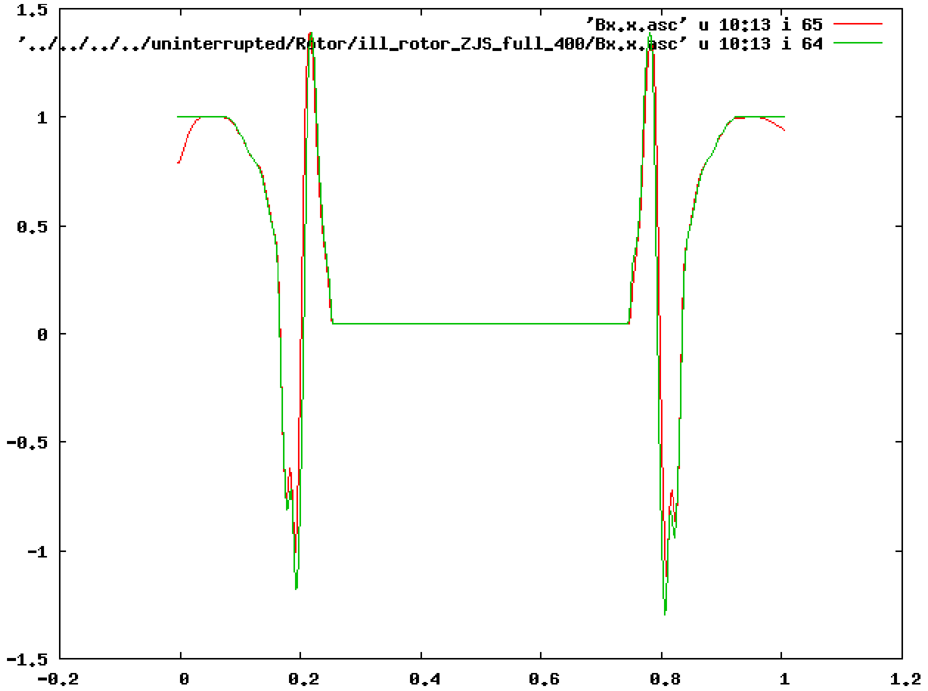


Begin from a uniform density fluid with a horizontal magnetic field

Spin a circular disk with uniform angular velocity so that the outer edge is moving at 0.99c.

The magnetic field is dragged along with the disk.

Convert A-field to B-field exactly, then back to A-field numerically in the middle of the run.

