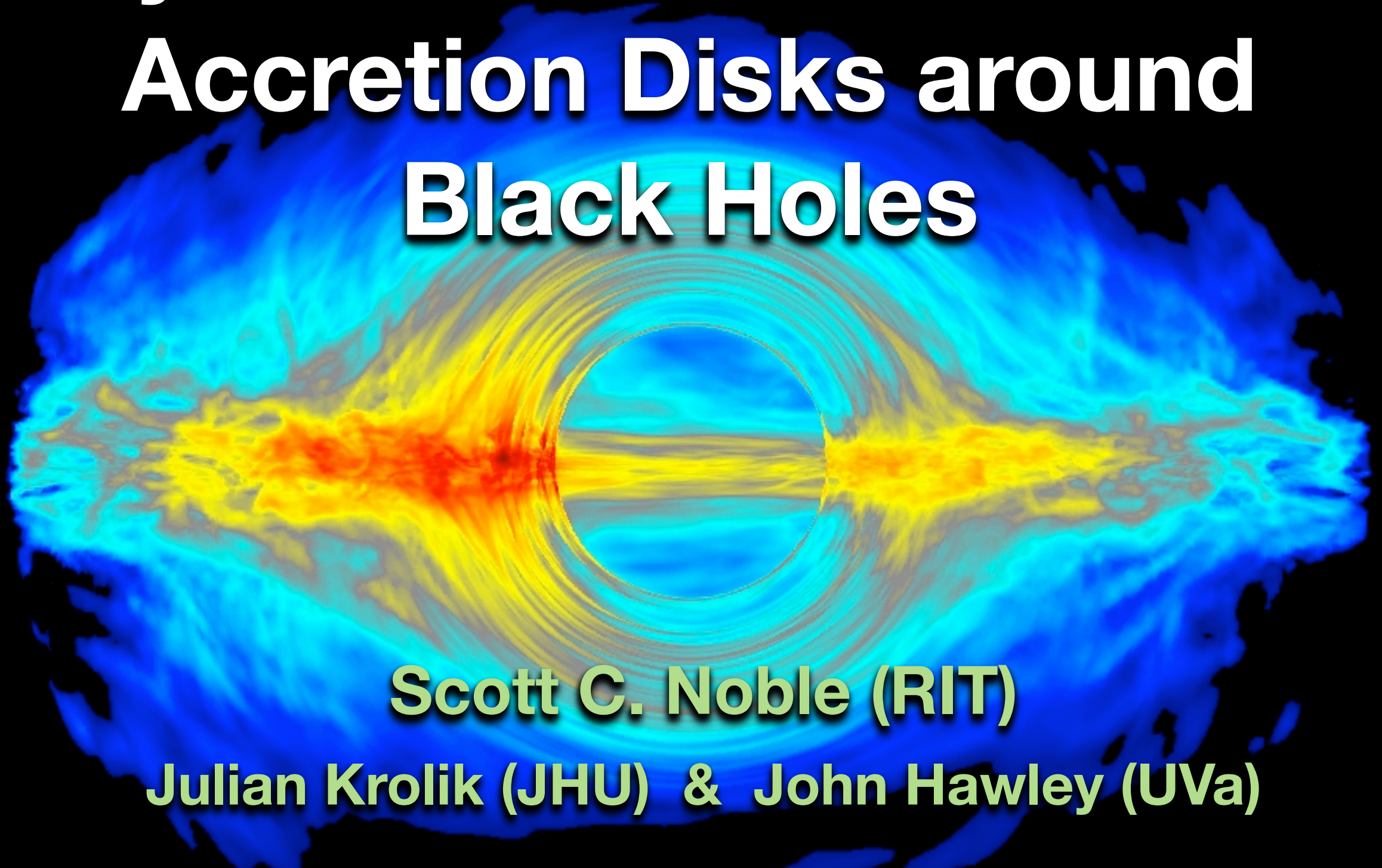


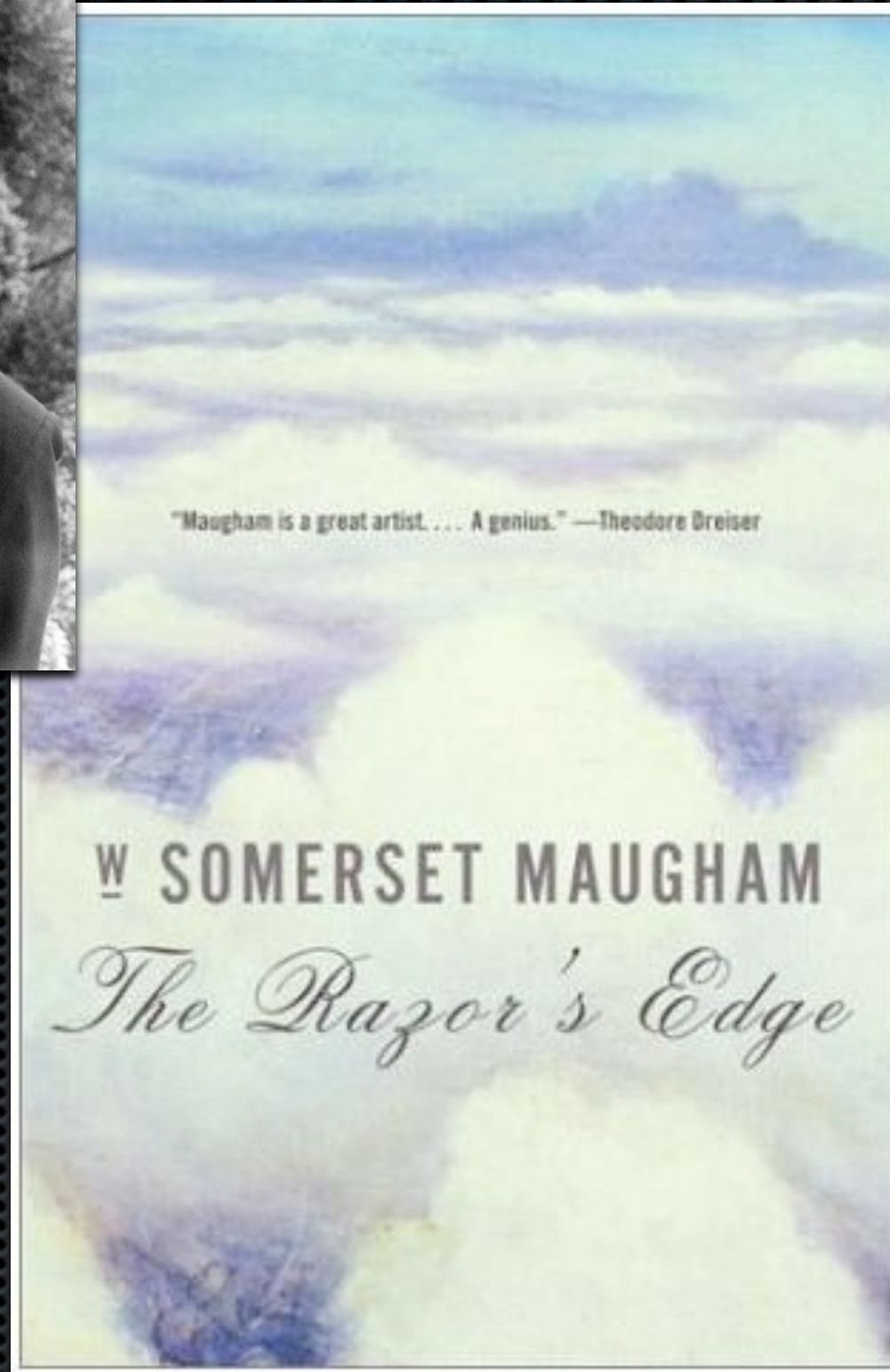
The Razor's Edge: Dynamical Models of Thin Accretion Disks around Black Holes



Scott C. Noble (RIT)

Julian Krolik (JHU) & John Hawley (UVa)

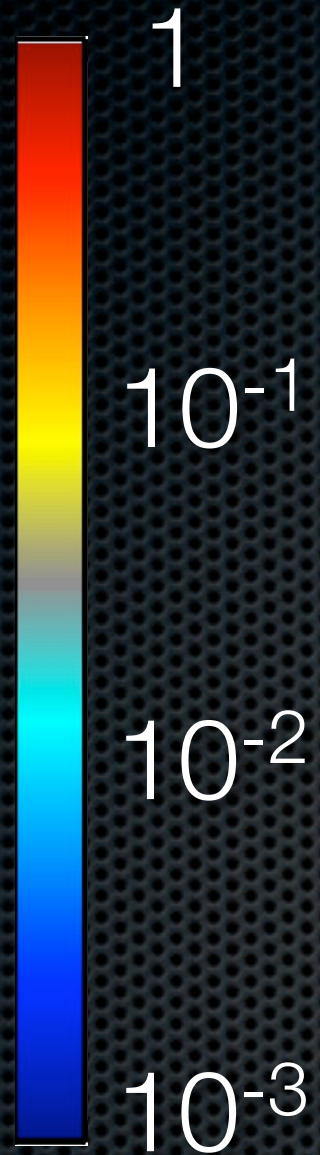
TAPIR Seminar -- Caltech -- May 28, 2010



"The sharp edge of a razor is difficult to pass over; thus the wise say the path to Salvation is hard." -- M. Somerset Maugham

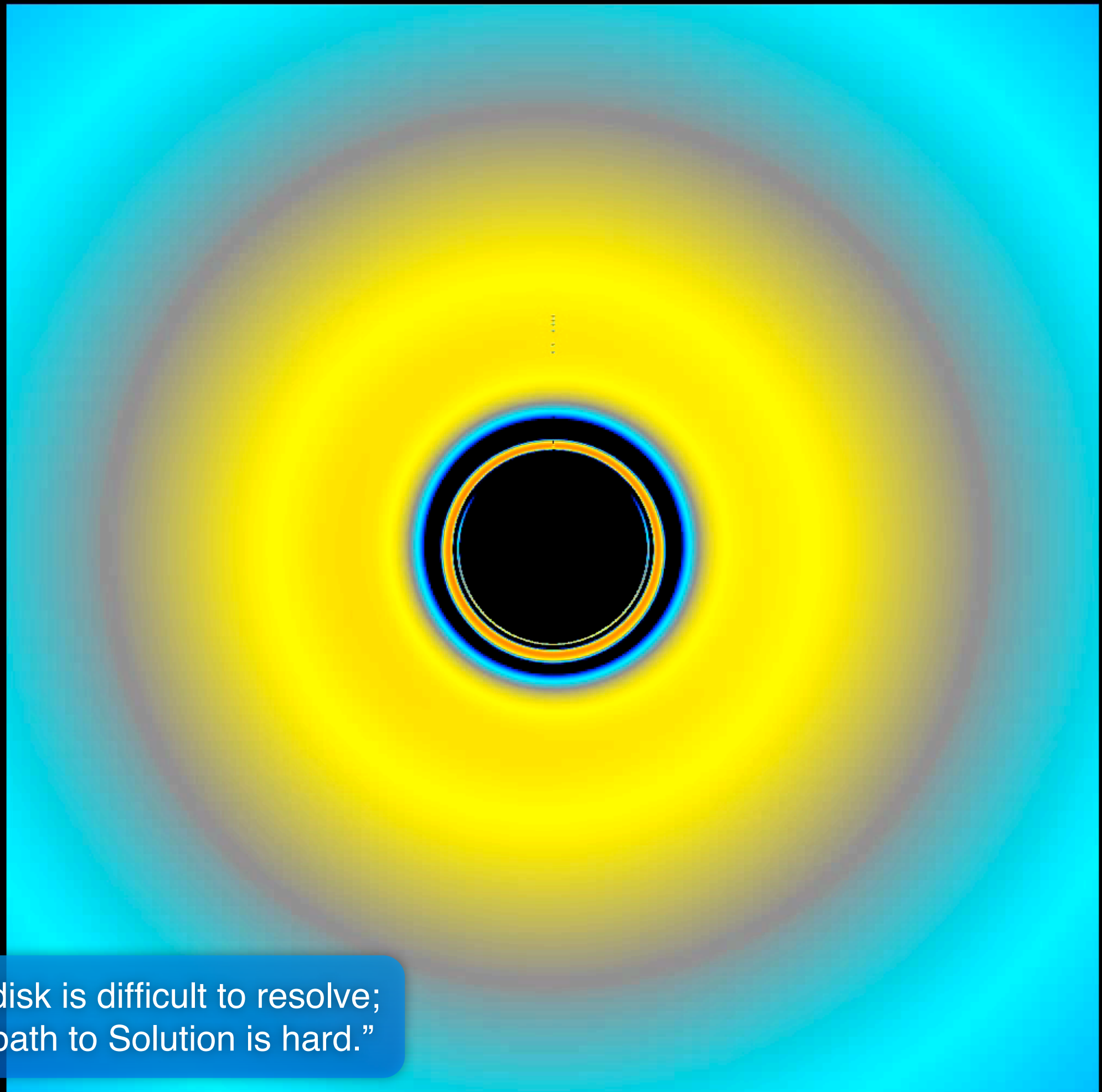
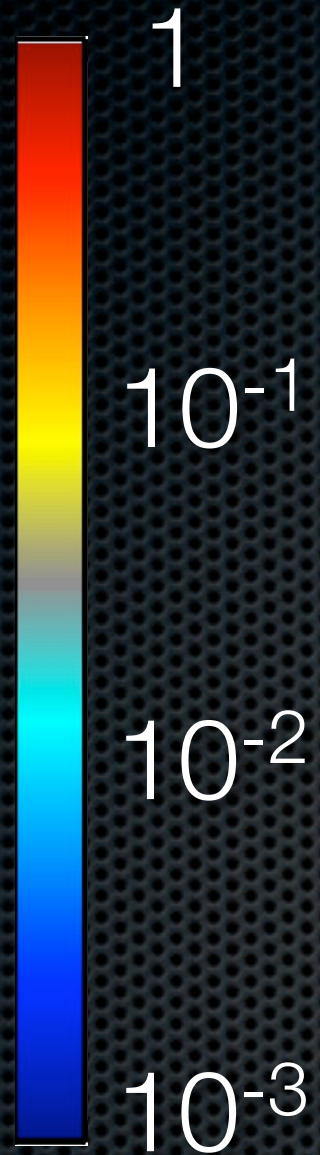


