A semianalytic Fisher matrix for precessing BH-NS binaries

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2015 April APS, Baltimore MD

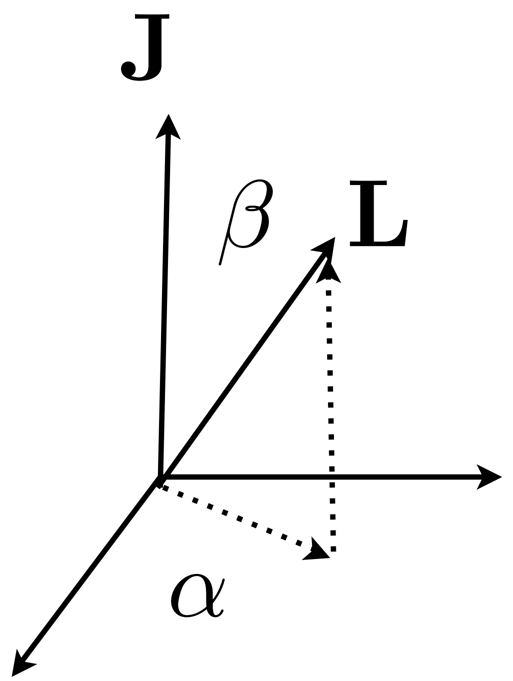
R·I·T

Method: exploit corotating frame

- **Expand waveform**

$$
\begin{aligned}
h(t,\hat{n},\lambda) &= e^{-2i\Psi_J}\frac{M}{d_L}\sum_{lm\bar{m}} h^{(C)}_{l\bar{m}}(t,\lambda) D^{l}_{m\bar{m}}(R(t)) Y^{(-2)}_{lm}(\hat{n}) \\
&= e^{-2i\Psi_J}\frac{M}{d_L}\sum_{lm\bar{m}} e^{-i\bar{m}(\Phi_{\rm orb}+\gamma)} e^{-im\alpha} d^{l}_{m\bar{m}}(\beta) A^{c}_{l\bar{m}}(t) Y^{(-2)}_{lm}(\hat{n})
\end{aligned}
$$

Arun et al 2009



- **SPA term by term**
  - As in SpinTaylorF2

Lundgren and ROS PRD 89 44021 (2013)

$$
\begin{aligned}
\hat{\mathbf{L}} &= \sin\beta_{JL}\cos\alpha_{JL}\hat{x}' + \sin\beta_{JL}\sin\alpha_{JL}\hat{y}' \\
&+ \cos\beta_{JL}\hat{\mathbf{J}}
\end{aligned}
$$

$$
\frac{d\hat{\mathbf{L}}}{dt} \simeq \frac{\mathbf{J}}{r^3}\left(2+\frac{3m_2}{2m_1}\right)\times\hat{\mathbf{L}}
$$

Apostolatos et al 1994: one spin

$$
\gamma = -\int \cos\beta_{JL} d\alpha_{JL}
$$

- **Substitute into inner products, Fisher**

$$
\Gamma_{ab} = \langle \partial_a h | \partial_b h \rangle = \sum_{l,m,\bar{m}} \sum_{l',m',\bar{m}'} \ldots
$$

Example SPA: SpinTaylorF2

- Time-domain signal

$$
\begin{aligned}
h_+(t) - i h_\times(t) &= e^{-2i\psi} \sum_{lm} h_{lm}(t)\ {}_{-2}Y_{l,m}(\theta,\phi) \\
&= e^{-2i\psi} \sum_{lm'} \sum_{m} D^l_{m',m}(\alpha(t),\beta(t),\zeta(t)) h^{\mathrm{ROT}}_{l,m}(t) {}_{-2}Y_{l,m'}(\theta,\phi)
\end{aligned}
$$

$$\underleftrightarrow{\qquad R(t) \qquad}$$

- Fourier-transform term-by-term

$$
\begin{aligned}
X(t) &\equiv D^l_{m',2}(R(t)) \times \frac{\eta v^2}{d_L} e^{-i2\Phi(t)} \times {}^{(-2)}Y_{l,m'}(\theta,\phi) \\
\tilde{X}(\omega) &\simeq D^l_{m',2}(R(t(\omega))) \times \frac{\eta v^2}{d_L} \frac{e^{i\Psi(\omega)}}{\sqrt{id^2\Phi/dt^2/\pi}} \times {}^{(-2)}Y_{l,m'}(\theta,\phi)
\end{aligned}
$$

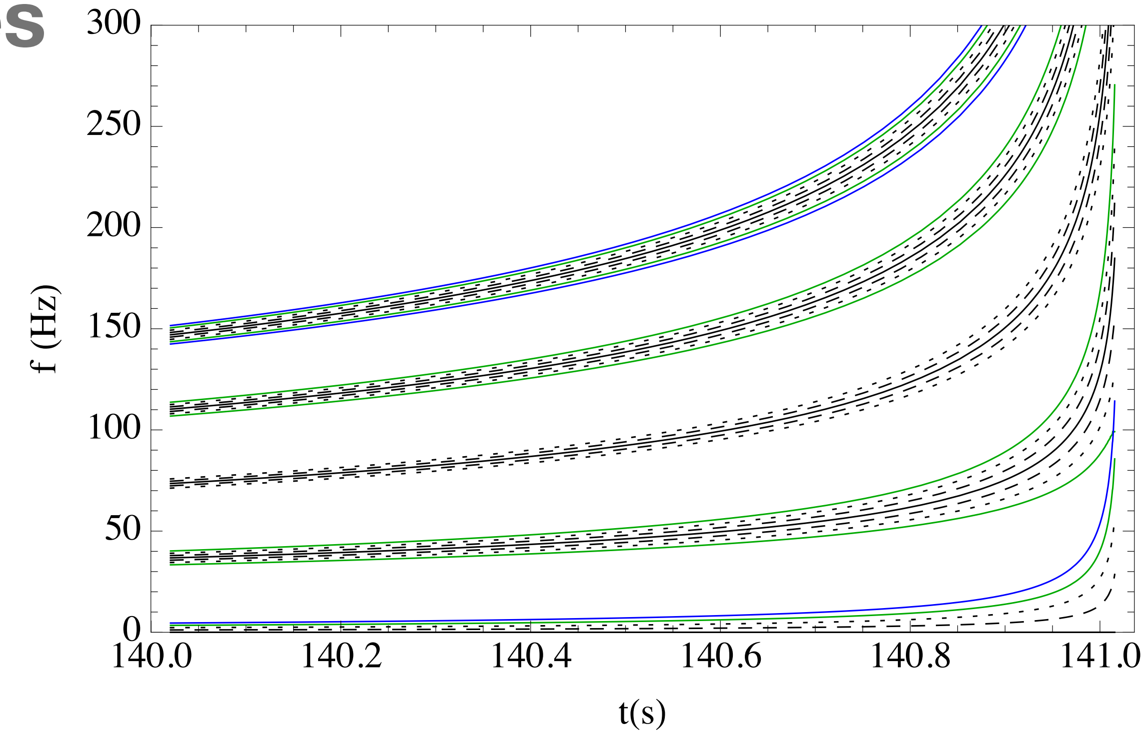
- Regroup terms: “carrier+sideband” [restricted to (l,m’)=(2,2) + (2,-2)]

