STAT 406-01: Mathematical Statistics II

Problem Set 2

Assigned 2016 February 2
Due 2016 February 9

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Hogg 11.2.4

2 Hogg 11.2.6

3 Hogg 11.2.9

Also verify that Y_4 *is* a sufficient statistic for θ by showing that $f_{\mathbf{X}|\Theta}(x_1, x_2, x_3, x_4|\theta) \propto f_{Y|\Theta}(y_4|\theta)$, where the proportionality constant can depend on $\mathbf{x}$ but not θ.

4 Poisson Process Rate Estimate

Consider a Poisson process with an unknown rate Θ whose prior pdf is a Gamma(α, β) distribution.

(a) Suppose we observe y events of this process, whose arrival times have spacings $\{x_1, \cdots, x_y\}$, i.e., the first event comes at time x_1, the second at time $x_1 + x_2$, etc, up to the yth event at time $x_1 + x_2 + \cdots + x_y$. If $\Theta = \theta$, $\{x_1, x_2, \ldots, x_y\}$ is a sample of size y drawn from some distribution. What is it?

(b) Work out the posterior distribution $f_{\Theta|\mathbf{X}}(\theta|x_1, \ldots, x_y)$ associated with the information in part (a).

(c) Suppose now that we observe for a specified time interval t and count a total of y events. Write the likelihood function $p_{Y|\Theta}(y|\theta)$ corresponding to this observation.

(d) Work out the posterior distribution $f_{\Theta|y}(\theta|y)$ and compare your result to what you found in part (b).

5 Hogg 11.3.2

Extra Credit: numerically determine the 90% HDR credible interval, as well as the 90% credible interval with equal tail probabilities.

6 Hogg 11.3.9