Modeling 1/10 to 1/100 Black-Hole Binaries in Full Numerical Relativity with Lazev /Carpet

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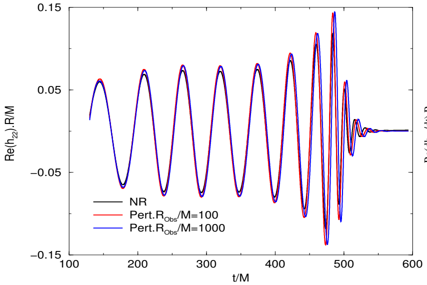
Intermediate-Mass-Ratio Binaries
The Science Driver for our project

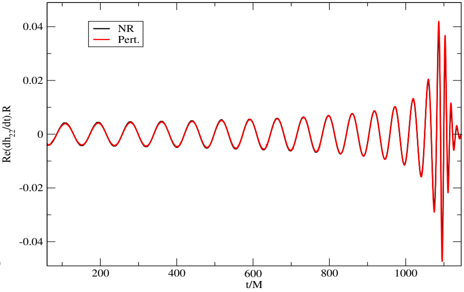
- SMBH have masses in the range $10^5 \lesssim M \lesssim 10^{10}$
- SMBHBs should have mass ratios $10^{-5} \lesssim q \lesssim 1$ ($1/10 \leq q \leq 1$ most likely)
- SMBH/BH binaries will have mass ration $10^{-9} \lesssim q \lesssim 10^{-2}$
- In general SMBH seem to be highly spinning.
- So we need accurate models for the waveform and dynamics from IMR mergers.
- These simulations are very challenging numerically.
- Low q limit first explored in Baker et al. PRD 2008 ($q = 1/6$), Gonzalez et al PRD 2009 ($q = 1/10$), Lousto and YZ PRD 2009 ($q = 1/8$ spinning).
- Small q gauge condition: Jena (Meuller et al CGQ 2010 and Meuller et al arxiv:2010), Schnetter 2010, AEI (Alic et al arxiv:2010).
- RIT worked on small q limit 2 years, worked on modifications to Jena gauge (beginning summer 2009).
- It is crucial to develop a model for the waveform and dynamics (Recoils, precession, $\cdots$) of these binaries

Modeling IMR Merger waveforms

[Lousto, Nakano, YZ, Campanelli PRL 2010], Lousto et al arXiv:1008.4360 [gr-qc]

- Model trajectories for BHB inspirals with small, but numerically feasible mass ratios ($q = 1/10$ downto $q = 1/100$?).
- Extrapolate trajectory to smaller q
- Model the waveform, perturbatively (of Schwarzschild), as a function of trajectory (and q).
- Remnant spin introduced perturbatively, magnitude predicted by empirical formula (Lousto CQG 2010)

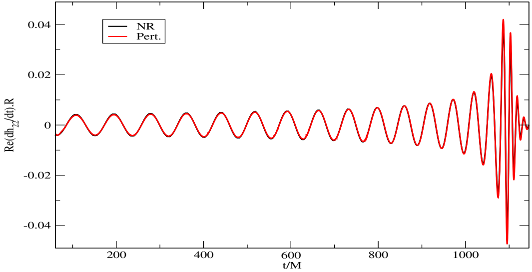


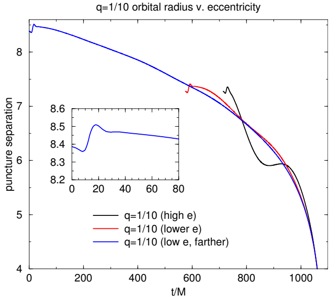


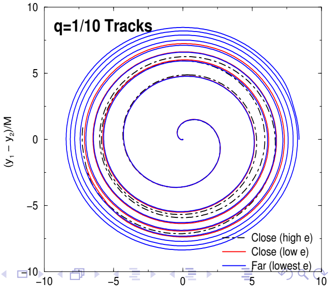
Lower eccentricity q=1/10 Runs

- BY ID leads to a burst of radiation that distorts orbit (also gauge effects). the given ID parameters don't correspond to the actual binary momenta and positions.
- Can generate low eccentricity BHBs, but it's expensive.
- Get good agreement between NR and Pert waveforms
- Can get better ID with PN-inspired ID with lower radiation content

  and 'correct' wave content (Mundim et al. Work in Progress).