Expanding isolates distinct time-frequency tracks

- **Distinct time-frequency trajectories**

$$2\pi f_{m,\bar{m}} \equiv m(\dot{\Phi}_{\rm orb} - \dot{\alpha}\cos\beta_{JL}) + \bar{m}\dot{\alpha}$$

  - Orthogonal (!) :
    - Known [Brown, Lundgren, O'Shaughnessy 2012]
  - Unique angular dependence for each term



- **(Intrinsic) Fisher matrix separates, simplifies**
  - Each term: (amplitude) * (fisher from **one** mode)
  - Leading order result:

$$\Gamma_{ab} \quad = \sum_{m=2}^{2} \sum_{s=\pm 1} \rho_{2ms}^2 \hat{\Gamma}_{ab}^{ms}$$

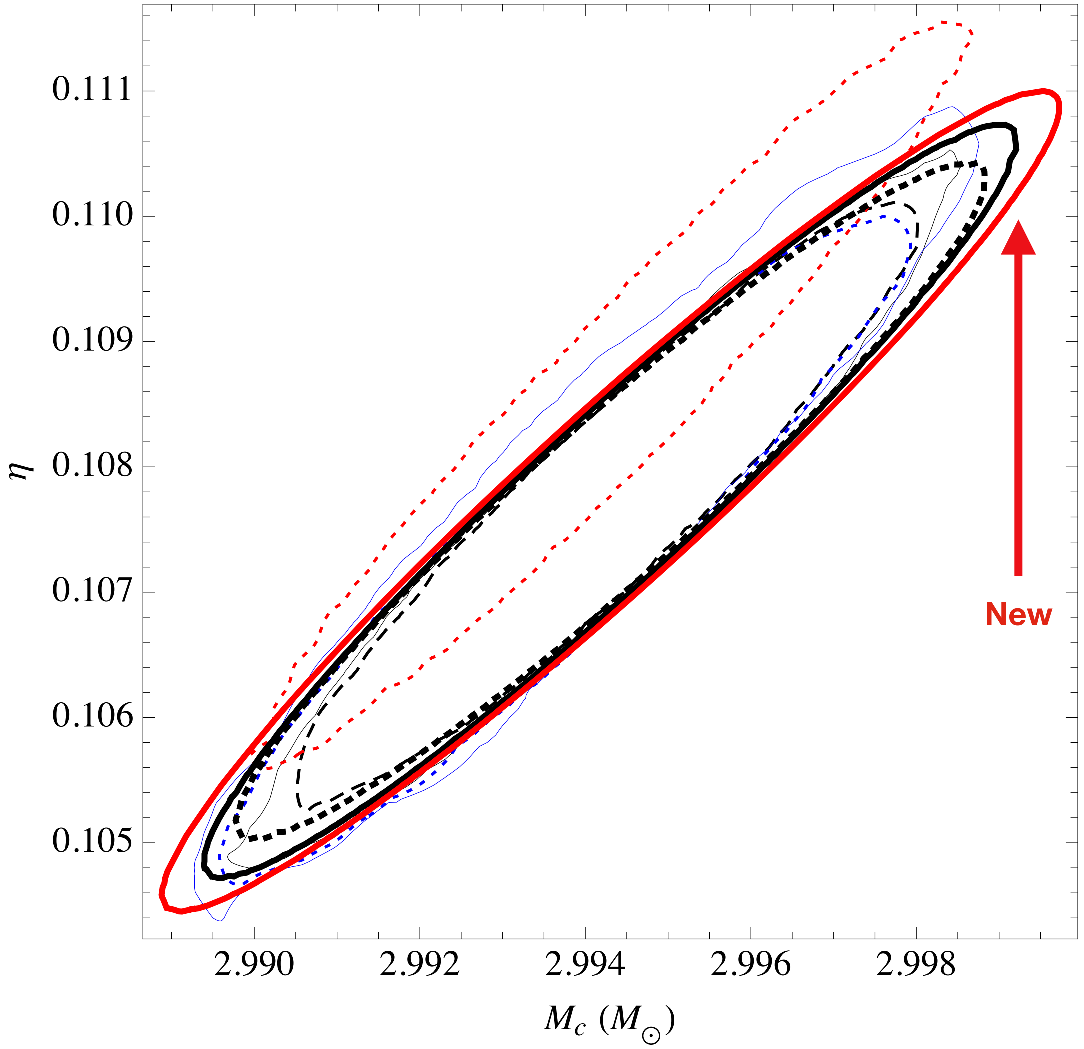
Simple approximate (intrinsic) Fisher matrix

$$\rho^2_{2ms} \equiv |_{-2}Y_{2m}(\theta_{JN}) d^2_{m,2s}(\beta)|^2 \int_0^\infty \frac{df}{S_h(f)} \frac{4(\pi \mathcal{M}_c{}^2)^2}{3d_L^2} (\pi \mathcal{M}_c f)^{-7/3}$$

$$\hat{\Gamma}^{(ms)}_{ab} = \frac{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3} \partial_a(\Psi_2 - 2\zeta - ms\alpha)\partial_b(\Psi_2 - 2\zeta - ms\alpha)}{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3}}$$

- Amplitude
- Angular dependence
- Phase

- Good:
  - Easy to calculate
  - Similar to nonprecessing (weighted average)
  - Intuition about separating parameters
- “Bad”
  - Ansatz / approximation
  - At best, retains all degeneracies of full problem (phases, …)



9d MCMC vs 4d Fisher
ROS et al 2014 (PRD 89 064048)