View of a razor-thin disk



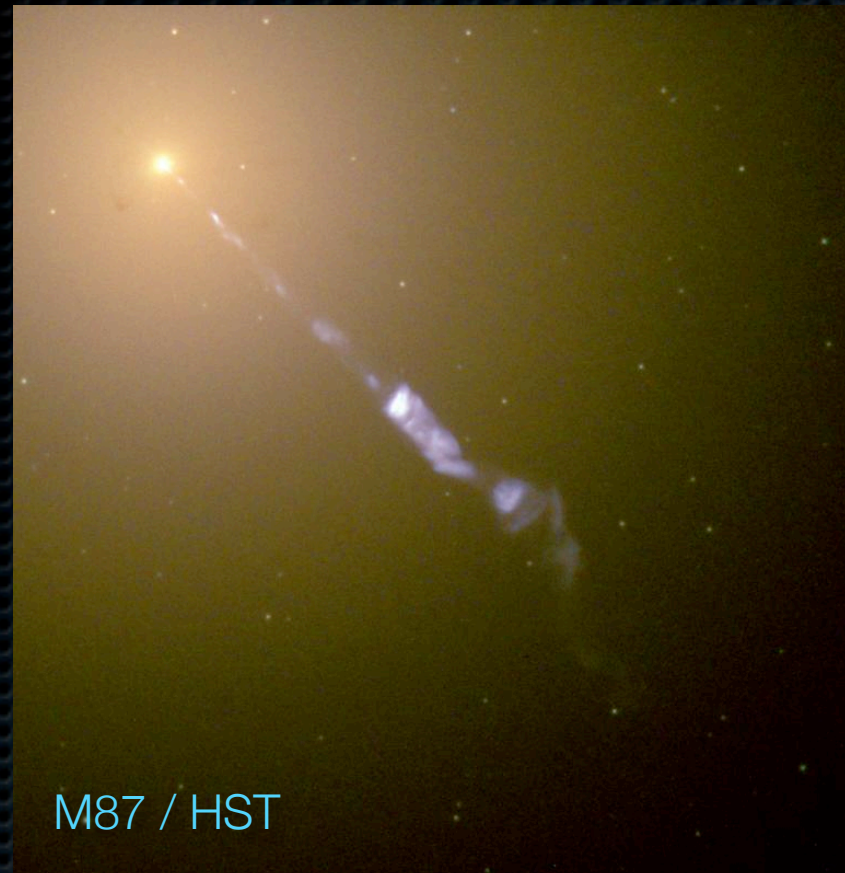
"The sharp edge of a disk is difficult to resolve;
thus the wise say the path to Solution is hard."

View of a razor-thin disk

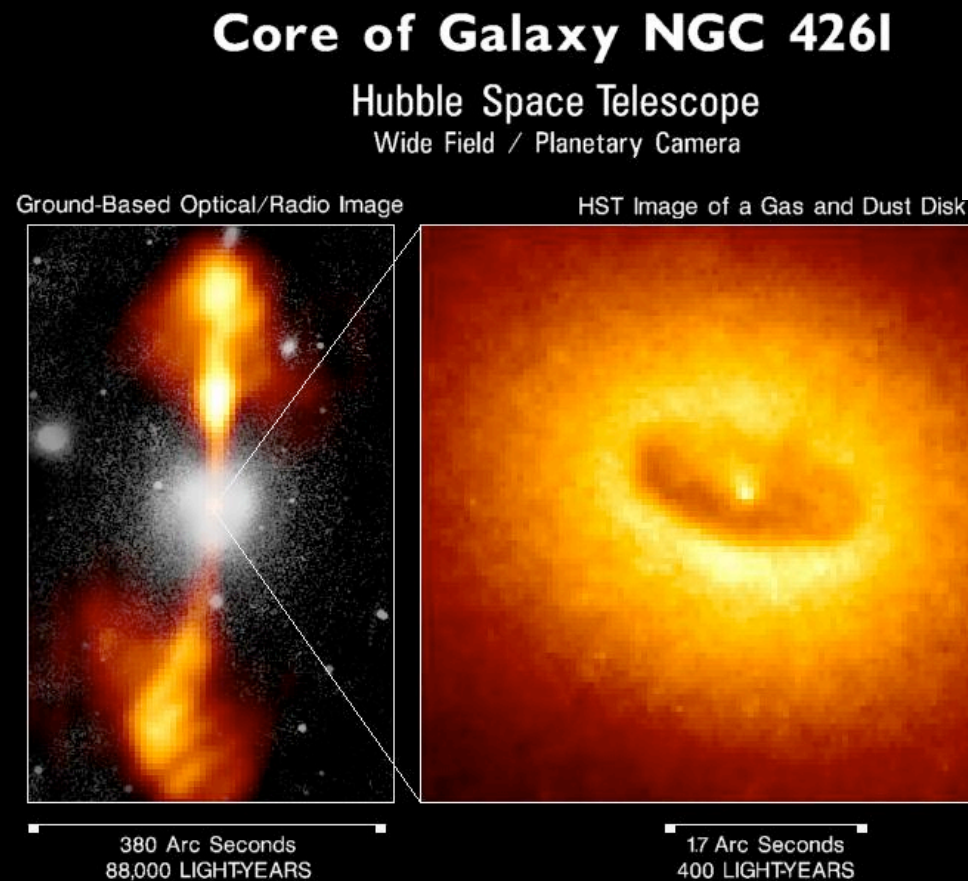
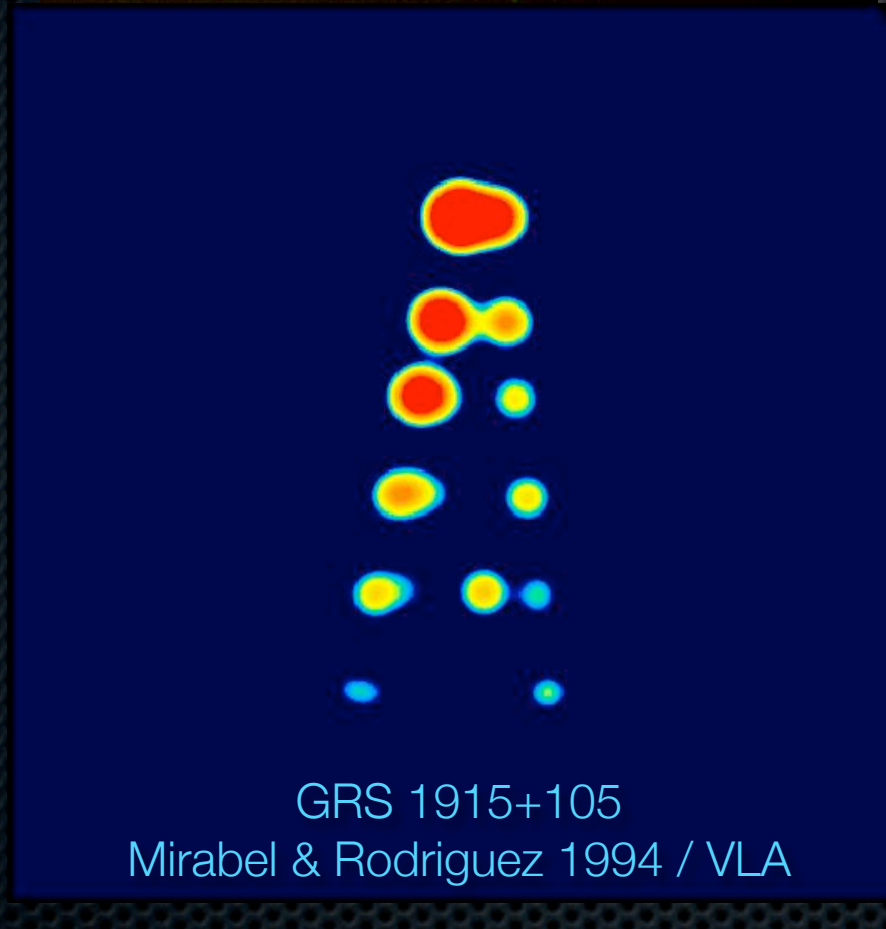
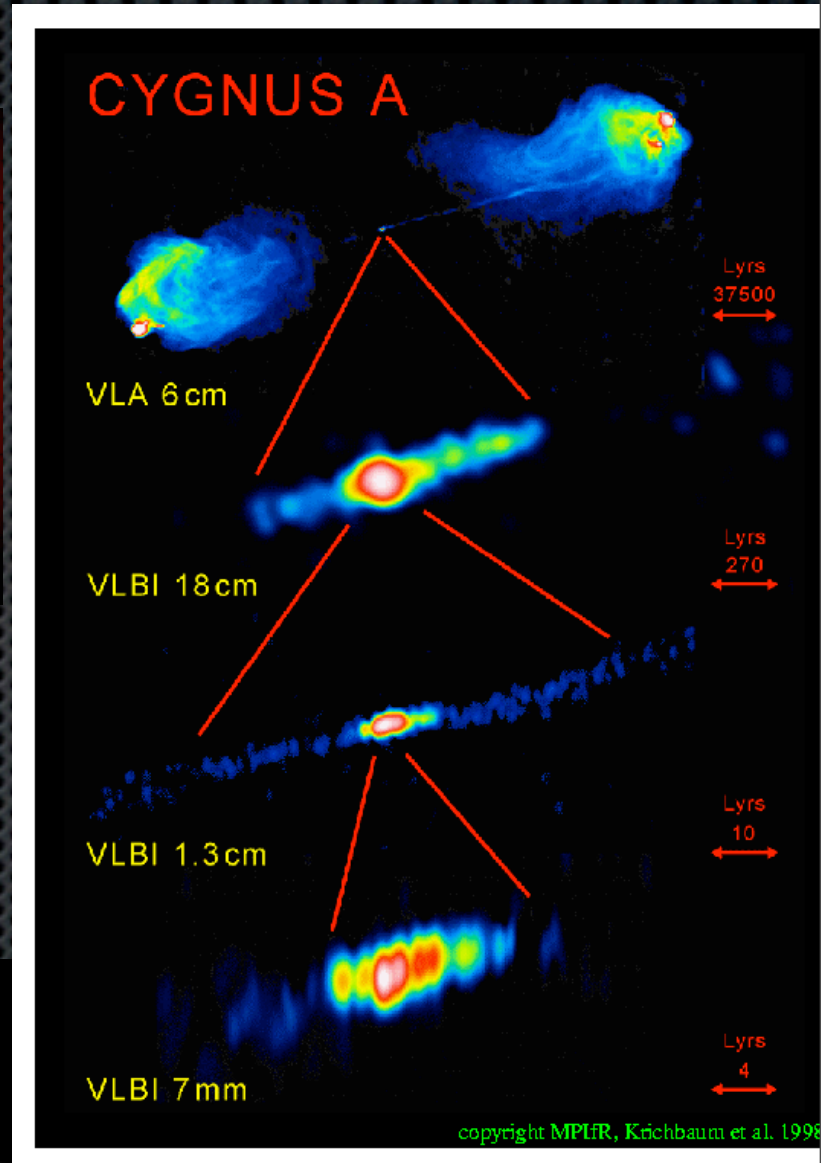


"The sharp edge of a disk is difficult to resolve;
thus the wise say the path to Solution is hard."

The Exciting World of Black Hole Accretion!



AGN!!
XRBs!!



Feedback!!