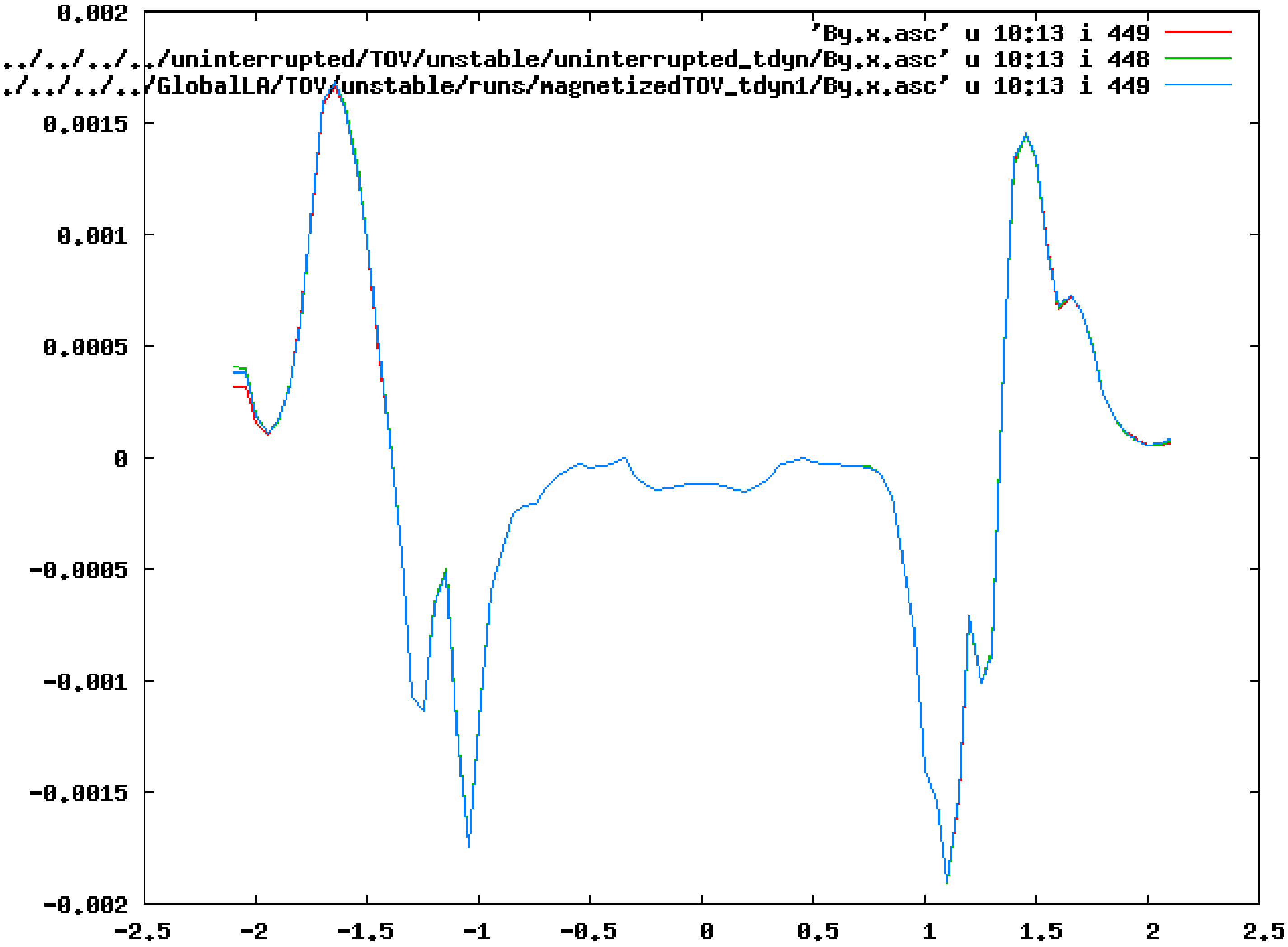


Test 2: The TOV bomb

Take a magnetized neutron star

Turn up the pressure by a lot at T=0.

It explodes!



Future work

Public code release

Will be included within Einstein Toolkit or as standalone code for general electromagnetic evolution codes

Multigrid code

Modern numerical relativity codes use adaptive mesh refinement – thing very small well-resolved boxes within bigger, more coarsely resolved boxes

Requires more complicated solvers

Live simulations – from circumbinary disks through merger!

What does a binary black hole merger look like for electromagnetic telescopes?