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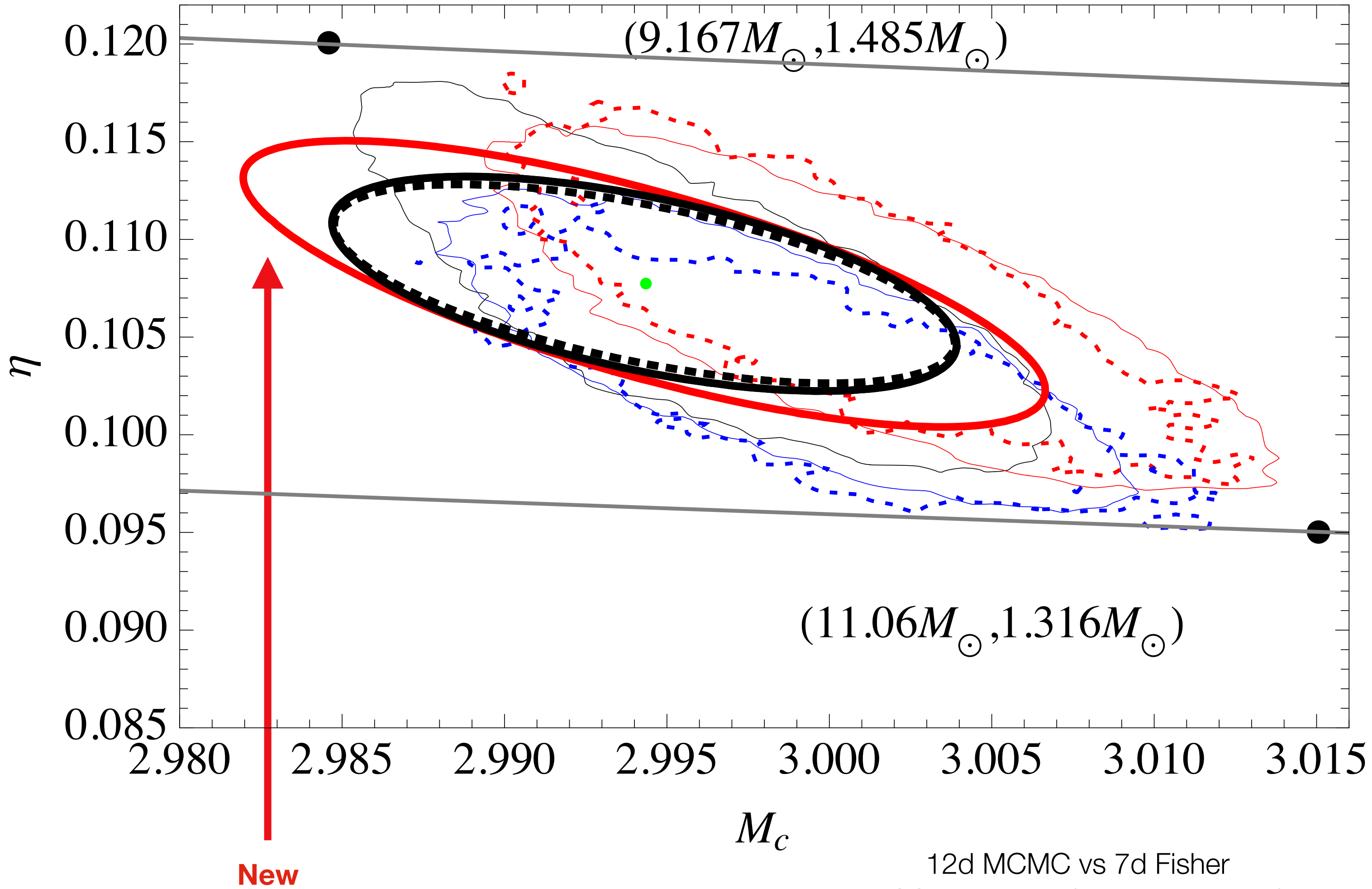


12d MCMC vs 7d Fisher
ROS et al 2014 (PRD 89 102005)

Conclusions

- Approximate Fisher matrix for single-spin binaries
  - Simple, intuitive
    - Recover nonprecessing limit
    - Tractable analytic calculations: small tilt angle; small spins; …
- Practical applications
  - Trends in parameter measurement vs source masses and viewing angle
  - What can we measure for a "face on" binary, $\theta_{JN} \simeq 0$: ~ no modulations?
  - **Big picture: Quickly** assess ability of GW measurements to extract information and decide between astronomical scenarios
- Additionally:
  - (Another) Frequency-domain precessing template
    - Easy to understand: SPA term by term

Klein et al 2014 (here: M7)
Lundgren and ROS ("SpinTaylorF2")
PRD 89 44021 (2013)
Schmidt et al ("IMRPhenomP")

Implications

- Illustrate measurements possible via GW [vs astrophysics]
  - Masses [vs NS, BH mass distribution] [e.g, Aasi et al 2013; Vitale et al 2014]
  - Spins [vs X-ray binaries] [e.g., Aasi et al 2013; Vitale et al 2014]
  - Geometry of merger [vs SN kicks; short GRBs...]
- **Understand** results via simple calculation
  - Separation of scales appears in observables
  - Calibrated analytic calculation against high-resolution, complete PE
- Future directions
  - Production scale: validate over parameter space; advanced instruments
  - More physics: 2 spins (Kesden et al); higher harmonics ; merger phase; high mass; …
    - Transition & corner cases: ansatz breaks if precession too slow
  - Usable: portable code for users
    - simplified analysis & formulae [SpinTaylorF2: Lundgren and ROS PRD 89 44021 (2013)]

Technical details: How to make the sausage

- Standard approximations
  - “Restricted” amplitude
  - Phase change (not amplitude) dominates overlap

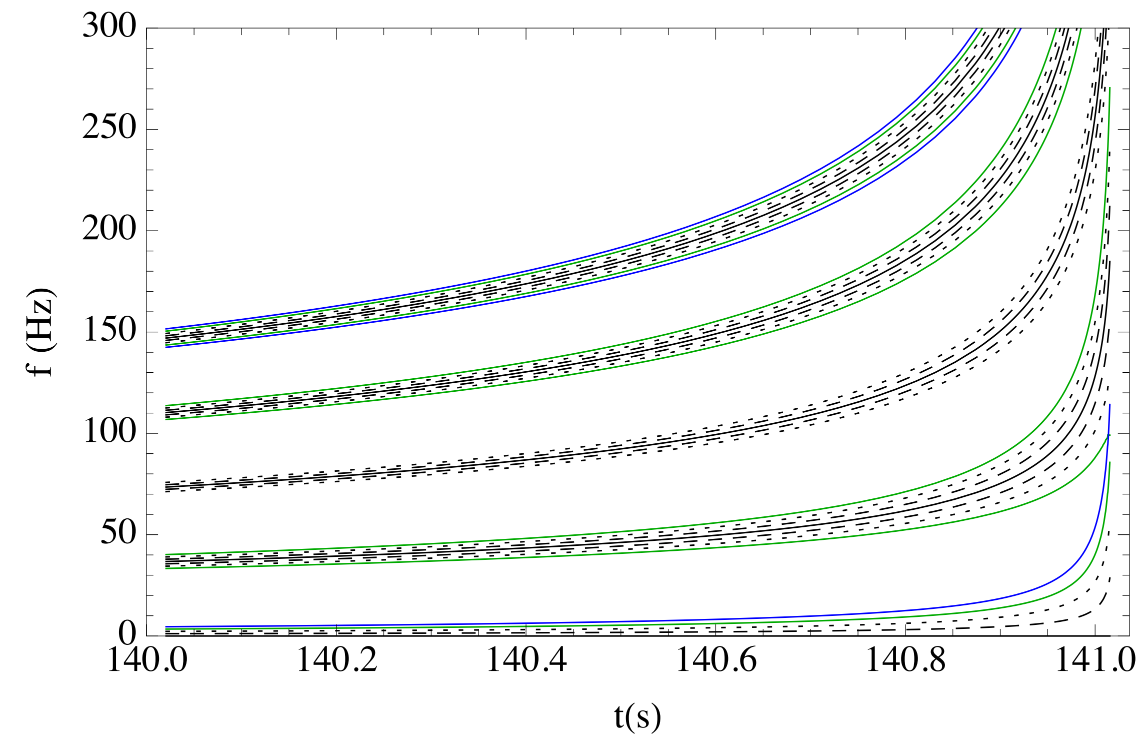
$$\langle \partial_a h | \partial_b h \rangle \simeq \langle h \,|(\partial_a \Psi)(\partial_b \Psi)|\, h \rangle$$

