Physics A300: Classical Mechanics I

Problem Set 5

Assigned 2006 February 21
Due 2006 March 3

Show your work on all problems!

1 Fourier Analysis

Consider a square wave of period $T$ and amplitude $x_0$, which is given inside the interval $0 < t < T$ by

$$F(t) = \begin{cases} F_0 & \text{when } 0 < t < T/2 \\ -F_0 & \text{when } T/2 < t < T \end{cases}$$

and which is defined outside that interval by its periodicity: $F(t + T) = F(t)$.

a) Use the methods described in section 2.11 of Symon to find the coefficients $A_n$ and $B_n$ in the Fourier series

$$F(t) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} \left( A_n \cos \frac{2\pi nt}{T} + B_n \sin \frac{2\pi nt}{T} \right)$$

Express your answers in a form involving no sines or cosines. (To do this, you’ll need to write different expressions for odd and even $n$.)

b) Use the coefficients $\{A_n\}$ and $\{B_n\}$ to write the force in the form

$$F(t) = \sum_{n=0}^{\infty} A_{0n} \cos(\omega_n t + \theta_{0n}) .$$

What are $A_{0n}$, $\omega_n$, and $\theta_{0n}$?

c) If the square wave is applied to a harmonic oscillator of natural frequency $\omega_0$ and damping parameter $\gamma$, i.e.: 

$$\ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = F(t) ,$$

use the principle of superposition to find the values of $A_{sn}$, and $\beta_n$ in the expansion

$$x(t) = \sum_{n=0}^{\infty} A_{sn} \sin(\omega_n t + \theta_{0n} + \beta_n)$$

of the steady-state solution.
2 Vector Practice

When doing the following problems from Symon, be sure to put an arrow over each vector (except for unit vectors, which get a hat). Symon uses boldface, but that’s easy to lose track of, so for this course we’ll insist on the arrow notation. So for example, Symon writes his equation (3.10) as

\[ \mathbf{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \]

while we will write

\[ \vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \]

2.1 Explicit Calculation

Consider the vectors

\[ \vec{A} = \hat{x} + 2\hat{y} - \hat{z} \quad \vec{B} = -2\hat{x} + 3\hat{y} + \hat{z} \]

Calculate:

a) \( \vec{A} - \vec{B} \) and its magnitude \( |\vec{A} - \vec{B}| \)

b) \( \vec{B} \cdot \vec{A} \)

c) The angle between \( \vec{A} \) and \( \vec{B} \)

d) \( \vec{A} \times \vec{B} \)

e) \( (\vec{A} - \vec{B}) \times (\vec{A} - \vec{B}) \)

2.2 Symon Chapter Three Problem 2

2.3 Symon Chapter Three Problem 5