Physics A300: Classical Mechanics I

Problem Set 3

Assigned 2002 September 16
Due 2002 September 23

Show your work on all problems!

1 Volume Integral (M & T 1-37)

Find the value of the integral
\[ \iiint_S \vec{A} \cdot d^2a \]

where
\[ \vec{A} = (x^2 + y^2 + z^2)(xe_x + ye_y + ze_z) \]

and the surface \( S \) is a sphere of radius \( R \) centered on the origin. Do the integral
a) directly, and also
b) by using Gauss’s theorem

2 Curls and Gradients

2.1 Curl of a Gradient

Let \( \varphi \) be any scalar field and consider the vector field
\[ \vec{\nabla} \times (\vec{\nabla} \varphi) \]

Write the expression for the \( i \)th component of this vector field using the Levi-Civita symbol \( \varepsilon_{ijk} \) and, use a standard property of partial derivatives to simplify the expression. Write your final result in vector notation.

2.2 Integral of a Gradient Around a Closed Loop

Consider a curve \( C_1 \) which begins at point \( P \) and ends at point \( Q \) and another curve \( C_2 \) which begins at \( Q \) and ends at \( P \). Let \( C \) be the closed curve which goes from \( P \) to \( Q \) along \( C_1 \), then comes back from \( Q \) to \( P \) along \( C_2 \). Calculate the integral
\[ \oint_C \vec{\nabla} \varphi \cdot d\ell \]

of an arbitrary scalar field \( \varphi \) along the closed curve \( C \) by two methods:
a) Break up the integral along $C$ into the piece along $C_1$ and the piece along $C_2$, evaluate each separately, and combine them:

$$\oint_C \vec{∇}\varphi \cdot d\vec{ℓ} = \int_{C_1} \vec{∇}\varphi \cdot d\vec{ℓ} + \int_{C_2} \vec{∇}\varphi \cdot d\vec{ℓ}$$

(You should be able to simplify this expression.)

b) Let $S$ be any surface whose boundary $\partial S$ is $C$, use Stokes’s theorem to rewrite the line integral along $C$ as a surface integral over $S$, and evaluate that integral.

Verify that the two methods give the same answer.

3 Drill Problem on Dimensional Analysis

3.1 Dimensionally Meaningful Expressions

Which of the following expressions or relations are sensible from a dimensional point of view? For the ones which don’t, state the reason why not.

a) $5 \text{ m} + 100 \text{ in}$

b) $40 \text{ cm} + 100 \text{ kg}$

c) $x < 5$ where $x$ is a length

d) $F = mx^2$ where $F$ is a force, $m$ is a mass, and $x$ is a length

e) $\ddot{x} = g \sin t$ where $x$ is a coordinate distance, $g = 9.8 \text{ m/s}^2$, and $t$ is a time

f) $mv^2 = 5G \frac{Mm}{r}$ where $m$ and $M$ are masses, $r$ is a length, and $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

3.2 Conversion of Units

Convert the following expressions into the units requested

a) $\frac{15 \text{ cm} + 45 \text{ m}}{3 \times 10^6 \text{ m/s}}$ expressed in nanoseconds ($1 \text{ s} = 10^9 \text{ ns}$) (Your answer should be exact)

b) $1.25 \text{ in/yr}$ expressed in centimeters per second. (Your answer should be written to three significant figures.)

4 (M & T 2-17)

A softball player hits the ball at a height of $0.7 \text{ m}$ above home plate. The ball leaves the bat travelling in a direction which makes an angle $35^\circ$ with the horizontal, and sails towards a fence $2 \text{ m}$ high and $60 \text{ m}$ away in centerfield. What must the initial speed of the softball be to clear the centerfield fence? Ignore air resistance, and take the acceleration of gravity to be $9.8 \text{ m/s}^2$. 