- (Approximate) orthogonality of harmonics with different ‘m’

$$\left\langle h_{22}^{(C)} D^l_{2\bar{m}}(R) | h_{22}^{(C)} D^l_{2m}(R) \right\rangle \simeq 0$$

$$\omega = \frac{d}{dt}[\Phi_{orb} - 2\gamma - m\alpha] \simeq \frac{d}{dt}\Phi_{orb}$$

- Rotation is “slowly varying”
  - Implies time-frequency relationship ~ independent of ‘m’, rotation
  - [Easily relaxed]
- (Approximate) simple precession



Example SPA: SpinTaylorF2 [details]

- Time domain form

$$h_+ = \frac{2M\eta}{D} v^2 \mathrm{Re}\left[\sum_m z_m e^{im\alpha} e^{2i(\Phi-\zeta)}\right]$$

$$z_m = {}_{-2}Y_{2,m}(\beta,0)\frac{4\pi}{5}\left[e^{-2i\psi}\ {}_{-2}Y_{2m}(\theta,0) + e^{2i\psi}\ {}_{-2}Y_{2-m}(\theta,0)\right] .$$

- Kinematics

$$\gamma \equiv \frac{|\mathbf{S}_1|}{|\mathbf{L}|} = \left(\frac{m_1\chi}{m_2}\right) v \ ;$$

$$\Gamma_J \equiv |\mathbf{J}|/|\mathbf{L}| = \sqrt{1+2\kappa\gamma+\gamma^2} \ .$$

- Precession angles

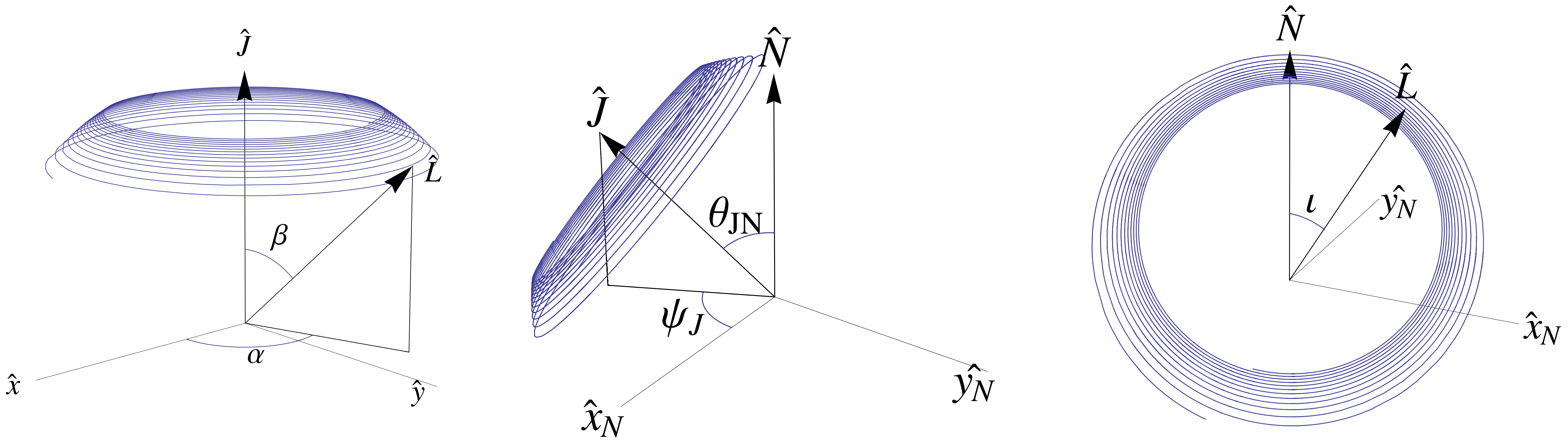
$$\alpha(v) = \eta\left(2+\frac{3m_2}{2m_1}\right)\int v^5\ \Gamma_J\left(\frac{dt}{dv}\right)\mathrm{d}v$$

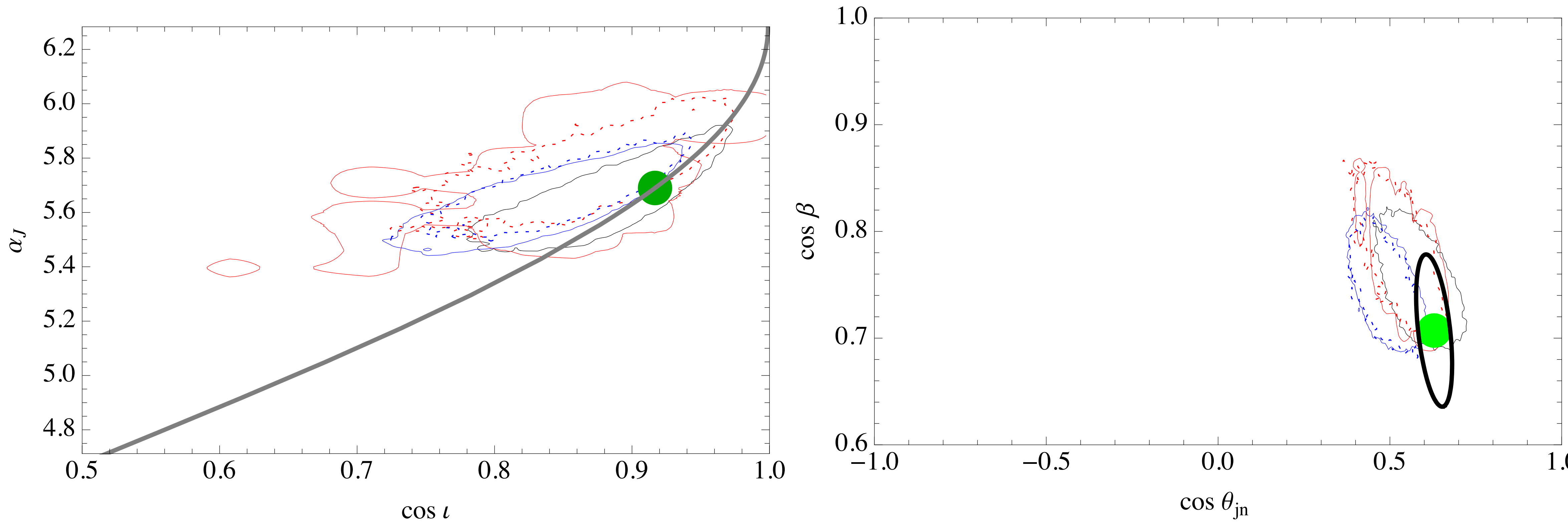
$$\zeta(v) = \eta\left(2+\frac{3m_2}{2m_1}\right)\int v^5(1+\kappa\gamma)\left(\frac{dt}{dv}\right)\mathrm{d}v \ .$$

