Physics A410: Thermal Physics

Problem Set 1

Assigned 2004 January 13
Due 2004 January 20

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems.

1 Schroeder 1.1

2 Schroeder 1.3

3 Schroeder 1.12

4 The Exponential Atmosphere

Consider an idealized atmosphere in which pressure P and density ρ depend only on height z and the acceleration of gravity is a constant g in the negative-z direction.

(a) Consider a "slab" of air of infinitesimal thickness dz, width L, and length L. Let dz be so small that we can neglect any terms proportional to dz^2. In particular, if the lower face of the slab is at $z = z_0$ and the upper face at $z = z_0 + dz$, we can approximate $P(z) \approx P(z_0) + P'(z_0)\,(z - z_0)$ for $z_0 \leq z \leq z_0 + dz$. (Here $P'(z) = \frac{dP}{dz}$ is the derivative.)

  (i) Assuming that the only forces on the slab are the gravitational force due to the earth and the force due to the pressure of the adjacent air, find the magnitude *and direction* of each of the individual forces on the slab (due to the Earth, the air above, the air below, the air to the left, etc). Be sure to drop any terms quadratic or higher in dz.

  (ii) Calculate the net component of force in each of the x, y, and z directions.

  (iii) By requiring the slab to be at rest, derive an equation which you can solve for $P'(z_0)$ in terms of $\rho(z_0)$, $P(z_0)$, and g. Since z_0 is arbitrary, you have derived a differential equation for $P'(z)$ in terms of $\rho(z)$, $P(z)$, and g.

(b) Assuming that the atmosphere is made of an ideal gas, write the density in terms of the pressure, temperature, and the average mass m of an air molecule. Use this to show that your differential equation from part **(a)** is equivalent to

$$\frac{dP}{dz} = -\frac{mg}{kT} P \tag{4.1}$$

which is called the *barometric equation*.

(c) Making the further (not terribly accurate) idealization that the temperature is independent of the height in the atmosphere, solve the barometric equation to obtain the pressure as a function of the height: $P(z) = P(0)e^{-mgz/kT}$. Derive a similar equation for the density.