Physics A300: Classical Mechanics I

Problem Set 1

Assigned 2002 August 28
Due 2002 September 4 (accepted thru September 6)

Show your work on all problems! Note that the answers to the first problem are in the appendix of Marion & Thornton, but they should only be used to check your work, since the answers themselves are an insufficient solution to the problem.

1 Drill Problem on Matrix Operations (M & T 1-14)

Consider the matrices

\[ A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix} \]

Calculate:

a) The determinant \( \det AB \)

b) \( AC \)

c) \( ABC \)

d) \( AB - B^T A^T \)

2 Drill Problem on Vector Operations

2.1 M & T 1-9

Consider the vectors

\[ \vec{A} = \vec{e}_1 + 2\vec{e}_2 - \vec{e}_3, \quad \vec{B} = -2\vec{e}_1 + 3\vec{e}_2 + \vec{e}_3 \]

Calculate:

a) \( \vec{A} - \vec{B} \) and its magnitude \( |\vec{A} - \vec{B}| \)

b) The component of \( \vec{B} \) along \( \vec{A} \)

c) The angle between \( \vec{A} \) and \( \vec{B} \)

d) \( \vec{A} \times \vec{B} \)

e) \( (\vec{A} - \vec{B}) \times (\vec{A} - \vec{B}) \)
3 Properties of Rotation Matrices

3.1 Composition
Show that if \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) are orthogonal (\( \Lambda^T = \Lambda^{-1} \)) and unimodular (\( \det \Lambda = 1 \)), then their product \( \mathbf{R} = \mathbf{R}_1 \mathbf{R}_2 \) also satisfies both of these properties.

3.2 Rotations About Coördinate Axes
a) Write the matrices \( \mathbf{R}_1(\theta_1) \), \( \mathbf{R}_2(\theta_2) \), and \( \mathbf{R}_3(\theta_3) \) describing rotations of \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \) about the \( x_1 \)-, \( x_2 \)-, and \( x_3 \)-axes, respectively.

b) Show explicitly that each of them is an orthogonal matrix (its transpose is equal to its inverse) and unimodular (its determinant equals one).

4 A Meaningful Vector Identity (M & T 1-24)
Let \( \vec{A} \) be an arbitrary vector, and let \( \vec{e} \) be a unit vector (i.e., a vector such that \( \vec{e} \cdot \vec{e} = 1 \)) in some fixed direction.

a) Show that
\[
\vec{A} = \vec{e}(\vec{A} \cdot \vec{e}) + \vec{e} \times (\vec{A} \times \vec{e})
\]

b) What is the geometrical significance of each of the two terms in the expansion?

Note: you may use the results of any of the previous book problems (1-1 to 1-23) in the demonstration.