- Frequency domain form

$$\bar{h}_+(f) \simeq \frac{2\pi\mathcal{M}_c^2}{D}\sqrt{\frac{5}{96\pi}}(\pi\mathcal{M}f)^{-7/6}\sum_m z_m e^{i(\Psi-2\zeta)+im\alpha}$$

Parameters and results 1: Instantaneous geometry



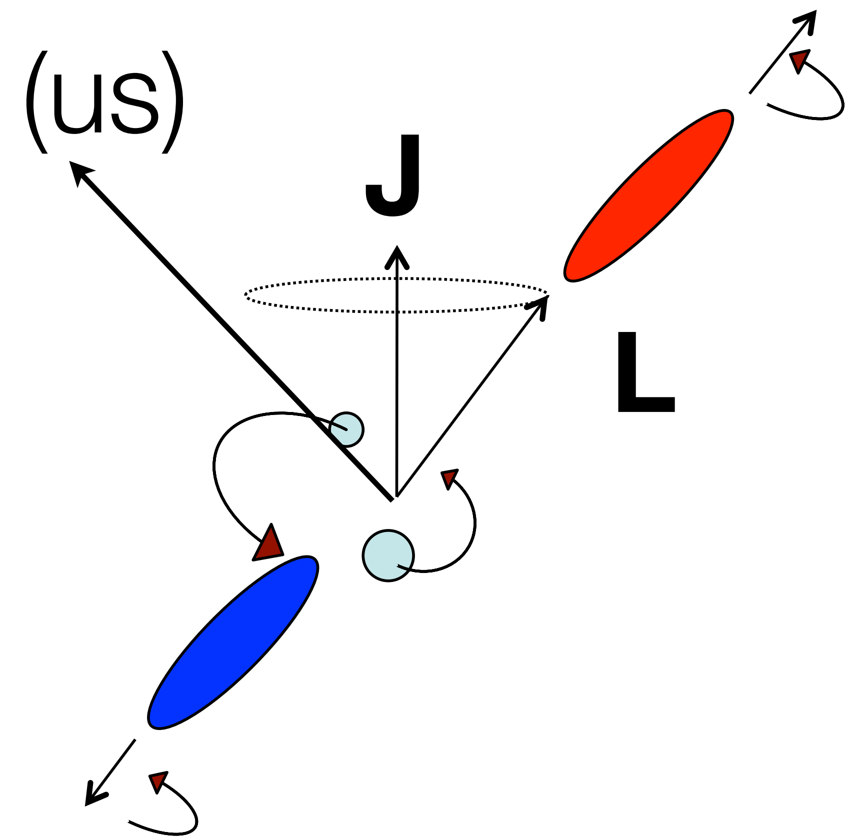


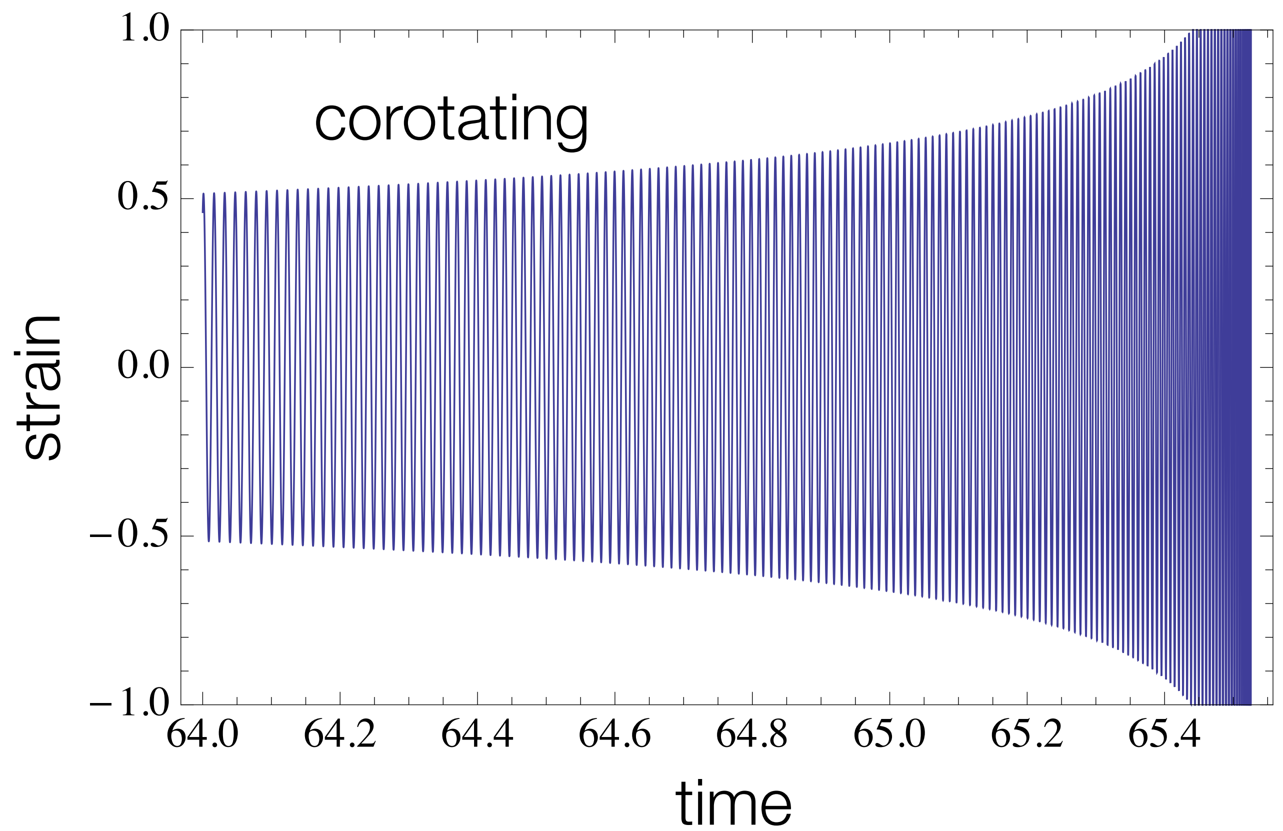
Dynamics of and GW from our BH-NS

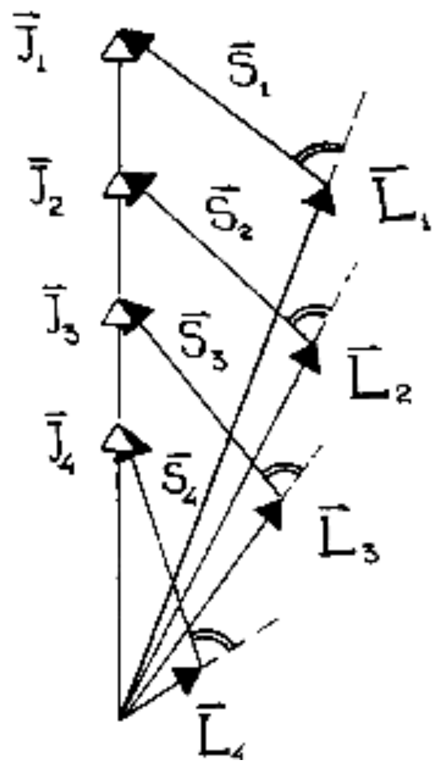
- What

$$m_1, m_2 = 10, 1.4 M_\odot \; ; \chi_1 = 1$$

- Dynamics:
  - spin, precession significant at >40 Hz
- GW
  - (corotating chirp) x (slow rotation)







