Show your work on all problems!

1. Devore Chapter 3, Problem 12
2. Devore Chapter 3, Problem 18
3. Devore Chapter 3, Problem 32
4. Devore Chapter 3, Problem 46
5. Computational Exercise (Extra Credit)

This exercise lets you apply the binomial distribution and Bayes’s theorem to consider the interpretation of a (somewhat) realistic experiment.

Suppose that you have a box containing ten six-sided dice. Nine of them are fair (1/6 chance of rolling each number), and one is loaded so that it has a 50% chance of rolling a six. Suppose you pick up one of the dice, roll it \( n = 30 \) times, and count how many sixes you get.

a. If you choose the fair die, the random variable \( X \) representing the number of sixes will obey a binomial distribution

\[
p(x|\text{fair}) = \binom{n}{x} \left( \frac{1}{6} \right)^x \left( \frac{5}{6} \right)^{n-x}
\]

Use a computer to plot \( p(x|\text{fair}) \) versus \( x \) for all of the possible values of \( x \). (Hint: if you’re using python, the binomial coefficient \( \binom{n}{x} \) can be calculated with the scipy function \( \text{comb}(n,x) \), so you need

```python
from scipy import comb
```
Also, it’s a good idea to use $1./6.$ rather than $1/6$ to avoid the gotchas of integer division.

b. If you choose the loaded die, $X$ will obey

$$p(x|\text{loaded}) = \binom{n}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x}$$

Use a computer to plot $p(x|\text{loaded})$ versus $x$ for all of the possible values of $x$.

c. If you choose a die at random, the a priori probability of choosing a fair die is $p(\text{fair}) = .9$ while $p(\text{loaded}) = .1$. Using the law of total probability, you can find

$$p(x) = p(x|\text{fair})p(\text{fair}) + p(x|\text{loaded})p(\text{loaded})$$

use a computer to plot $p(x)$ vs $x$.

d. You can now use Bayes’s theorem to calculate

$$p(\text{fair}|x) = \frac{p(x|\text{fair})p(\text{fair})}{p(x)}$$

for each possible value of $x$. Use a computer to plot $p(\text{fair}|x)$ vs $x$.

e. Find the explicit value of $p(\text{fair}|15)$, the probability that you chose the fair die, given that 15 out of 30 rolls were sixes.