Probing the Spacetime of BHs

- ✦ Variability: e.g. QPOs, short time scale fluctuations

Done et al 2007

- ✦ Polarization
(e.g. Schnittman & Krolik 2009)

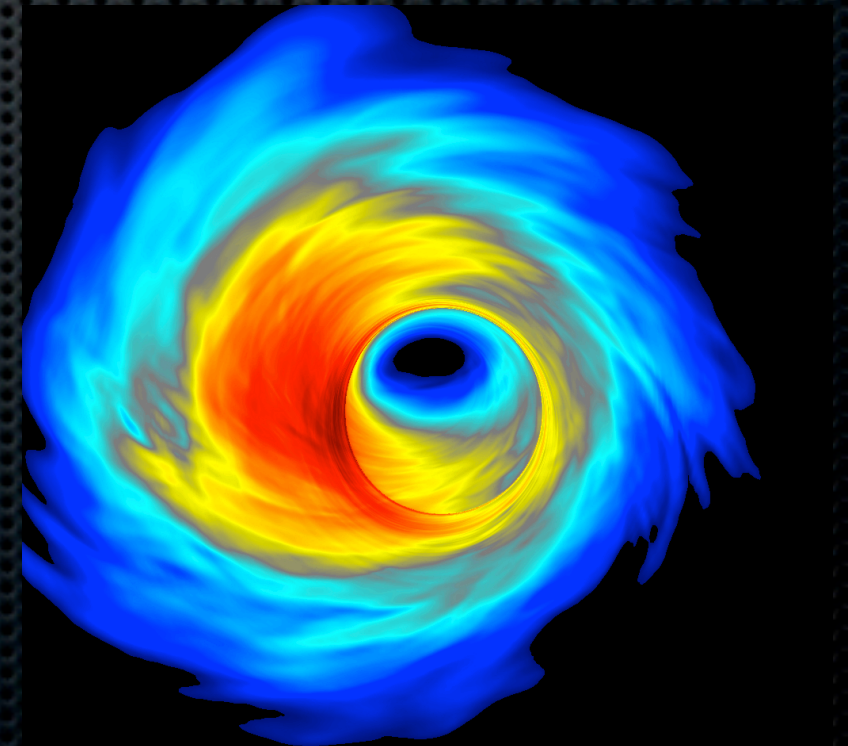
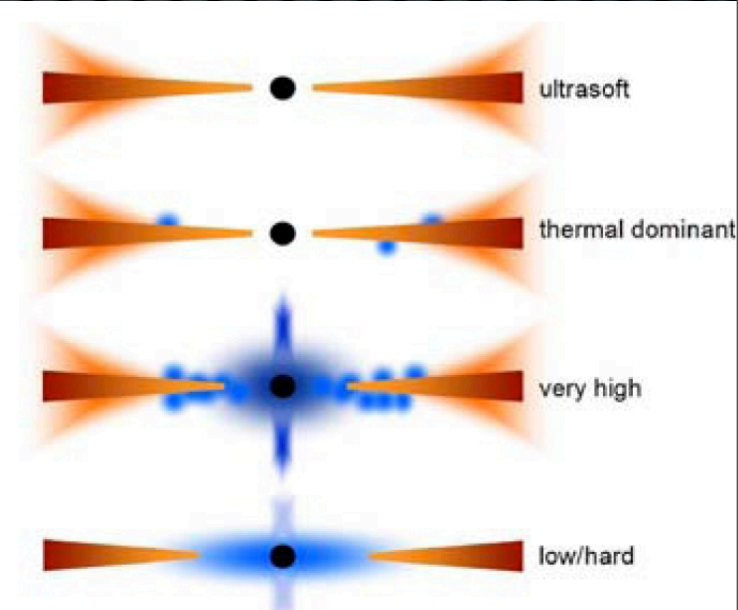
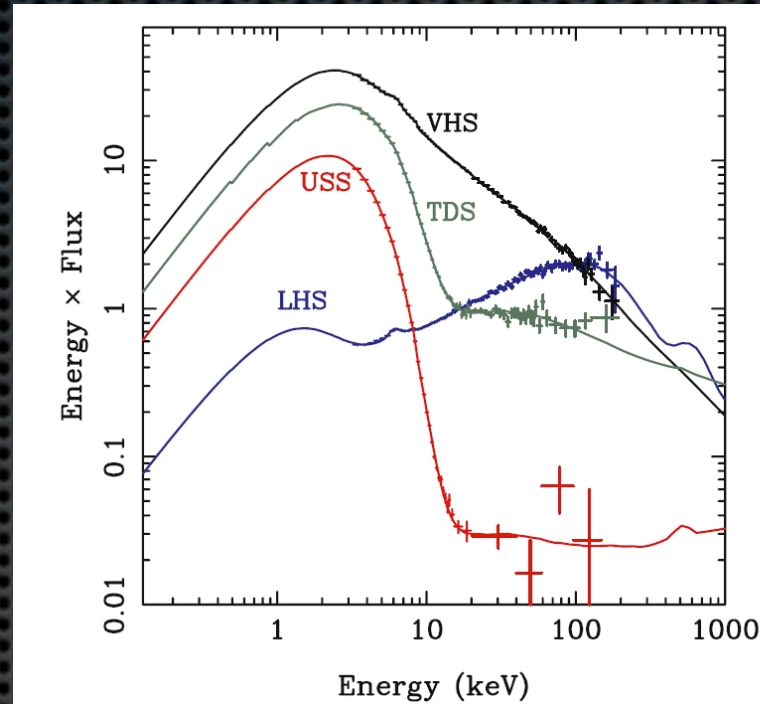
- ✦ Spectral Fitting of Thermal Emission

$$L = AR_{\text{in}}^2 T_{\text{max}}^4 \quad R_{\text{in}}^2 = f(a, M)$$

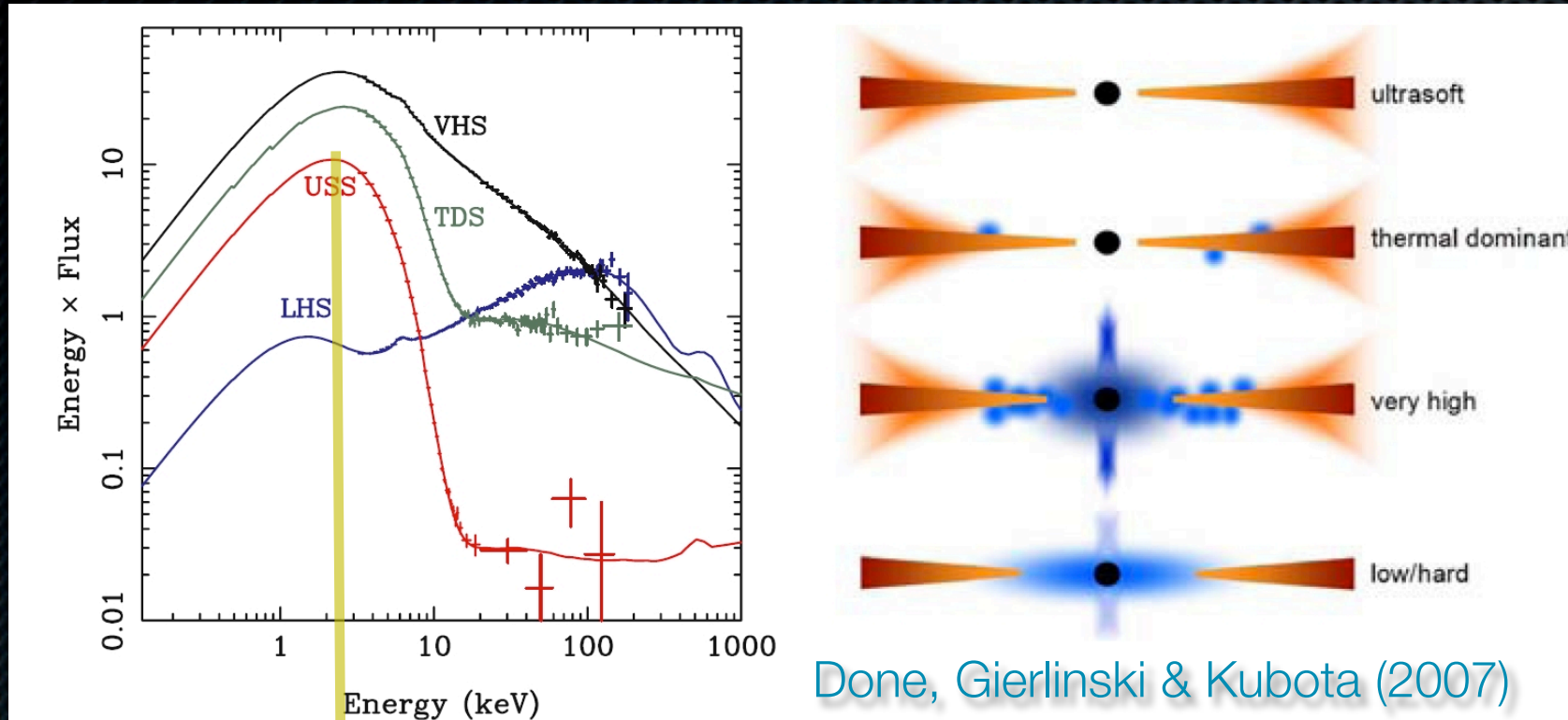
McClintock et al. 2006, Shafee et al. 2006

- ✦ Relativistic Iron Lines
- ✦ Directly Resolving the BH Silhouette
 - ✦ e.g. Sgr A* with sub-mm/mm VLBI

Noble et al. 2007, Mościbrodzka et al 2009,
Broderick et al 2006-2009, Doeleman et al. 2009

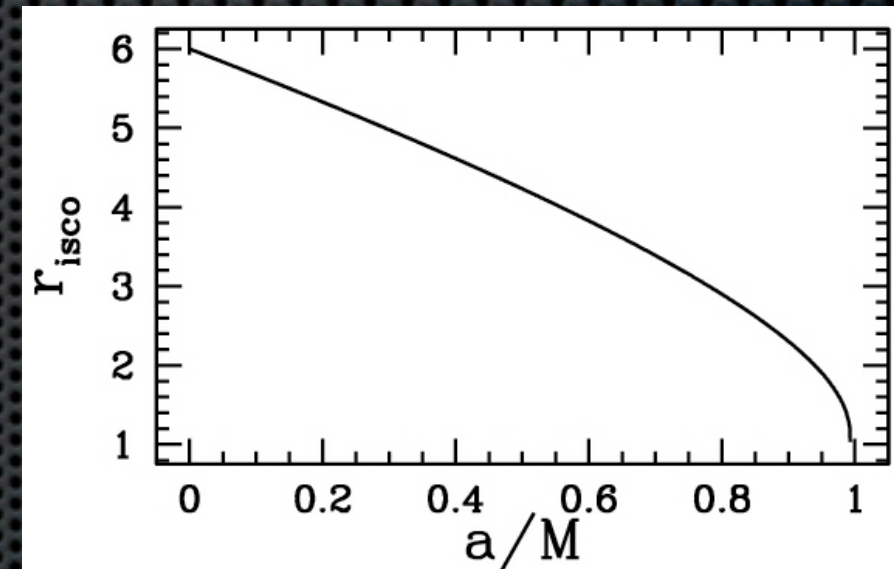


Thermal Spectral Fitting for BH Spin



$$L = AR_{\text{in}}^2 T_{\text{max}}^4$$

$$R_{\text{in}} = R_{\text{in}}(M, a) \simeq R_{\text{ISCO}}$$



T_{max}

TABLE 1

BLACK HOLE SPIN ESTIMATES USING THE MEAN OBSERVED VALUES OF M , D , AND i

Candidate	Observation Date	Satellite	Detector	a_* (D05)	a_* (ST95)
GRO J1655–40	1995 Aug 15	ASCA	GIS2	~0.85	~0.8
			GIS3	~0.80	~0.75
	1997 Feb 25–28	ASCA	GIS2	~0.75 ^a	~0.70
			GIS3	~0.75 ^a	~0.7
4U 1543–47	1997 Feb 26	RXTE	PCA	~0.75 ^a	~0.65
	1997 (several)	RXTE	PCA	0.65–0.75 ^a	0.55–0.65
	2002 (several)	RXTE	PCA	0.75–0.85 ^a	0.55–0.65

^a Values adopted in this Letter.