Apostolatos et al 1994

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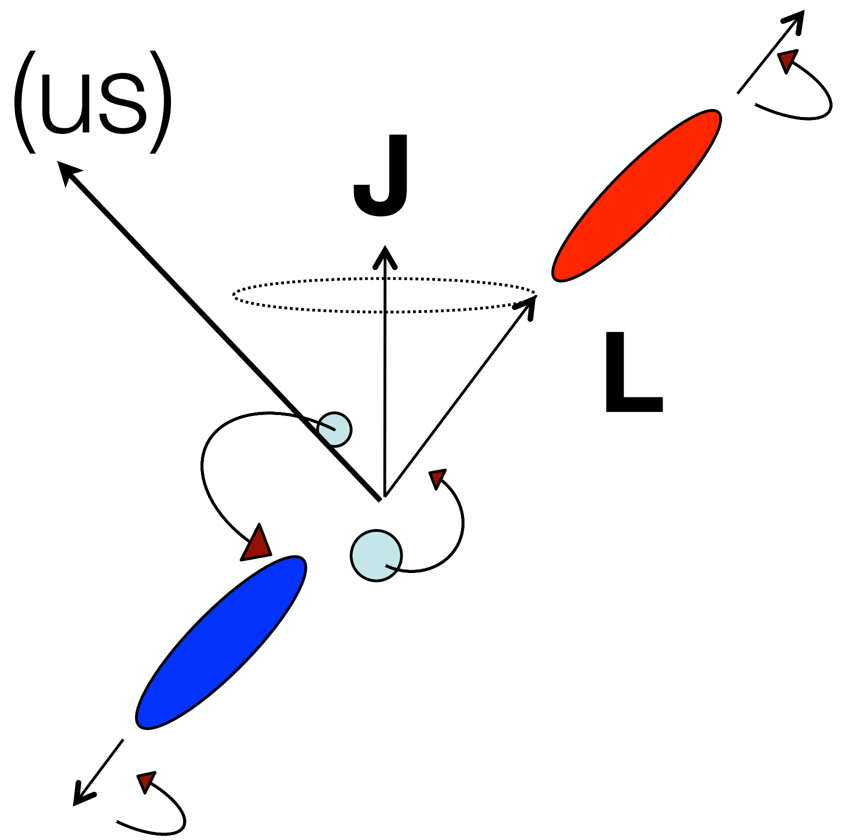
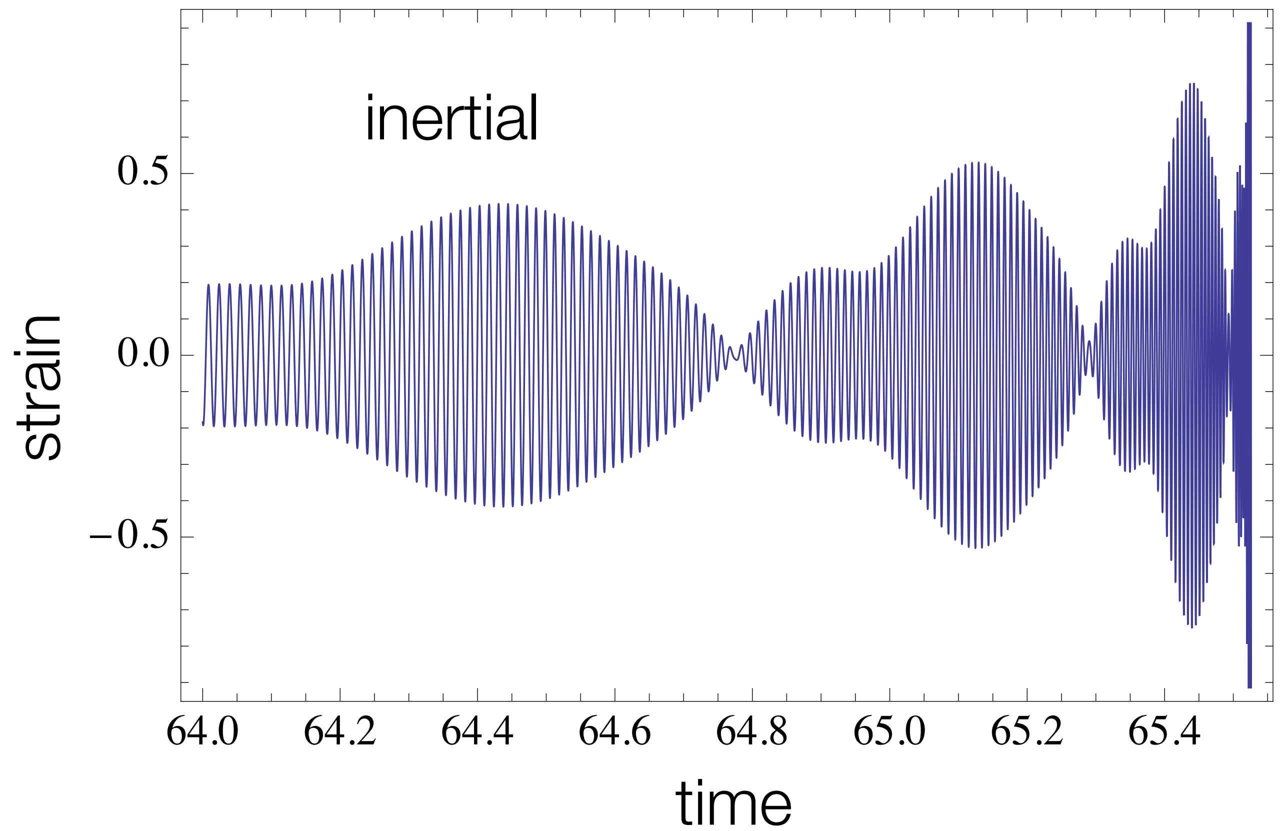
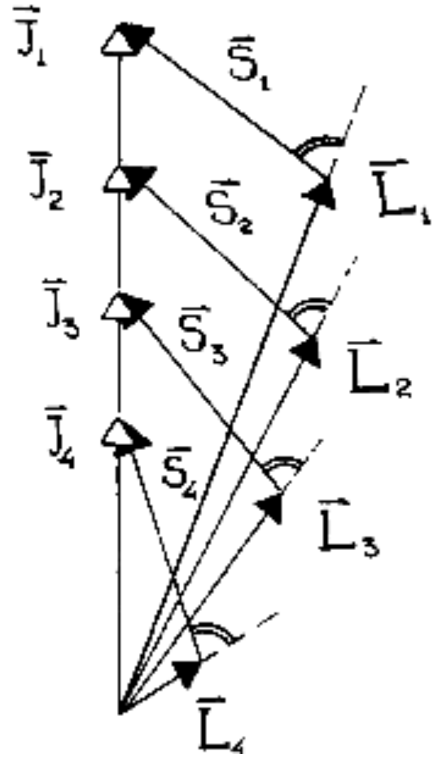






