STAT 406-01: Mathematical Statistics II

Problem Set 3

Assigned 2016 February 9
Due 2016 February 18

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Hogg 11.3.4

2 Hogg 11.3.7

3 Hogg 11.3.1

Extra credit: numerically estimate the parameters which maximize the posterior, to an accuracy of 1%.

4 Haldane Prior

Consider the problem of Bernoulli trials with probability parameter θ. Define a new parameter $\lambda = \ln \frac{\theta}{1-\theta}$, the *log odds ratio*.

(a) If the prior pdf on the log odds ratio is $f_\Lambda(\lambda)$, perform a change of variables to write $f_\Theta(\theta)$ in terms of $f_\Lambda\left(\ln \frac{\theta}{1-\theta}\right)$.

(b) Show that a uniform prior for λ is equivalent to the improper Haldane prior $f_\Theta(\theta) \propto \frac{1}{\theta(1-\theta)}$ for θ.

5 Bayesian Hypothesis Testing

Let $X_1, \ldots X_n$ be a sample of size n drawn from a $b(1,\theta)$ distribution (i.e., the results of n Bernoulli trials with probability θ.

(a) Write the likelihood function $f_{\mathbf{X}|\Theta}(x_1, \ldots, x_n|\theta)$ and show $y = \sum_{i=1}^n x_i$ is a sufficient statistic for the parameter θ.

(b) Let H_0 be the simple hypothesis that $\theta = 0.5$. Calculate the evidence $f(\mathbf{x}|H_0)$.

(c) Let H_1 be the composite hypothesis that assigns a uniform prior to Θ for $0 < \theta < 1$. Calculate the evidence $f(\mathbf{x}|H_1) = \int_0^1 f_{\mathbf{X}|\Theta}(\mathbf{x}|\theta)\, f_\Theta(\theta|H_1)\, d\theta$

(d) Construct the Bayes factor $f(\mathbf{x}|H_1)/f(\mathbf{x}|H_0)$ relating the two models.

(e) Suppose you hear that six out of six coin flips have come up heads. Use the Bayes factor to relate the change in relative plausibility of hypotheses H_1 and H_0 above.

(f) Suppose further that your a priori assessment was that hypothesis H_0 (fair coin) was 1000 times as likely as hypothesis H_1 (a coin which is unfair in some unknown way). What relative probabilities would you assign to the hypotheses after learning of the six straight heads?