Shafee et al. (2006)

McClintock et al. (2006)

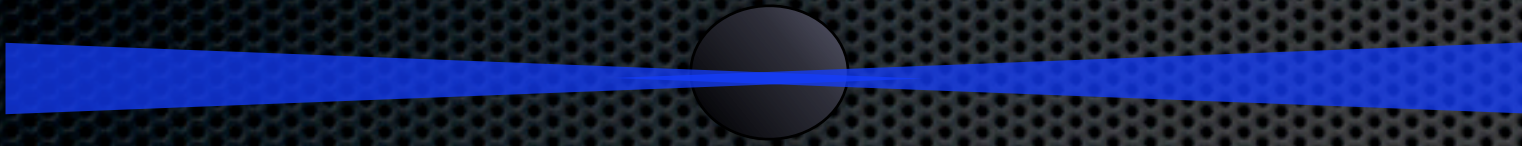
OBJECT	POWER LAW	
	Mean	Standard Deviation
GRS 1915+105 ^a	0.998	0.001
GRS 1915+105 ^b	0.998	0.001

Disk “Dichotomy”

Thin Disks:

- Shakura & Sunyaev (1973)
- Novikov & Thorne (1973)
- Page & Thorne (1974)

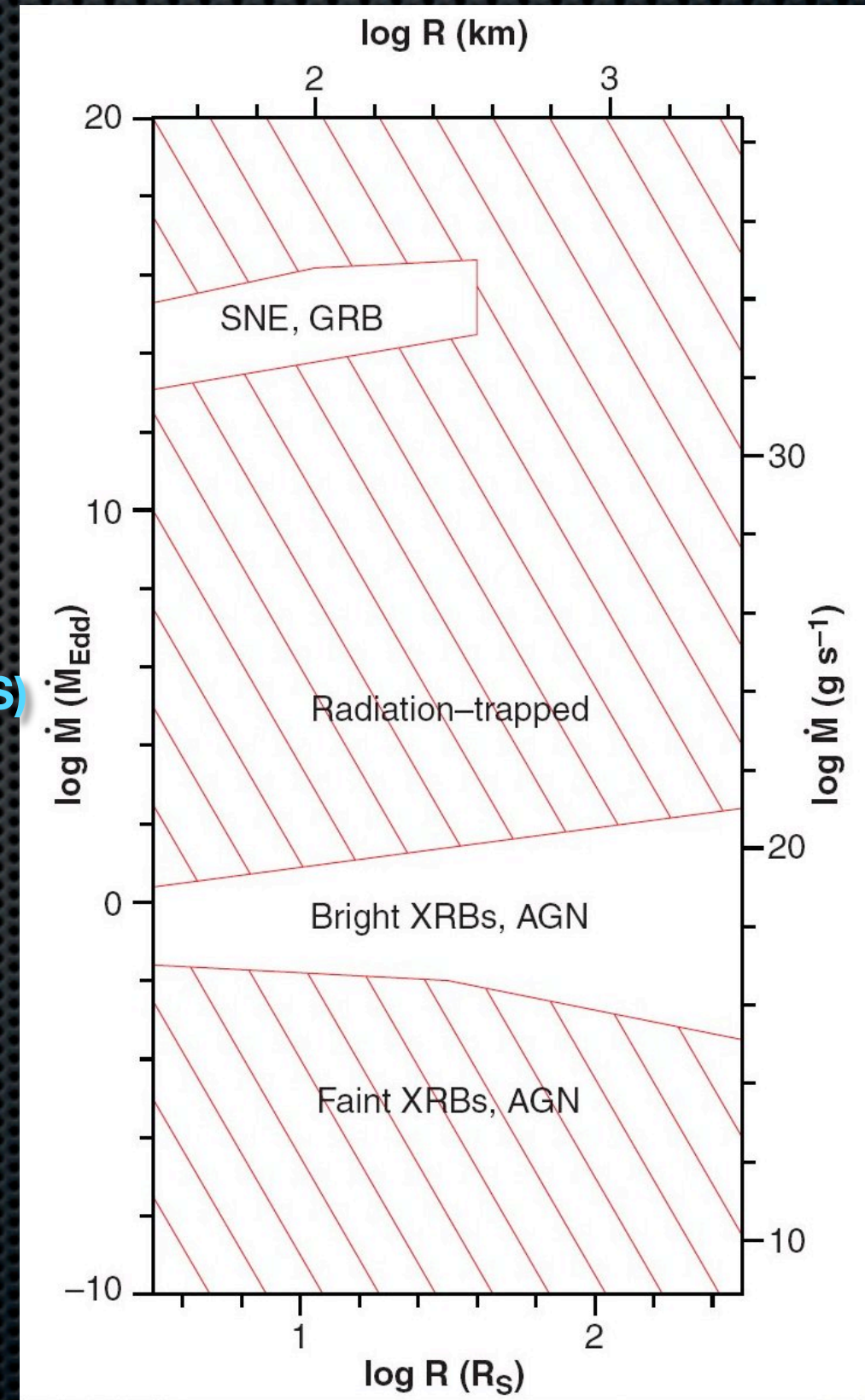
- Dissipation Rate < Cooling Rate
- “Cold”, Optically Thick
- Thermal or Multi-temperature black body



Thick Disks:

- Narayan & Yi (1994-5) (ADAF)
- Blandford & Begelman (1999) (ADIOS)
- Quataert & Gruzinov (2000) (CDAF)

- Dissipation Rate > Cooling Rate
- “Hot”, optically thin, outflows
- 2 Temperature flow, advected heat



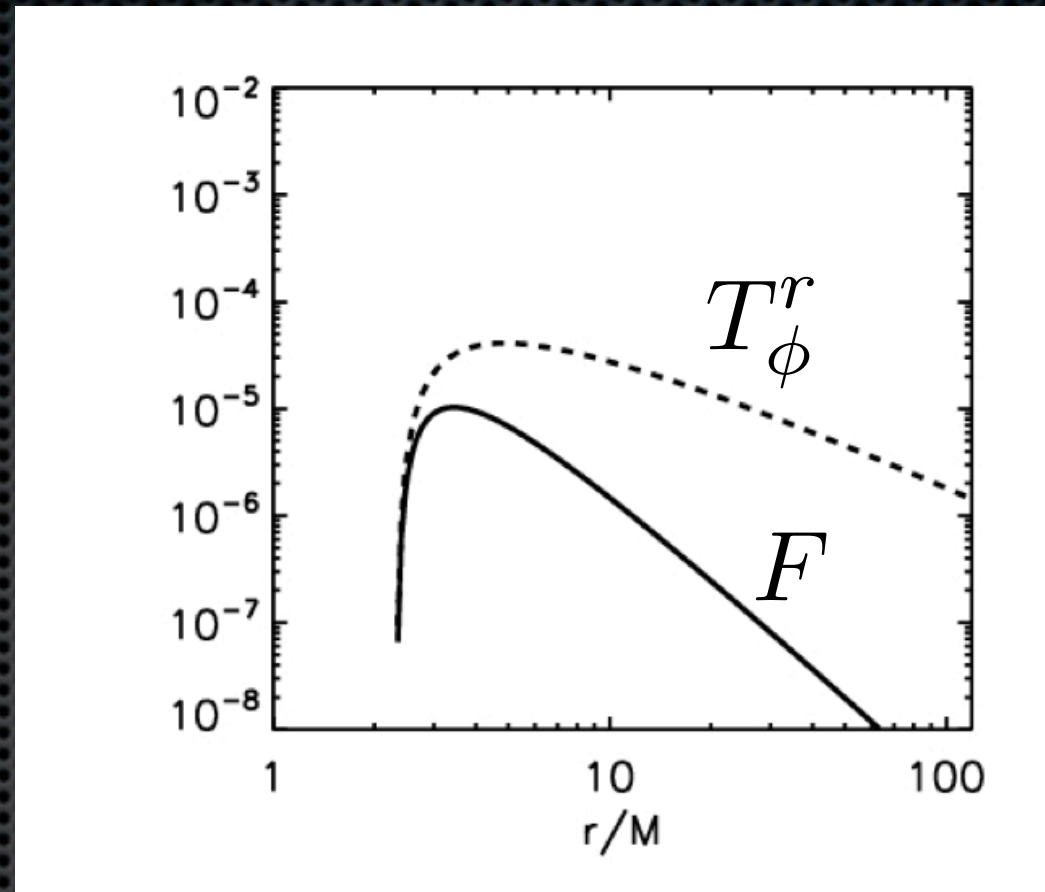
Narayan & Quataert (2005)

Steady-state Thin Disk Models

- Novikov & Thorne (1973)
- Stationary gravity
 - Perfect radiator
 - Work done by stress locally dissipated & radiated
 - Zero stress at ISCO as boundary condition
 - Luminosity as total liberation of binding energy up until plunge into ISCO

$$L = \eta \dot{M} c^2$$

$$\eta = 1 - \epsilon_{\text{ISCO}}$$



Shakura & Sunyaev (1973)

$$T_{\phi}^r = -\alpha P \quad P = \rho c_s^2$$

$$t_{\phi}^r = -\alpha c_s^2$$

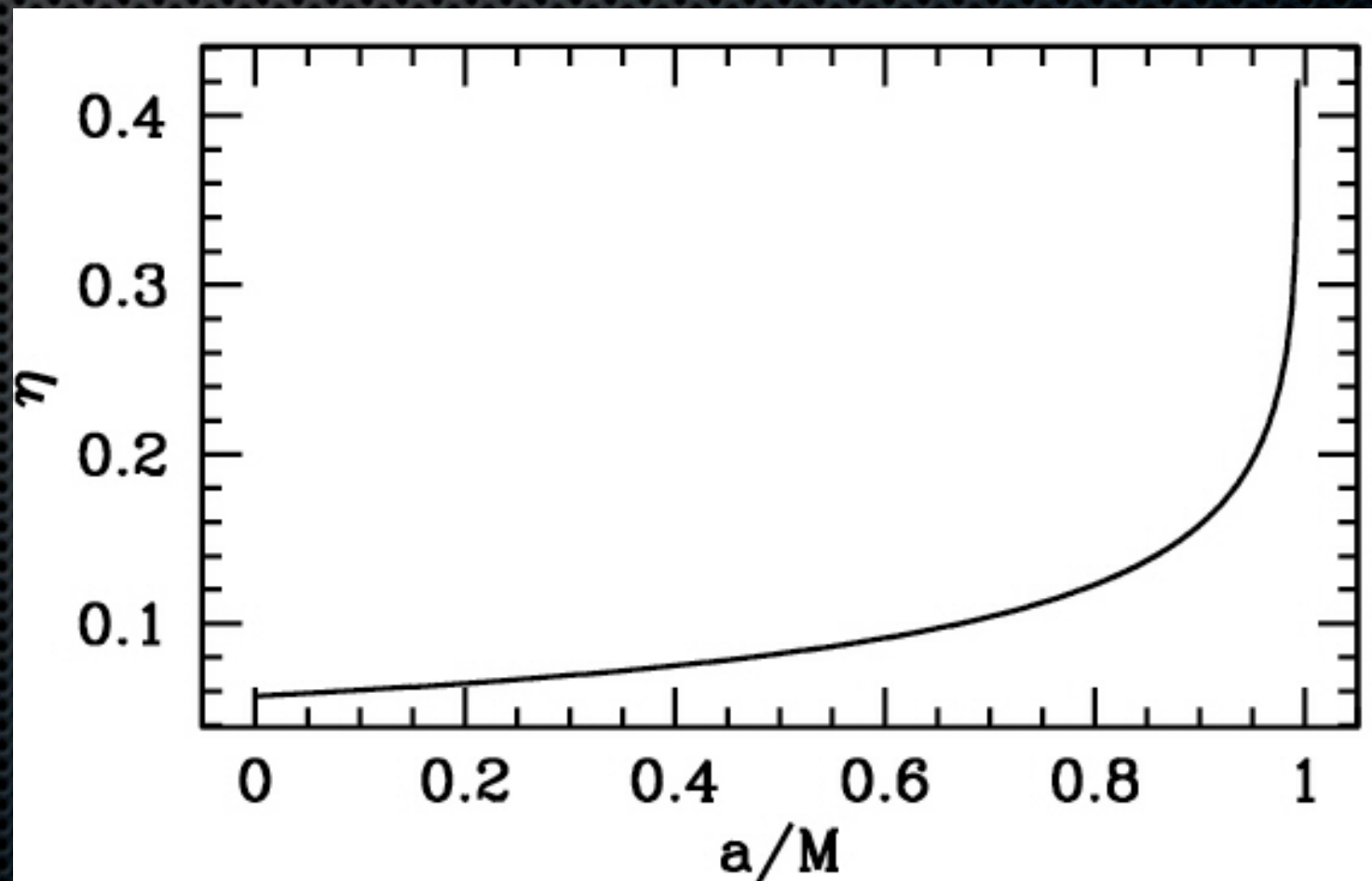
No stress at sonic point:

$$\rightarrow R_{\text{in}} = R_s \simeq R_{\text{ISCO}}$$

Muchotzeb & Paczynski (1982)

Abramowicz et al. (1988)

Afshordi & Paczynski (2003)



² It is conceivable that the disk material might contain extremely strong magnetic fields, and that these fields might transport a torque from the infalling material at $r < r_{\text{ms}}$ to the disk at $r \geq r_{\text{ms}}$. In this case the boundary condition at r_{ms} would be modified, and the solution for f would be changed. It seems to us unlikely that the changes would be substantial, except very near r_{ms} (i.e., at $r - r_{\text{ms}} \lesssim 0.1r_{\text{ms}}$). But when constructing explicit disk models, one should examine this possibility carefully.

Page & Thorne (1974)

In these three cases it seems almost certain that the ultimate, limiting value of a_* will not exceed our value of 0.998—and, hence, that the efficiency for converting rest mass into escaping radiation will not exceed 30 percent.

Other ways in which our assumptions may fail are these:

- i) Magnetic fields attached to the disk may reach into the horizon, producing a torque on the hole (Ya. B. Zel'dovich and V. F. Schwartzman, private communication).
- ii) The disk will recapture some of the photons it emits, thereby preventing them from going down the hole.
- iii) The time-averaged, radial disk structure will be changed by photon recapture and resultant heating, and by magnetic torques that couple the innermost parts of the disk to the hole and couple them to matter that has fallen out of the disk and is plunging down the hole. The result will be deviations of the emitted photon flux $F(r)$ from the law derived in Paper I, and deviations of the specific energy and angular momentum of the infalling matter from E_{ms}^+ and L_{ms}^+ .