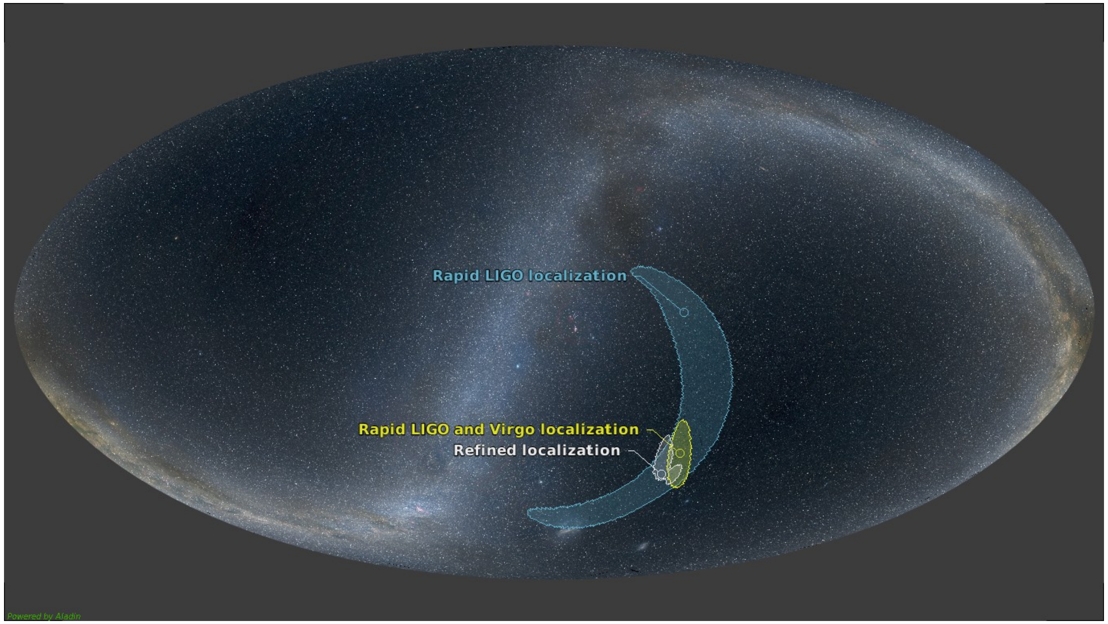
Our future: Multimessenger Astrophysics

LIGO has yet to announce its first detection of merging neutron stars

- Rumors abound that this may soon change

3 interferometers (LIGO-H/LIGO-L/VIRGO) can narrow down locations for telescopes to follow-up

Immediate ramifications for understanding gamma-ray bursts, medium term for nuclear physics, medium/long term for stellar evolution

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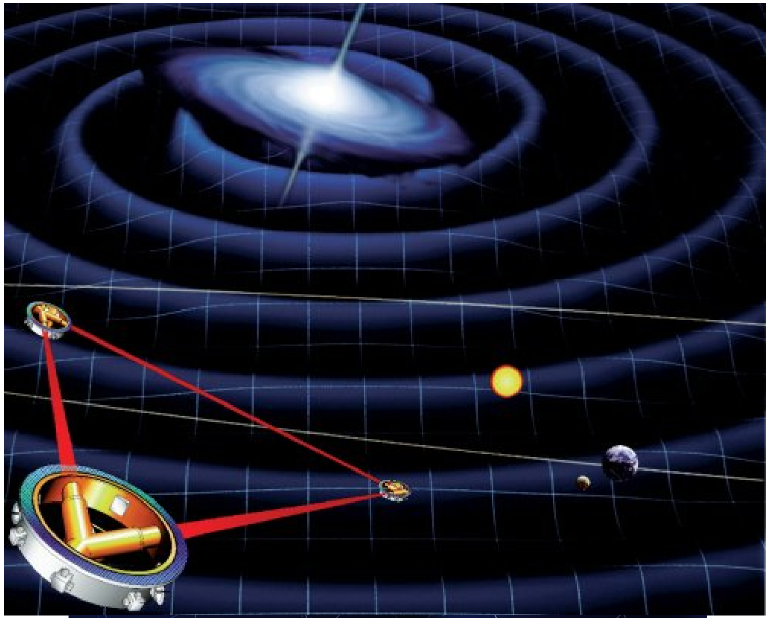
Rumors swirl that LIGO snagged gravitational waves from a neutron star collision

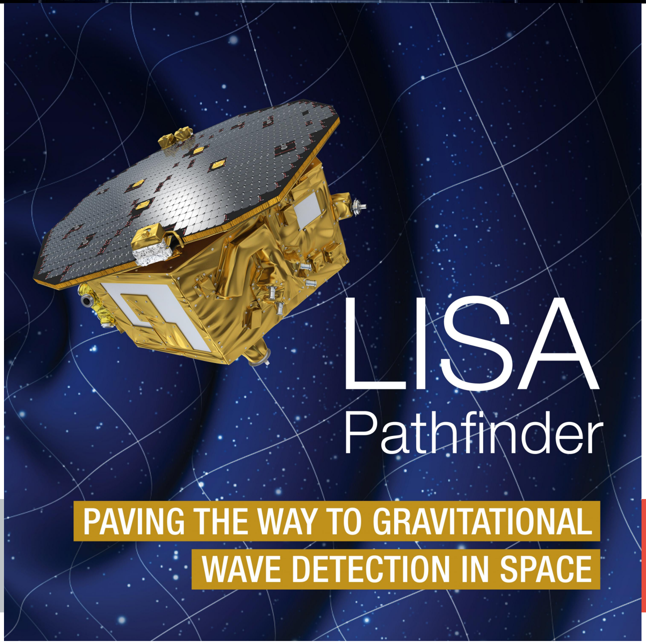
As latest search ends, there's speculation of a detected neutron star smashup

BY EMILY CONOVER 3:14PM, AUGUST 25, 2017

Top: Virgo Team

Our future: Multimessenger Astrophysics





From a theory/computation perspective:

- Simulating matter is relatively straightforward
- Simulating the emission of photons is somewhat straightforward
- Simulating the interplay between radiation, matter, magnetic fields, etc. is VERY COMPLICATED!

