# Physics A301: Classical Mechanics II

Problem Set 3

Assigned 2006 May 18
Due 2006 May 25

**Show your work on all problems!** Be sure to give credit to any collaborators, or outside sources used in solving the problems.

## 1 Periodically Accelerating Reference Frame

Symon Chapter Seven, Problem Two. Call the unstretched length of the spring $\ell_0$.

## 2 Rotational Oblateness of the Earth

Consider the Earth, rotating at a fixed angular velocity $\vec{\omega}$. Define two sets of Cartesian coördinates, $(x, y, z)$ and $(x^*, y^*, z^*)$, each with its origin at the center of the Earth and its positive $z$ (or $z^*$) axis pointed towards the North Pole along the Earth's rotation axis, so that $\vec{\omega} = \omega\hat{z} = \omega\hat{z}^*$. Let the unstarred coördinate axes be fixed in space while the starred ones rotate along with the Earth.

a) Define spherical coördinates, $(r, \theta, \phi)$ and $(r^*, \theta^*, \phi^*)$, respectively, corresponding to the non-rotating and rotating Cartesian coördinate systems. Explain (or show) why $r^* = r$ and $\theta^* = \theta$, and also $\hat{r}^* = \hat{r}$ and $\hat{\theta}^* = \hat{\theta}$.

b) Assume that most of the matter in the Earth is spherically symmetric about its center, so that the gravitational field is well approximated by

$$\vec{g} = -g(r)\,\hat{r} \tag{2.1}$$

where the magnitude $g(r)$ is a function of $r$ alone. Find the effective gravitational field $\vec{g}^{\text{eff}}$ including centrifugal effects in the co-rotating coördinate system, in terms of $g(r)$, $\omega$, $r$, $\theta$, and the basis vectors $\hat{r}$ and $\hat{\theta}$.

c) In the limit $\omega \to 0$, the Earth is just a sphere of radius $R$ and mass $M$, and the gravitational acceleration at its surface is

$$g_0 = \frac{GM}{R^2}\ . \tag{2.2}$$

Construct the dimensionless combination of $\omega$, $g_0$, and $R$ which is proportional to $\omega^2$, and call this $\varepsilon$.

d) Let the actual shape of the surface of the rotating Earth be given by

$$r = R + \delta R(\theta) \tag{2.3}$$

where $\delta R(\theta)$ is a small correction of order $\varepsilon$. If $\vec{n}$ is a vector normal (perpendicular) to this surface, what is the ratio of components $n_\theta/n_r$?

e) Using the results of part b), construct the ratio $g_\theta^{\text{eff}}/g_r^{\text{eff}}$. This should be proportional to $\omega^2$ and therefore also of order $\varepsilon$.

f) In your answer to part e), set $r$ to $R$ and neglect the $\omega^2$ term appearing in the *denominator*. (These approximations are justified because anything more accurate would just add a correction of order $\varepsilon^2$.) Express this approximate ratio in terms of $g_0$, $R$, $\omega$, and $\theta$. Verify that you get sensible results at the North Pole ($\theta = 0$), the Equator ($\theta = \pi/2$) and the South Pole ($\theta = \pi$).

g) By requiring $\vec{g}^{\text{eff}}$ to be normal to the surface of the Earth, obtain and solve a differential equation for $\delta R(\theta)$. (Don't forget to include the integration constant in the solution.) What is the difference between the polar and equatorial radii, in terms of $g_0$, $R$, and $\omega$?

h) Using the actual values for the Earth ($g_0 = 9.8\,\text{m/s}^2$, $\omega = 2\pi/(24\,\text{hr})$, and $R = 6.4 \times 10^3\,\text{km}$), evaluate the following ratios to two significant figures:

  i) $\varepsilon$, the "small" parameter defined in part c);

  ii) The fractional decrease of the magnitude of $\vec{g}^{\text{eff}}$ at the equator relative to the poles;

  iii) $\frac{\delta R(\pi/2) - \delta R(0)}{R}$, the size of the equatorial bulge as a fraction or the Earth's radius.

# 3 Deflection of a Falling Object Due to Coriolis Force

Do Symon Chapter Seven, Problem Seven. Also evaluate your expression for the displacement, for an object dropped from a height of 20 meters at a latitude of 30°N, and specify the direction of the deflection.