Thorne (1974)

Gammie (1999)

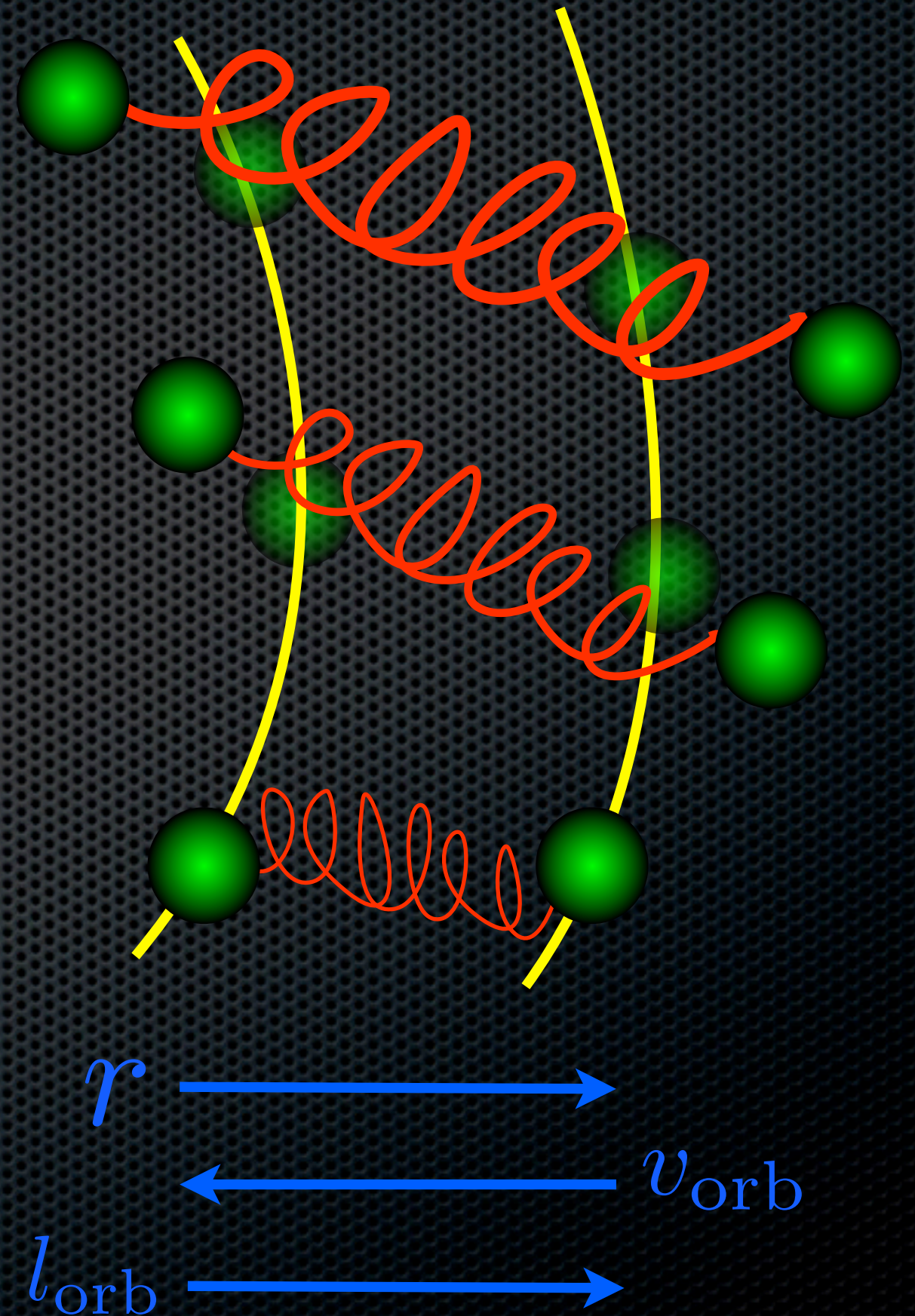
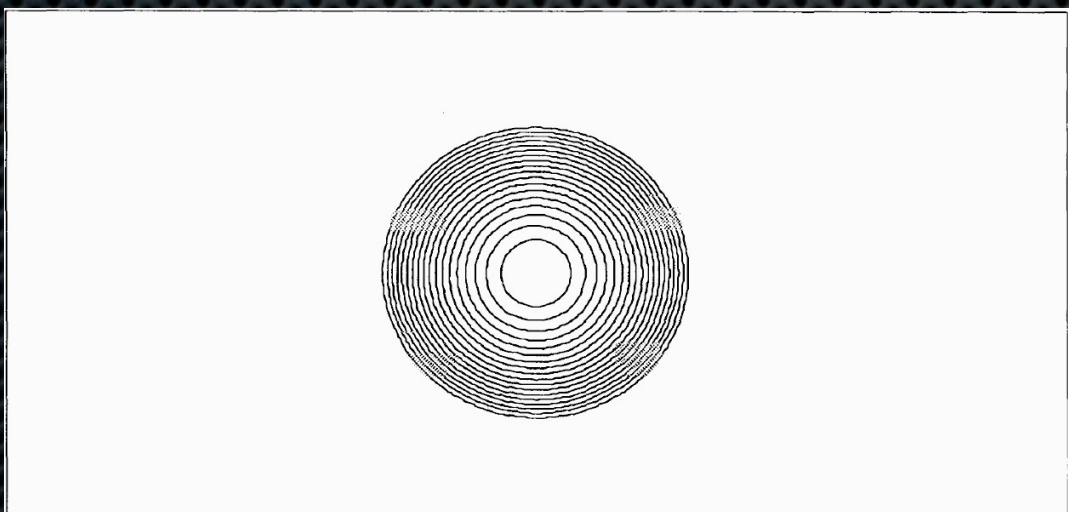
- Magnetized inflow model matched to thin disk
- Efficiency tied to mag. flux BC

Agol & Krolik (2000)

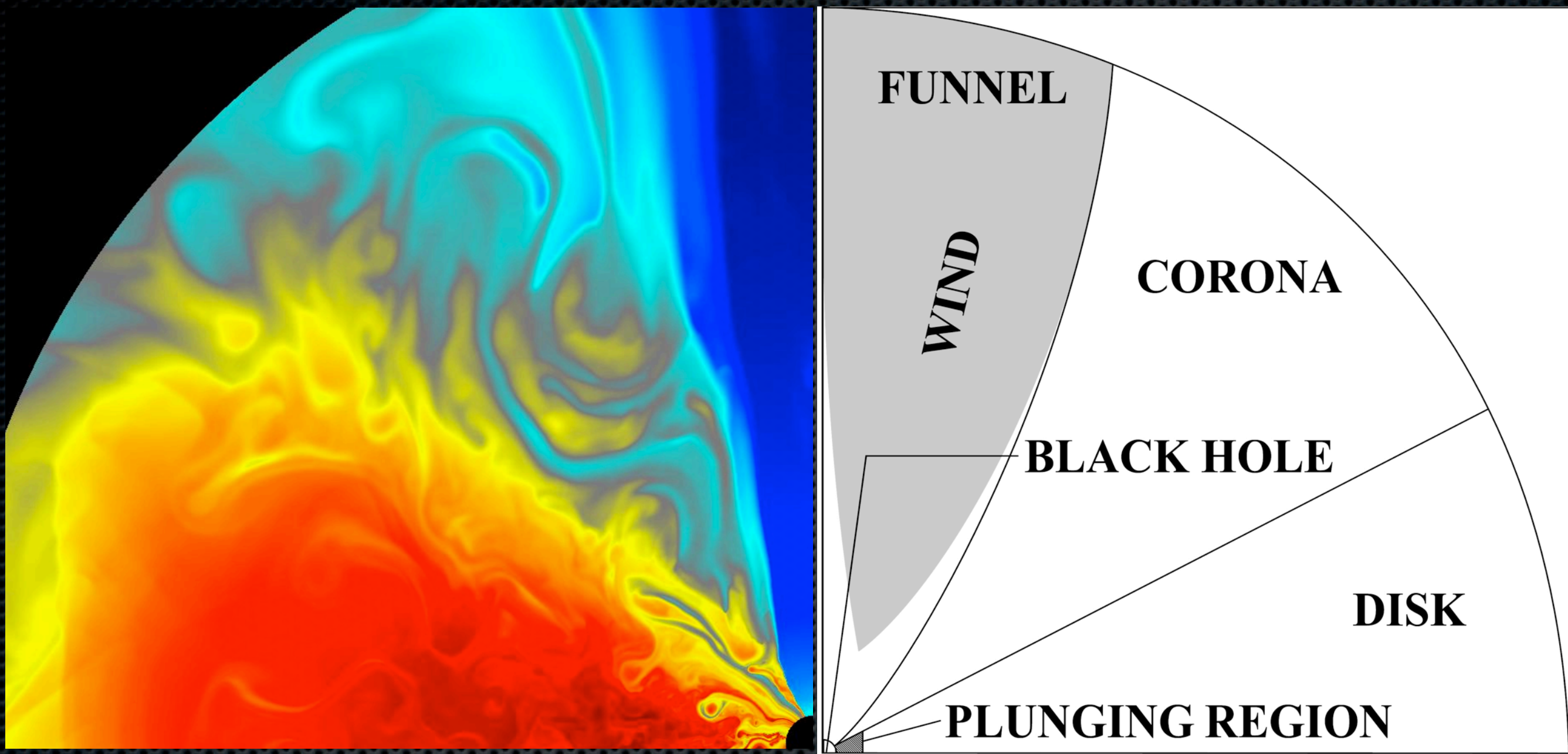
- Magnetic torques at ISCO can affect radiative efficiency

Magneto-rotational Instability (MRI)

- Velikhov (1959)
- Chandrasekhar (1960)
- Balbus & Hawley (1991)
- Growth on orbital time scale.
- MRI develops from weak initial field --- relevant for any (partially) ionized gas.
- Magnetic coupling over different radii is not well described by local viscosity.
- Can explain high accretion rates where hydrodynamic viscosity cannot.
- Fastest instability known that feeds off free energy of differential rotation.