From the observational side:

- LIGO/VIRGO operational and improving; KAGRA LIGO India to come…
- LISA will also open up new horizons
- We have made 4-5 detections to date – eventually several per day?

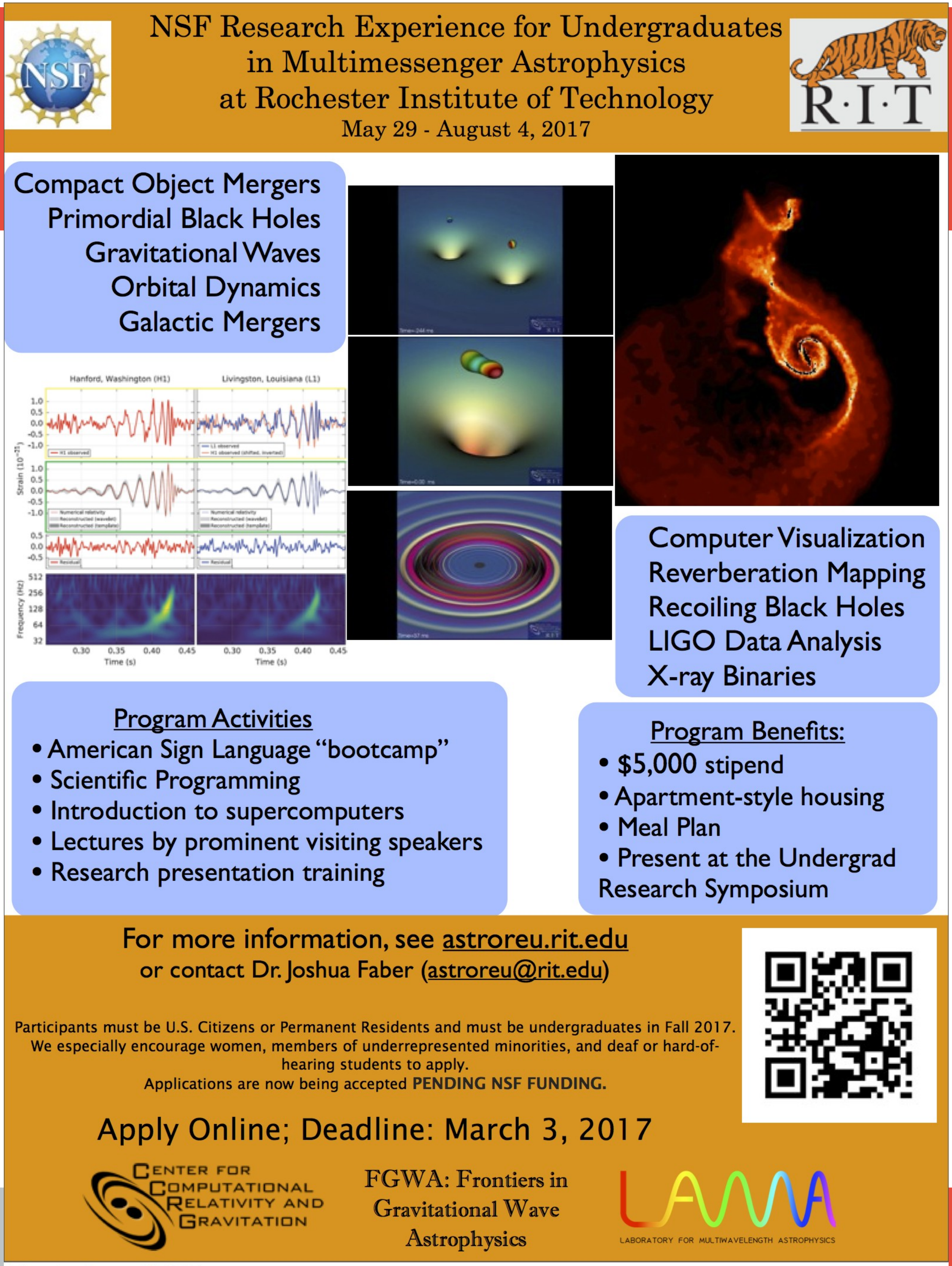
Shameless Advertising

We have an REU program in Multimessenger Astrophysics

Applications will open in December

8 students per year

2018 will be our second year



NSF Research Experience for Undergraduates
in Multimessenger Astrophysics
at Rochester Institute of Technology
May 29 - August 4, 2017

Compact Object Mergers
Primordial Black Holes
Gravitational Waves
Orbital Dynamics
Galactic Mergers

