1016-351-03
Probability

Problem Set 3

Assigned 2010 March 23
Due 2010 March 30

Show your work on all problems!

1 Devore Chapter 3, Problem 12

2 Devore Chapter 3, Problem 18

3 Devore Chapter 3, Problem 30

4 Devore Chapter 3, Problem 46

5 Computational Exercise (Extra Credit)

This exercise lets you apply the binomial distribution and Bayes's theorem to consider the interpretation of a (somewhat) realistic experiment.

Suppose that you have a box containing ten six-sided dice. Nine of them are fair (1/6 chance of rolling each number), and one is loaded so that it has a 50% chance of rolling a six. Suppose you pick up one of the dice, roll it $n = 30$ times, and count how many sixes you get.

a. If you choose the fair die, the random variable X representing the number of sixes will obey a binomial distribution

$$p(x|\text{fair}) = \binom{n}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{n-x}$$

Use a computer to plot $p(x|\text{fair})$ versus x for all of the possible values of x. (Hint: if you're using python, the binomial coëfficient $\binom{n}{x}$ can be calculated with the scipy function `comb(n,x)`, so you need

```
from scipy import comb
```

Also, it's a good idea to use `1./6.` rather than `1/6` to avoid the gotchas of integer division.

b. If you choose the loaded die, X will obey

$$p(x|\text{loaded}) = \binom{n}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{n-x}$$

Use a computer to plot $p(x|\text{loaded})$ versus x for all of the possible values of x.

c. If you choose a die at random, the a priori probability of choosing a fair die is $p(\text{fair}) = .9$ while $p(\text{loaded}) = .1$. Using the law of total probabilty, you can find

$$p(x) = p(x|\text{fair})p(\text{fair}) + p(x|\text{loaded})p(\text{loaded})$$

use a computer to plot $p(x)$ vs x.

d. You can now use Bayes's theorem to calculate

$$p(\text{fair}|x) = \frac{p(x|\text{fair})p(\text{fair})}{p(x)}$$

for each possible value of x. Use a computer to plot $p(\text{fair}|x)$ vs x.

e. Find the explicit value of $p(\text{fair}|15)$, the probability that you chose the fair die, given that 15 out of 30 rolls were sixes.