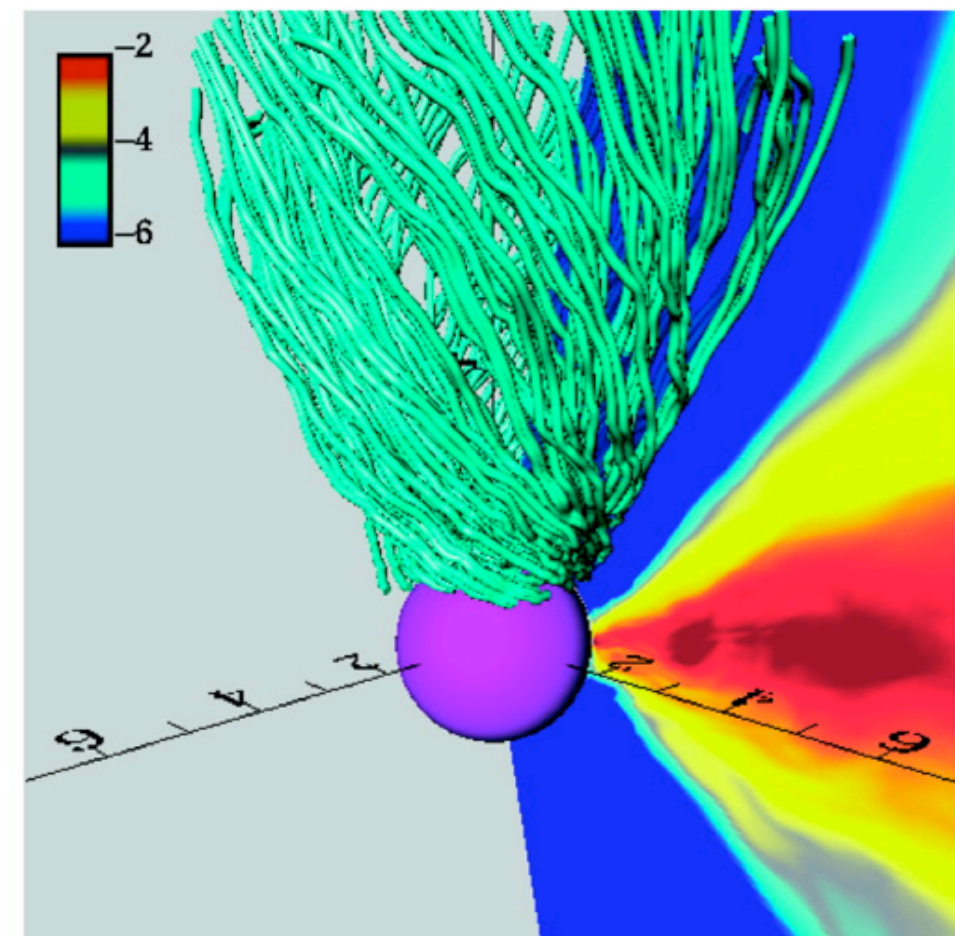
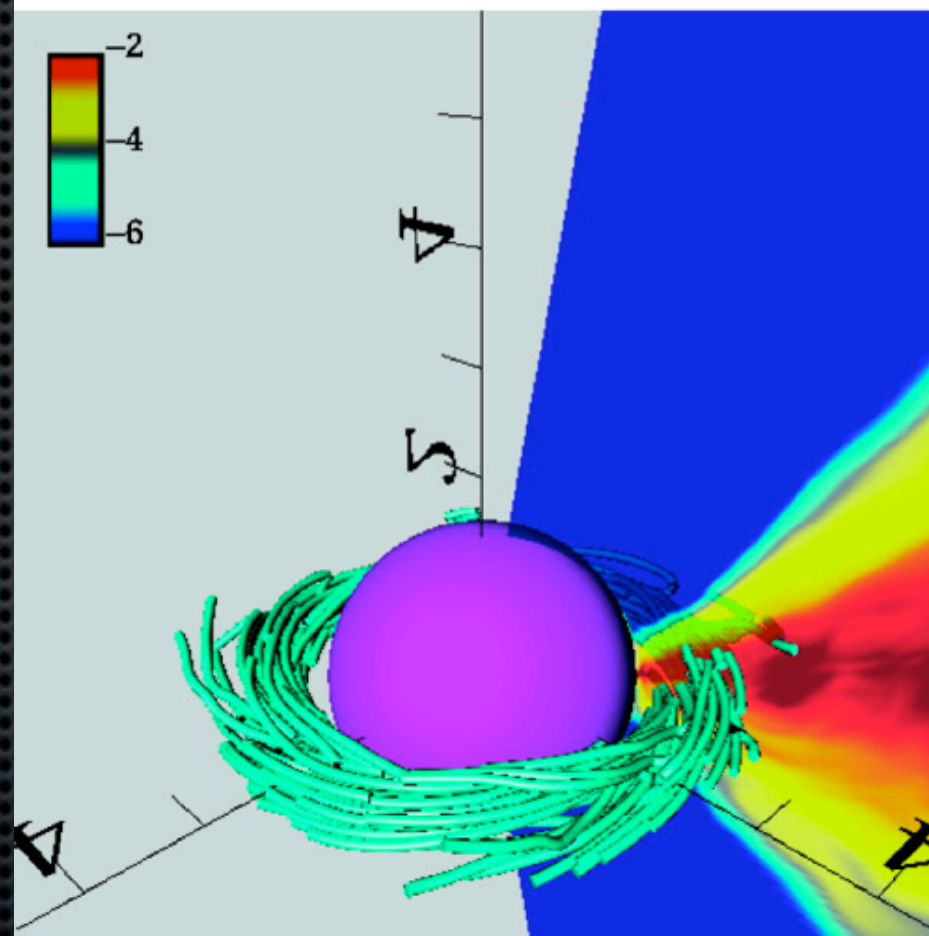
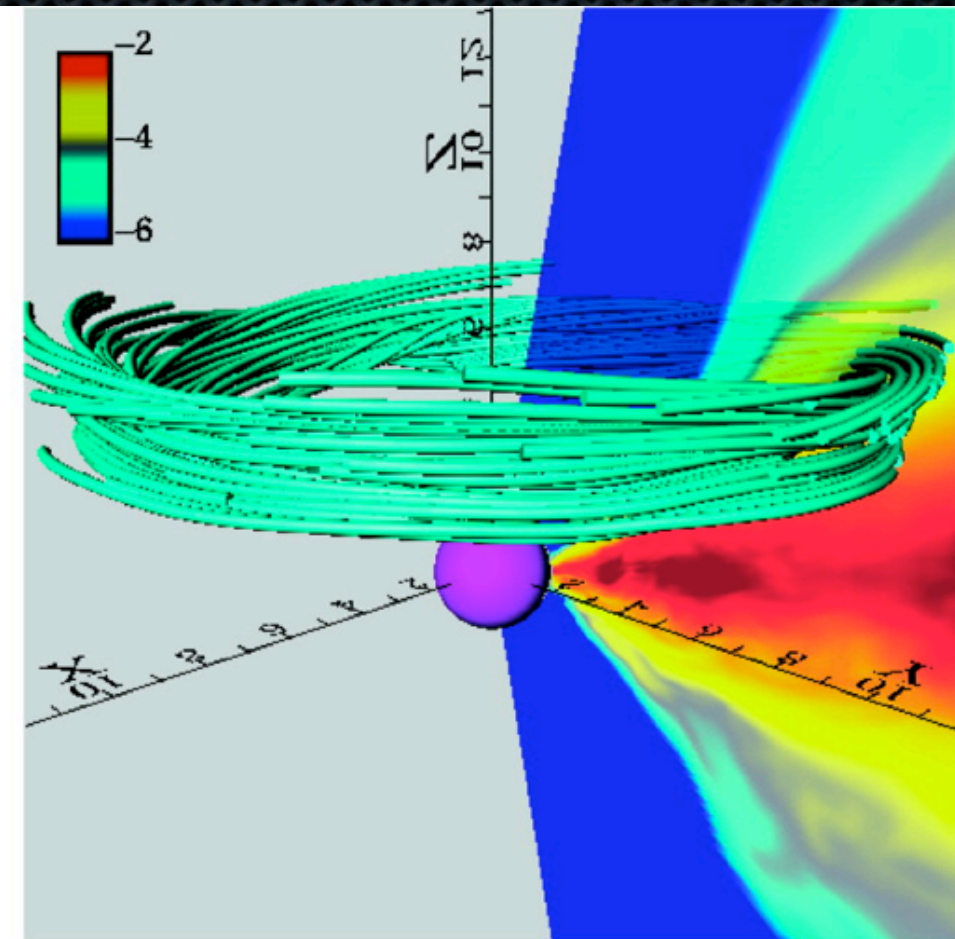
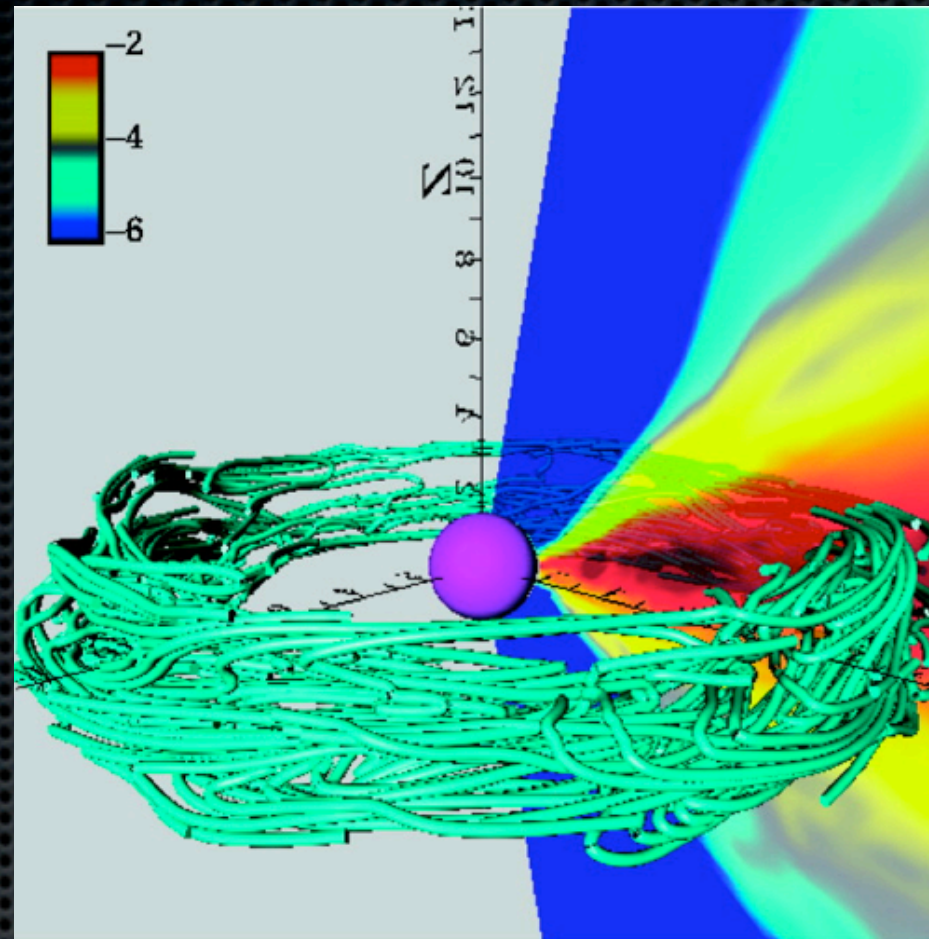
Disk Morphology



McKinney & Gammie (2004)

Hawley, De Villiers, Krolik, Hirose 2003+

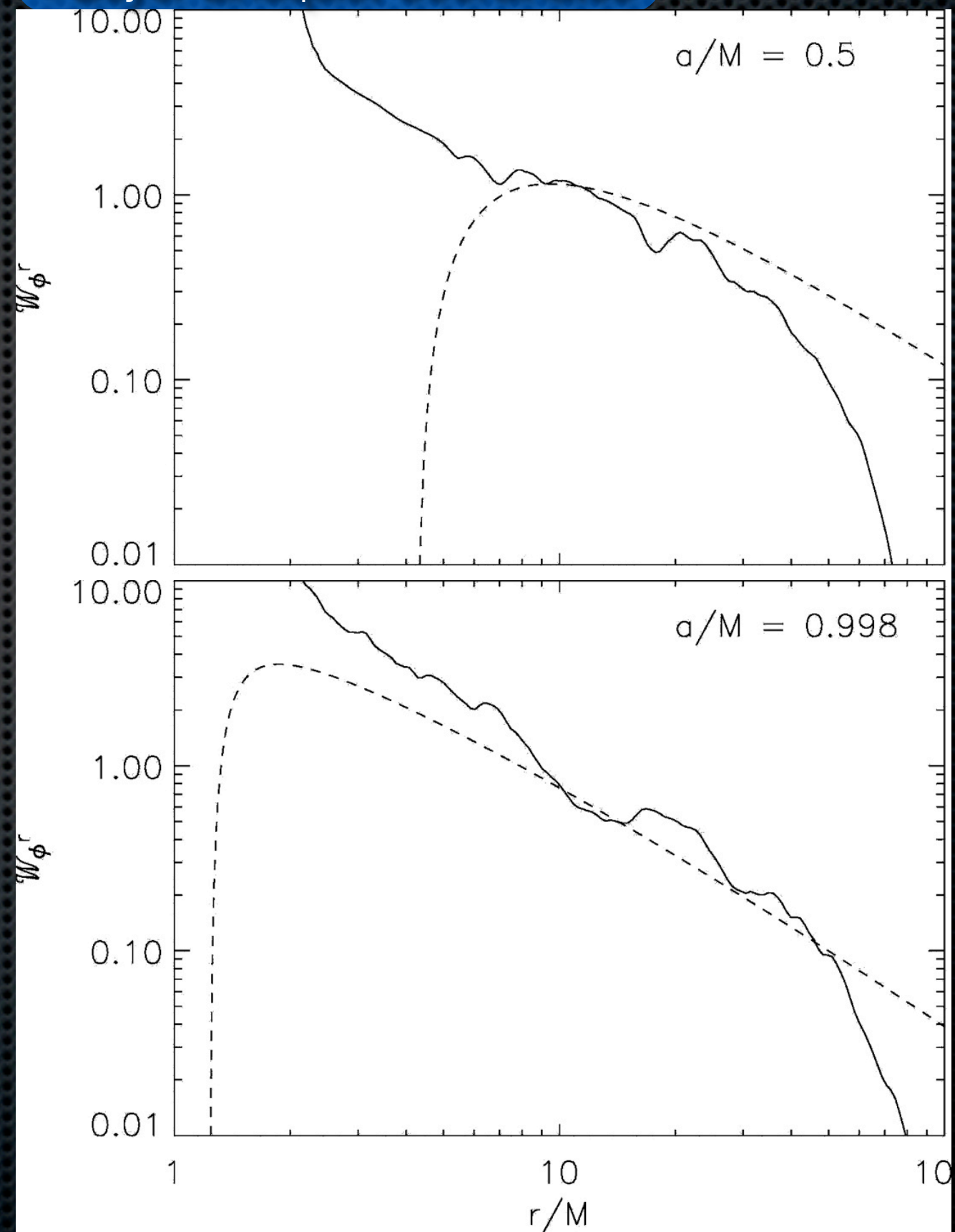
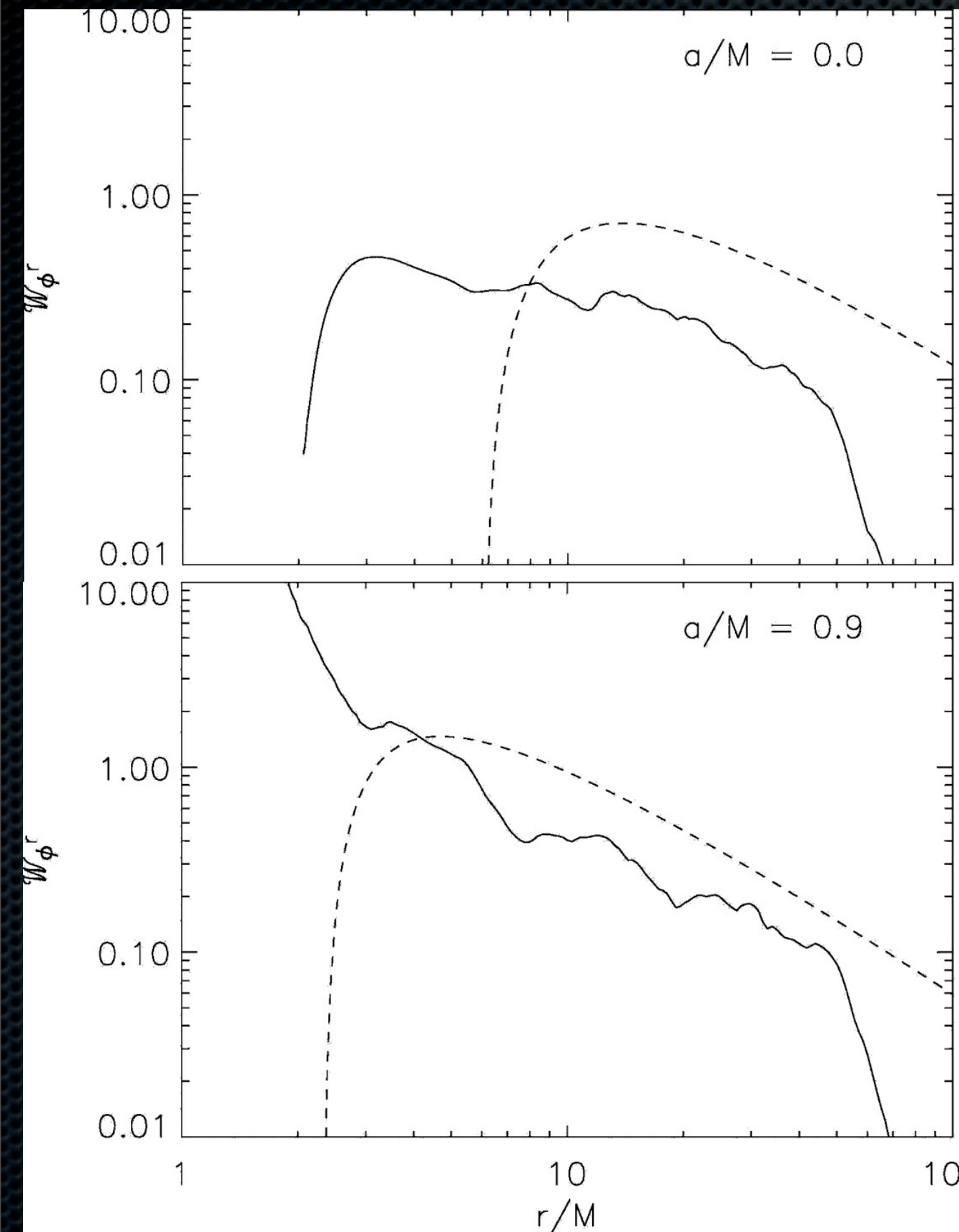
Canonical Magnetic Field Distribution