Apostolatos et al 1994

Measuring gravitational waves

Detector

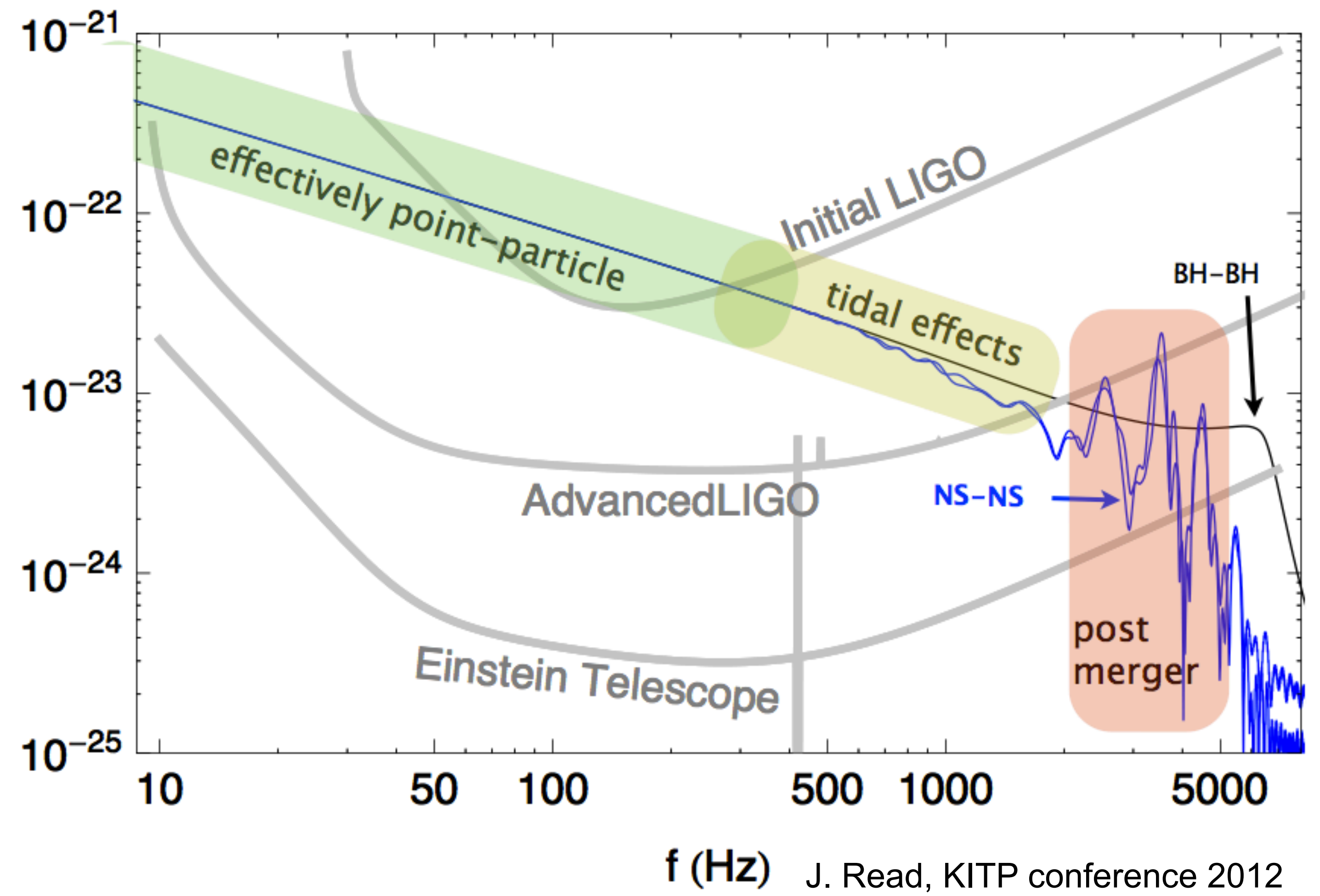
- Nearly gaussian, stationary

$$\langle n^*(f)n(f)\rangle = \frac{1}{2}S_h(|f|)\delta(f-f')$$

$$p(\{d\}|H_0) \propto \exp - \frac{\langle d|d\rangle}{2} dd_1 dd_2 \dots dd_N$$

$$\langle a|b\rangle \equiv 2\int_{-\infty}^{\infty} df \frac{a^*(f)b(f)}{S_h(|f|)}$$

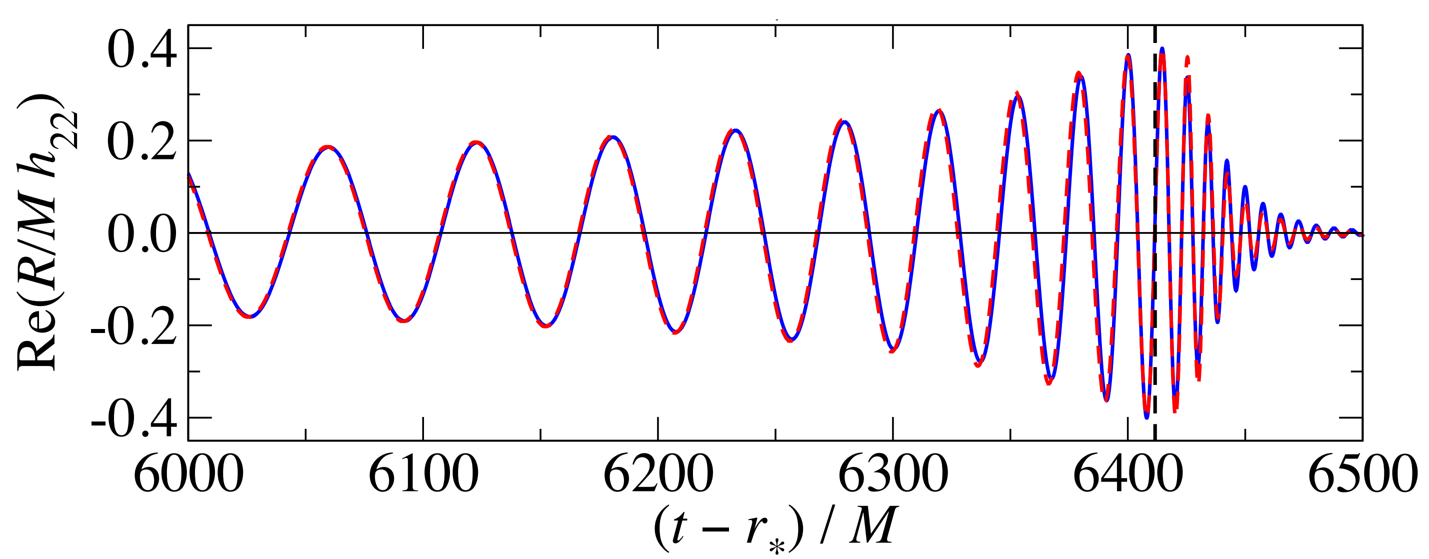
- Band limited



J. Read, KITP conference 2012

Signal

- More cycles at low frequency
- “Typical” merger physics not in band
  - “Input” binary dominates
- Orbital phase: degenerate evolution