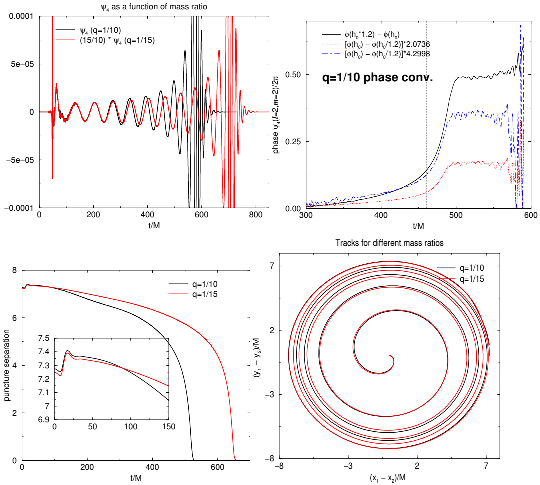






Comparing small q Runs

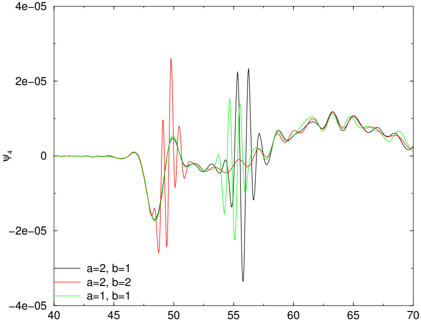
- Waveform amplitude scales with q and lower q gives more orbits ($q = 1/10$ and rescaled $q = 1/15$ waveforms shown).
- get a 'universal' plunge but last few orbits differ as $q \to 0$
- Require iterative procedure to get low eccentricity.
- Initial 'Jump' in the orbit roughly independent of q as $q \to 0$ (gauge + initial burst).
- Consistent results for $q = 1/10$, $E_{rad} = 0.0045$, $\Delta M = 0.0046$, $J_{rad} = 0.052$, $\Delta J = 0.050$, $V_{kick} = 60 \mathrm{km\,s}^{-1}$, $Vp = 62 \mathrm{km\,s}^{-1}$, $\delta\phi(\omega = 0.2) = 0.03 rad$ (pred: 8th order*)
- Consistent results for $q = 1/15$, $E_{rad} = 0.0022$, $\Delta M = 0.0023$, $J_{rad} = 0.023$, $\Delta J = 0.0229$, $V_{kick} = 33.5$, $Vp = 34.1$, $\delta\phi(\omega = 0.2) = 0.12 rad$ (pred: 8th order*)



The setup for $q = 1/100$

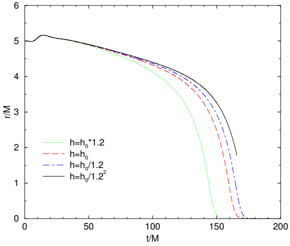
- Proof-of-Principle simulation that shows that existing codes can be modified to evolve small q binaries.
- Nonspinning BHs
- Brandt-Bruegmann Puncture ID
- 8th order centered FD
- 4th order RK in time
- 5th order spatial prolongation
- 2nd order time prolongation
- Outer Boundaries at 400 M
- 15 levels of refinement
- 2nd smallest grid has radius $4r_H$ (use RW/Z potentials to guide mesh locations)
- 1 level inside the AHs
- $W = \sqrt{\chi}$ conformal factor
- Gauge: $(\partial_t - \beta^i \partial_i)\alpha = -2\alpha K$, $\partial_t \beta^a = \frac{3}{4}\tilde{\Gamma}^a - \eta(x^k, t)$
- η modified version of Mueller et al $\eta(x^k, t) = R_0 \frac{\sqrt{\partial_i W \partial_j W \tilde{\gamma}^{ij}}}{(1 - W^a)^b}$, $a = 2$, $b = 2$, $R_0 = 1.3$ used in simulations