Computer Visualization
Reverberation Mapping
Recoiling Black Holes
LIGO Data Analysis
X-ray Binaries

Program Activities
- American Sign Language "bootcamp"
- Scientific Programming
- Introduction to supercomputers
- Lectures by prominent visiting speakers
- Research presentation training

Program Benefits:
- $5,000 stipend
- Apartment-style housing
- Meal Plan
- Present at the Undergrad Research Symposium

For more information, see astroreu.rit.edu
or contact Dr. Joshua Faber (astroreu@rit.edu)

Participants must be U.S. Citizens or Permanent Residents and must be undergraduates in Fall 2017.
We especially encourage women, members of underrepresented minorities, and deaf or hard-of-hearing students to apply.
Applications are now being accepted **PENDING NSF FUNDING.**

Apply Online; Deadline: March 3, 2017

Center for Computational Relativity and Gravitation

FGWA: Frontiers in Gravitational Wave Astrophysics

LAMA
Laboratory for Multiwavelength Astrophysics

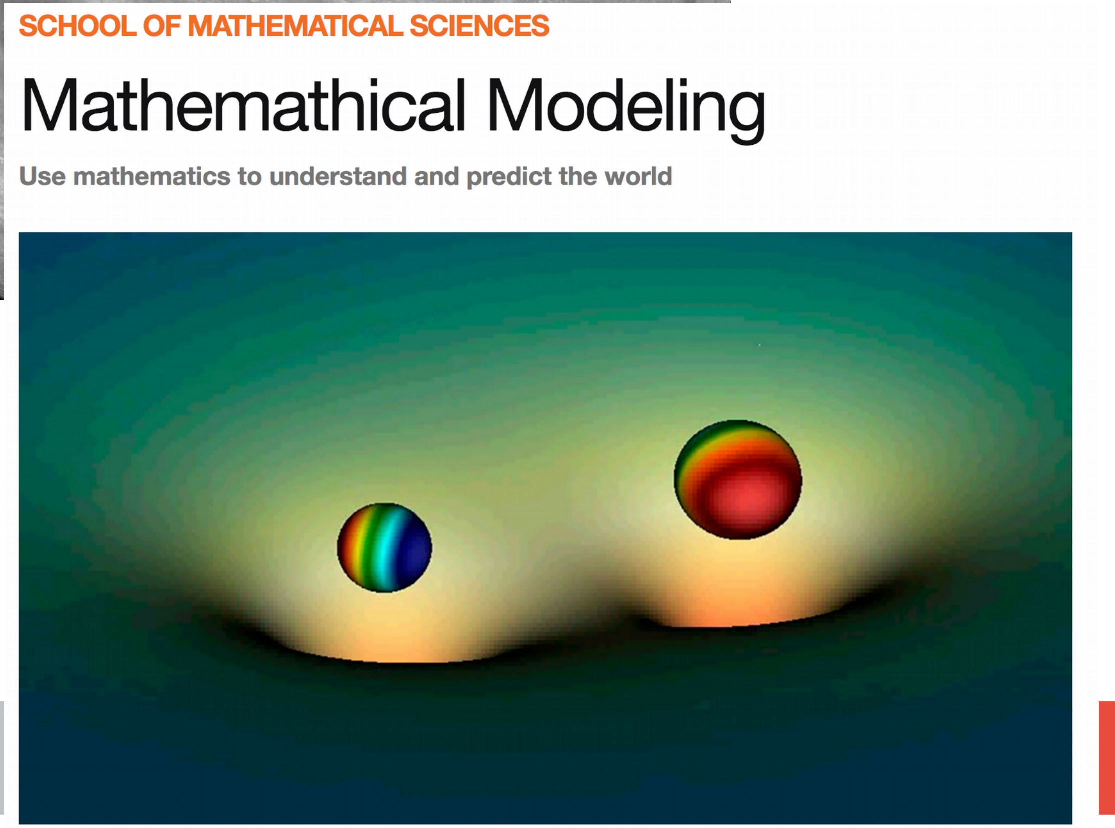
Thursday, January 26, 2017

PhD Programs

RIT has Masters/PhD programs in

- Astrophysical Sciences and Technology (AST)
- Mathematical Modeling

Ask me about both!


SCHOOL OF MATHEMATICAL SCIENCES

Mathemathical Modeling

Use mathematics to understand and predict the world