Taracchini et al 2013 (1311.2544)

Inferring source parameters

- Evidence for signal

$$Z(d|H_1) \equiv \frac{p(\{d\}|H_1)}{p(\{d\}|H_0)} = \int d\lambda p(\vec{\lambda}|H_1)\frac{p(\{d\}|\vec{\lambda}, H_1)}{p(\{d\}|H_0)}$$

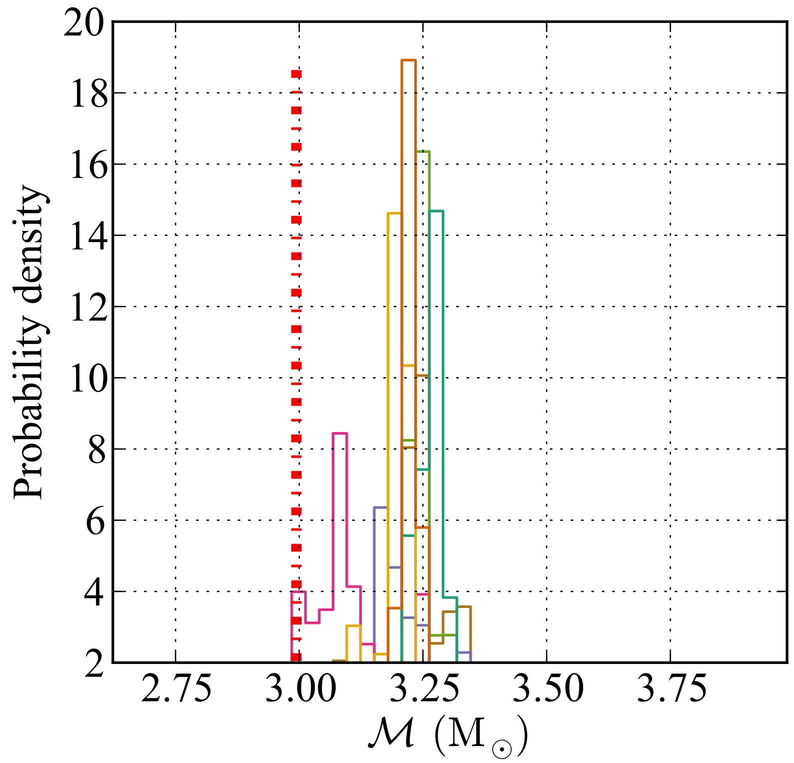
H_1 : with signal
H_0 : no signal

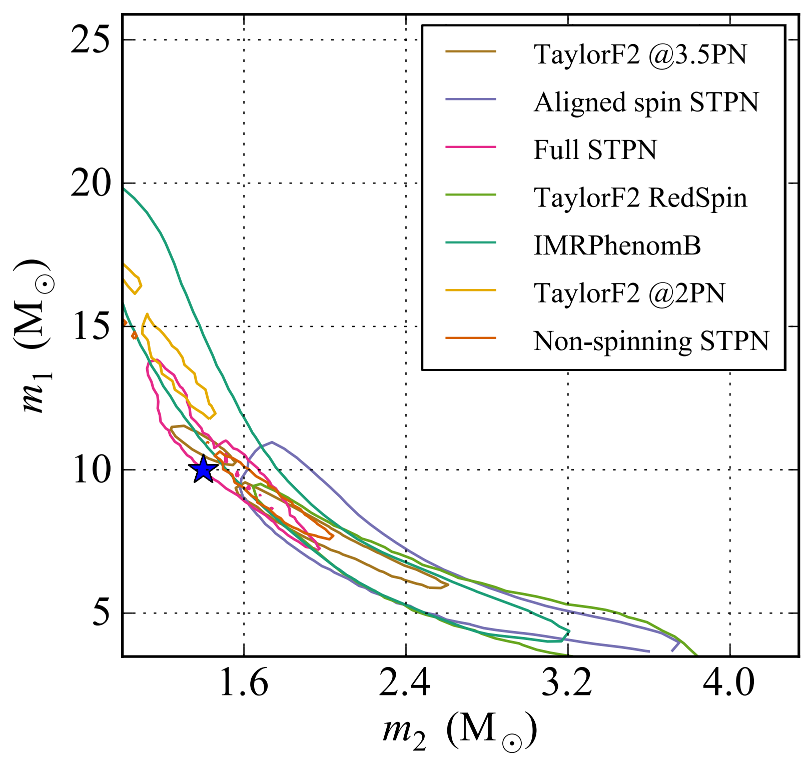
posterior distribution

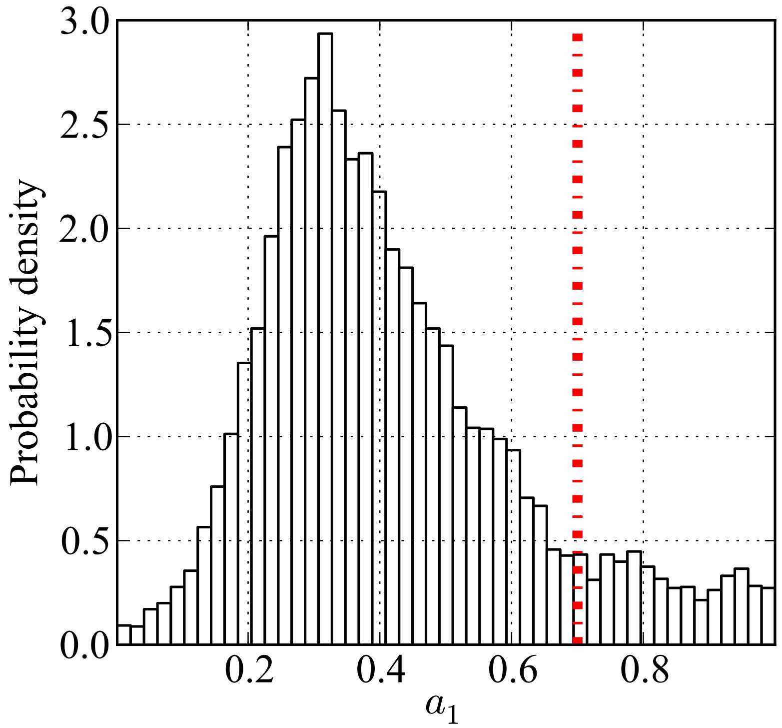
- Inputs:
  - Prior knowledge $p(\lambda|H_1)$ about distribution of λ
  - Signal model $h(\lambda)$
  - Noise model $p(\{d\}|H_0)$ $p(\{d\}|\vec{\lambda}, H_1) = p(\{d - h(\vec{\lambda})\}|H_0)$
  - Algorithm for integral/exploration in many dimensions

Comparison: S6 PE paper (BH-NS)

- Same framework [earlier], smaller study
- No analysis or geometry







Aasi et al 2013