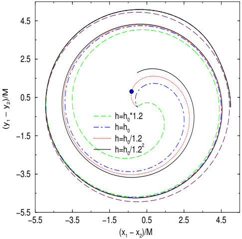
- Gauge speeds $V_{2,2} > V_{2,1} > V_{1,1}$
- $V_{1,2}$ (Jena's original form) unstable for our GH
- Gauge noise (reflections of GW at AMR boundaries) smallest for $(2, 2)$

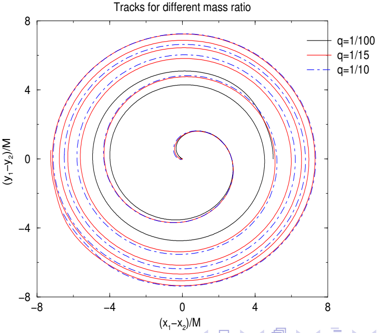


$q = 1/100$ QC Tracks

- obtain 2+ orbits *inside ISCO*
- Convergence 8th order.. but evidence for a 2nd order convergent error at very high resolutions near the end of the simulation (prolongation)
- See 'universal' plunge at end. The very last part of the plunge seems to be geodesic
- Low eccentricity $e \sim 0.003$

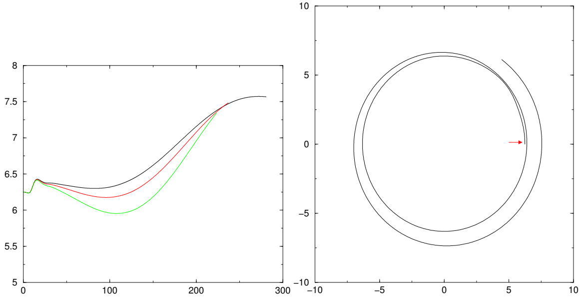




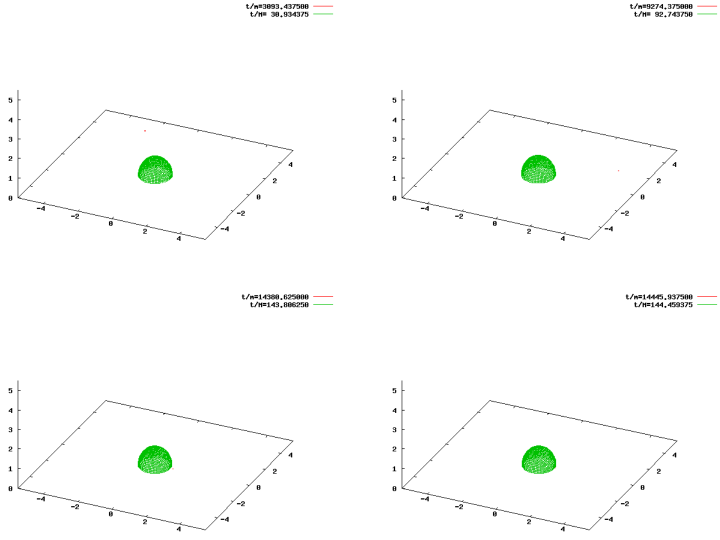


A note on ID

- Obtaining QC data tricky. Simple PN data leads to “zoom-whirl” look orbits (r increases and decreases on periods several orbits long).
- Require iterative procedure (Pfeiffer et al CGQ 2007).
- Closer orbits easier to obtain than farther out.



AHs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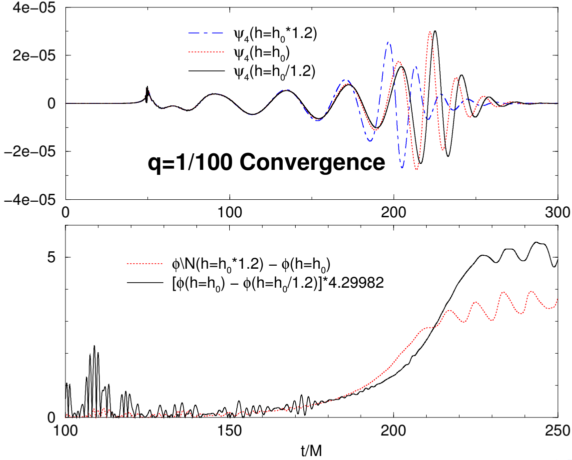
t/M= 92.743750
t/m=14380.625000
t/M=143.806250
t/m=14445.937500
t/M=144.459375

Waveform

- Phase convergent to 8th order at given resolutions
- Note amplitude (1/10 amplitude of $q = 1/10$).
- Prolongation effects at higher resolutions



Remnant

Remnant horizon parameters and radiated energy-momentum. Here we provide $\delta M_H^* = M_{\rm ADM} - M_H$ and $\delta S_H^* = J_{ADM} - S_H$, which are small numbers obtained by taking the difference between two much larger numbers. The calculation of δS_H^* is relatively inaccurate because it requires an extrapolation to infinite resolution.

E_{rad}	0.000060 ± 0.000001	δM_H^*	0.00007 ± 0.00001
J_{rad}	0.00050 ± 0.00002	δS_H^*	0.0003 ± 0.0002
α	0.0333 ± 0.0002		
$\eta^{-2} E_{rad}^{\omega>0.167}/M$	$0.489 \pm .010$	0.47 (P.L. 4.0%)	
$\eta^{-2} J_{rad}^{\omega>0.167}/M^2$	3.54 ± 0.10	3.44 (P.L. 2.9%)	

- Get very good agreement with perturbative plunge calculations

Remnant spin and total radiated mass (starting from infinite separation) as a function of mass ratio q as measured in our simulations and as predicted by our empirical formulae.

q	1/10	1/15	1/100
α (Comp)	0.2603	0.18875	0.0333
α (Pred)	0.2618	0.1903	0.03358
δM (Comp)	0.00826	0.00507	0.000618
δM (Pred)	0.00806	0.00498	0.000604

- Get very good agreement with empirical formula. No fitting.

Numerical Challenges

- $q = 1/100$ simulation is a Proof-of-Principle that existing codes can be modified to evolve small q binaries.
- Time prolongation errors become important at high resolutions.
- Higher-order time prolongation will be useful to get cumulative waveform phase errors to low levels post merger.
- Regridding is expensive
- HDF5 memory issues lead to the use of more cores than ideal, or long recovery times.
- AMR boundary reflection from strong ID and gauge pulses contaminate waveform.
- Will require efficient evolutions with more levels of refinement (15-16 levels for $q = 1/100$, ~ 20 for $q = 1/1000$).
- True AMR would better guide the placement and size of refined levels
- Multiple 'pre' evolutions required to specify QC data
- ID with less junk and a better method for finding QC parameters
- ID parameters from PN evolutions not superior to QC parameters. An iterative procedure is required.
- We showed 8th-order convergence of waveform phase (prior to prolongation contamination) for $q = 1/10$ to $q = 1/100$.
- Existing Carpet / EinsteinToolkit codes can handle small q limit$\cdots$ Now it's a question of making the codes more efficient in these regimes.

Open Discussion

We showed that q=1/100 is possible as a proof of principle. Our simulations indicate several key areas for Carpet improvements to make efficient simulations.

- Scaling to 10,000's cores. (Small q simulations require larger memory footprints and more grid points.)
- Higher-Order time prolongation
- Faster regridding
- True AMR or some better guided refinement
- Efficient evolutions with more levels of refinement.
- Use of perturbation techniques (e.g. RW/Z to guide mesh refinement and further insight into the spacetime near the small BH).
- Fine tuning of gauges is important for efficient evolutions.