Hirose et al. (2004)

Krolik, Hawley & Hirose (2005)

- Non-conservative
- 3D GRMHD
- $H/R \sim 0.12$
- Boyer-Lindquist Coordinates



SCN, Krolik & Hawley (2009)

- HARM3D:

- Based on Gammie's Harm (2D) and HAM (non-rel) codes
- 3D Ideal GRMHD
- Kerr-Schild coordinates
- Modern high-res. shock-capturing methods
- Flux (energy) conserving
- Constrained Transport scheme
- Optically-thin cooling function
- Maintains constant H/R
- Cooling on orbital timescale

$$\nabla_{\mu} T^{\mu}_{\nu} = -\mathcal{L} u_{\nu}$$

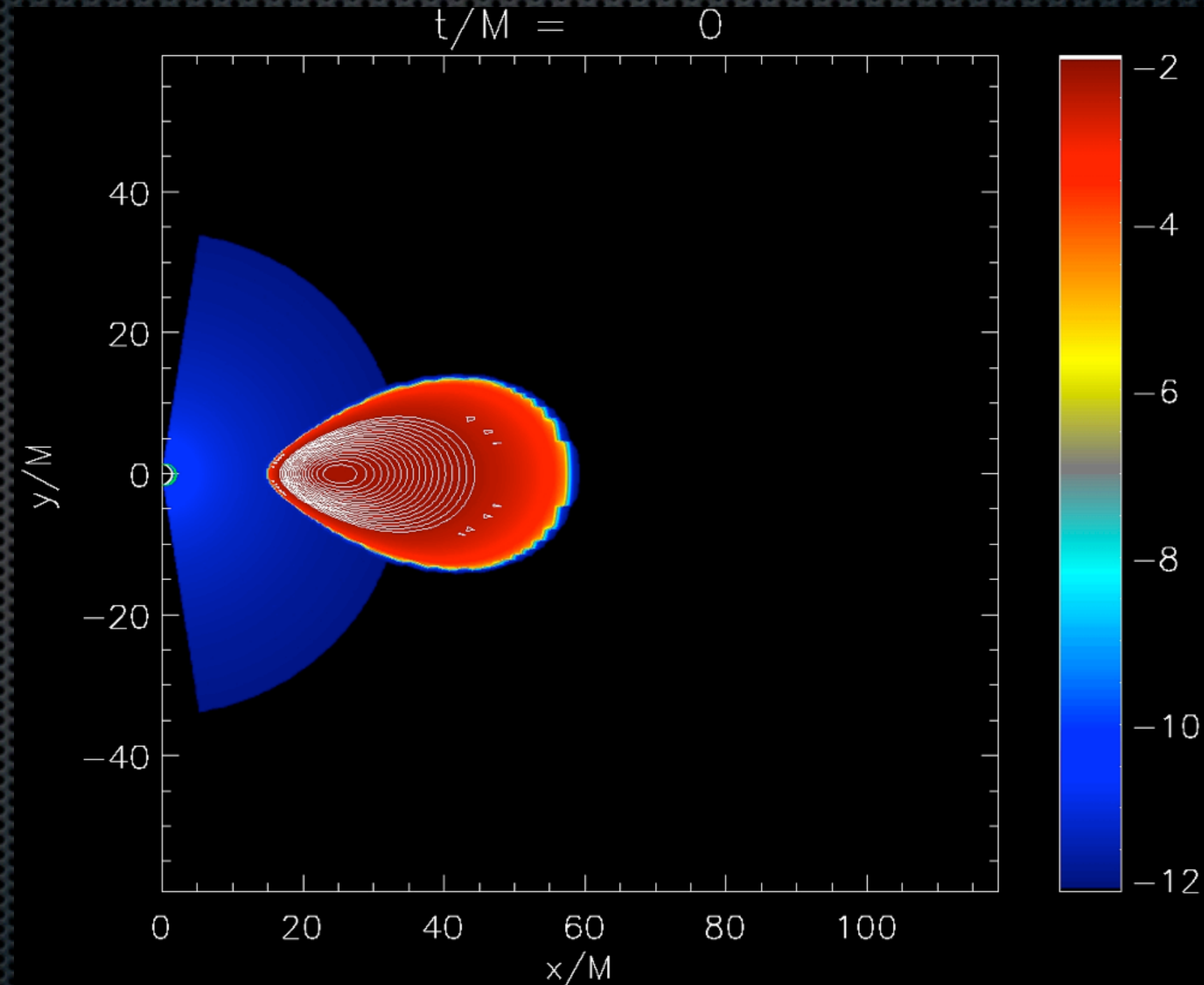
$$\mathcal{L} = \Omega_K u \Delta^q$$

$$T_{\circ} = \frac{\pi}{2} \left(\frac{H}{r} r \Omega_K \right)^2$$

$$N_r \times N_{\theta} \times N_{\phi} = 192 \times 192 \times 64$$

$$r \in [< r_{\text{hor}}, 120M] \quad \theta \in \pi [\delta, 1 - \delta] \quad \phi \in [0, \pi/2]$$

$$a = 0.9M$$



SCN, Krolik & Hawley (2009)

- HARM3D:

- Based on Gammie's Harm (2D) and HAM (non-rel) codes
- 3D Ideal GRMHD
- Kerr-Schild coordinates
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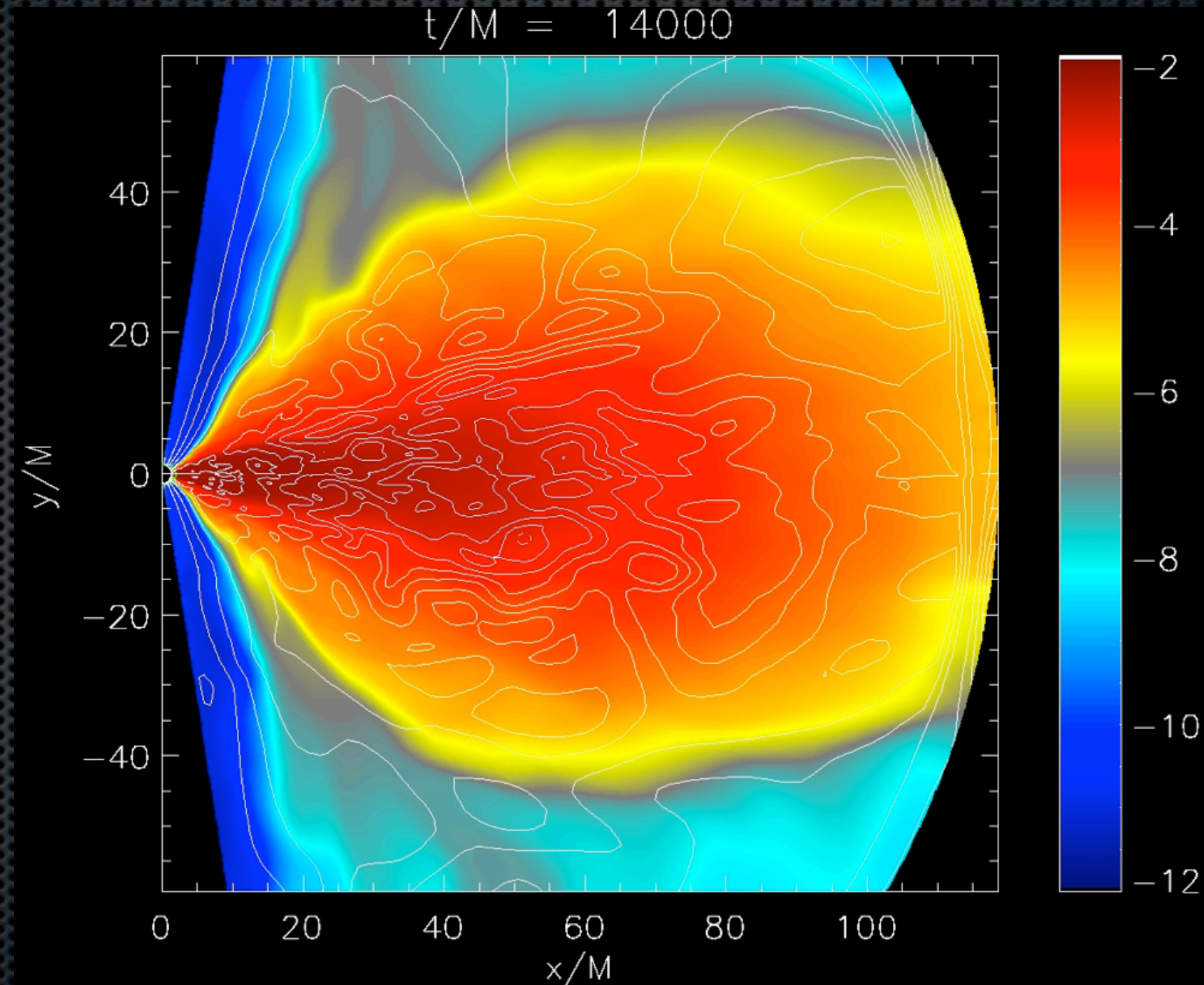
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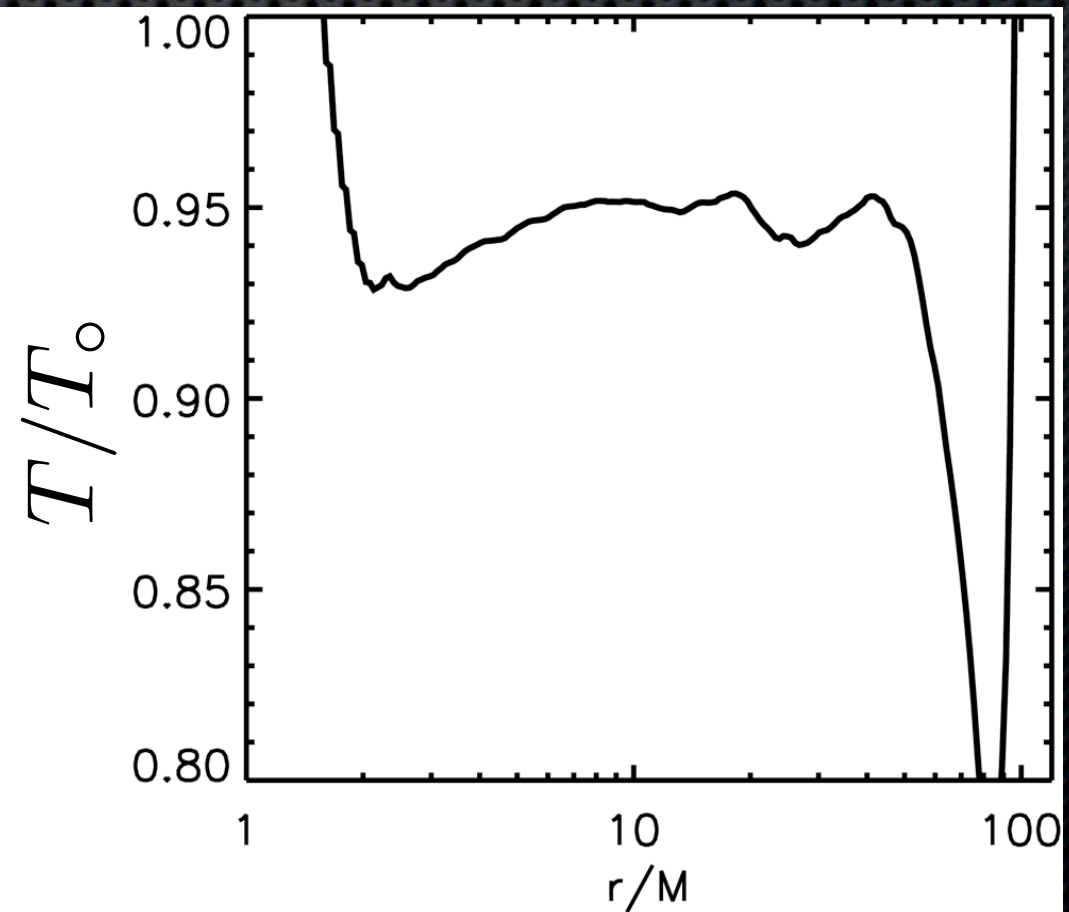
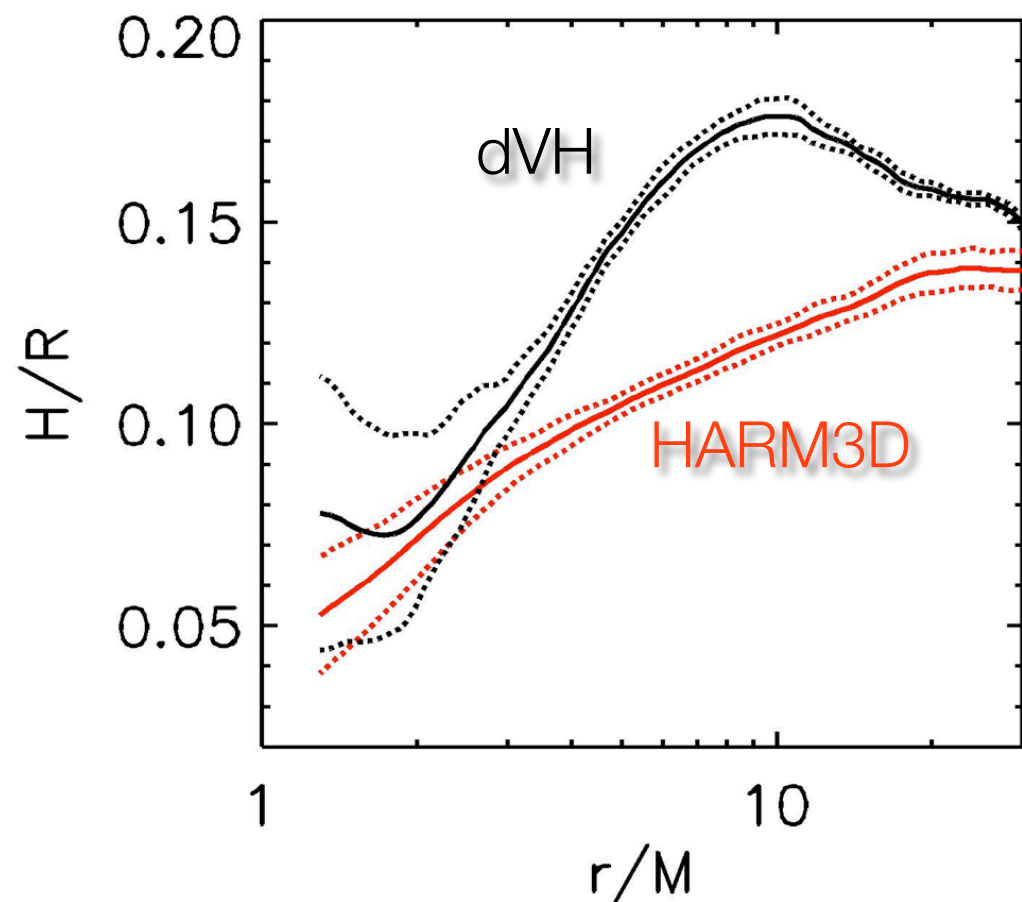
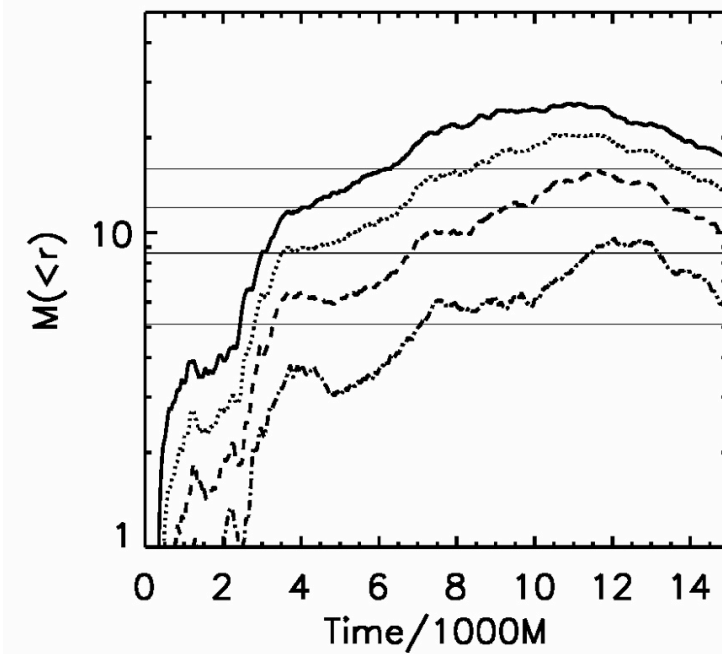
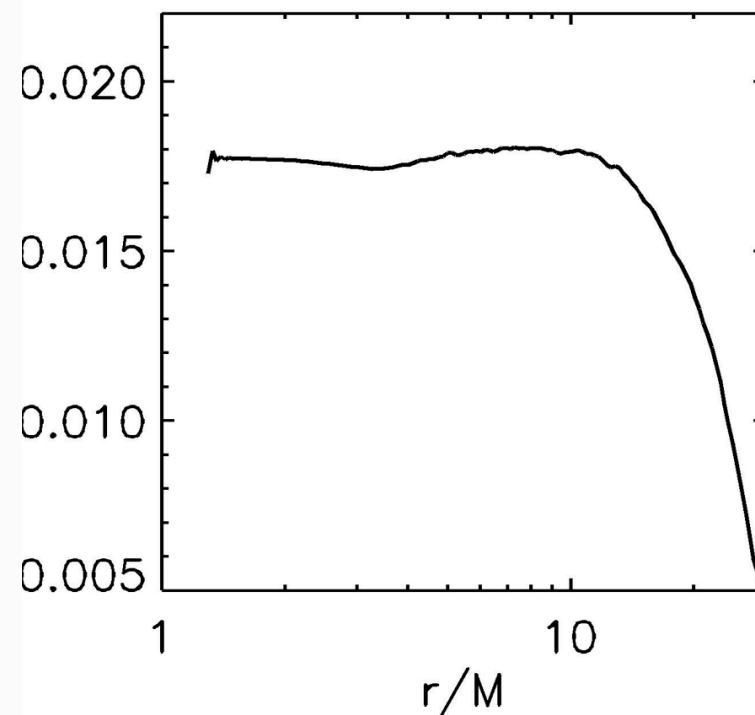
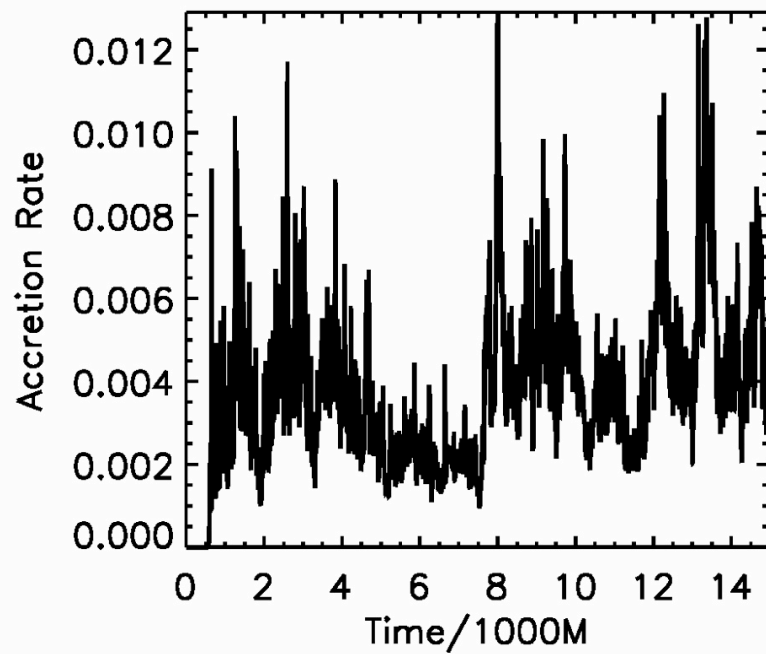
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$$a = 0.9M$$

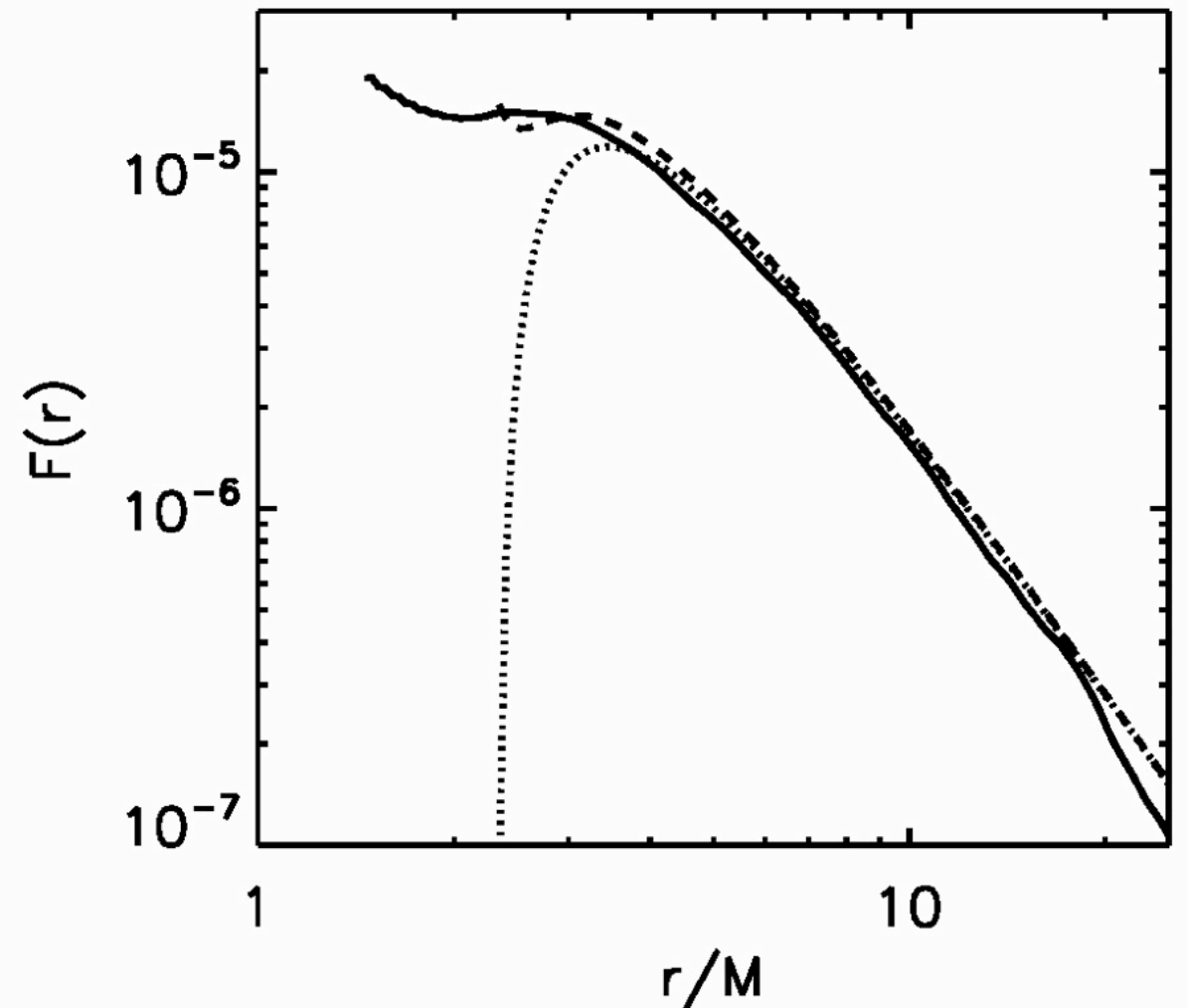
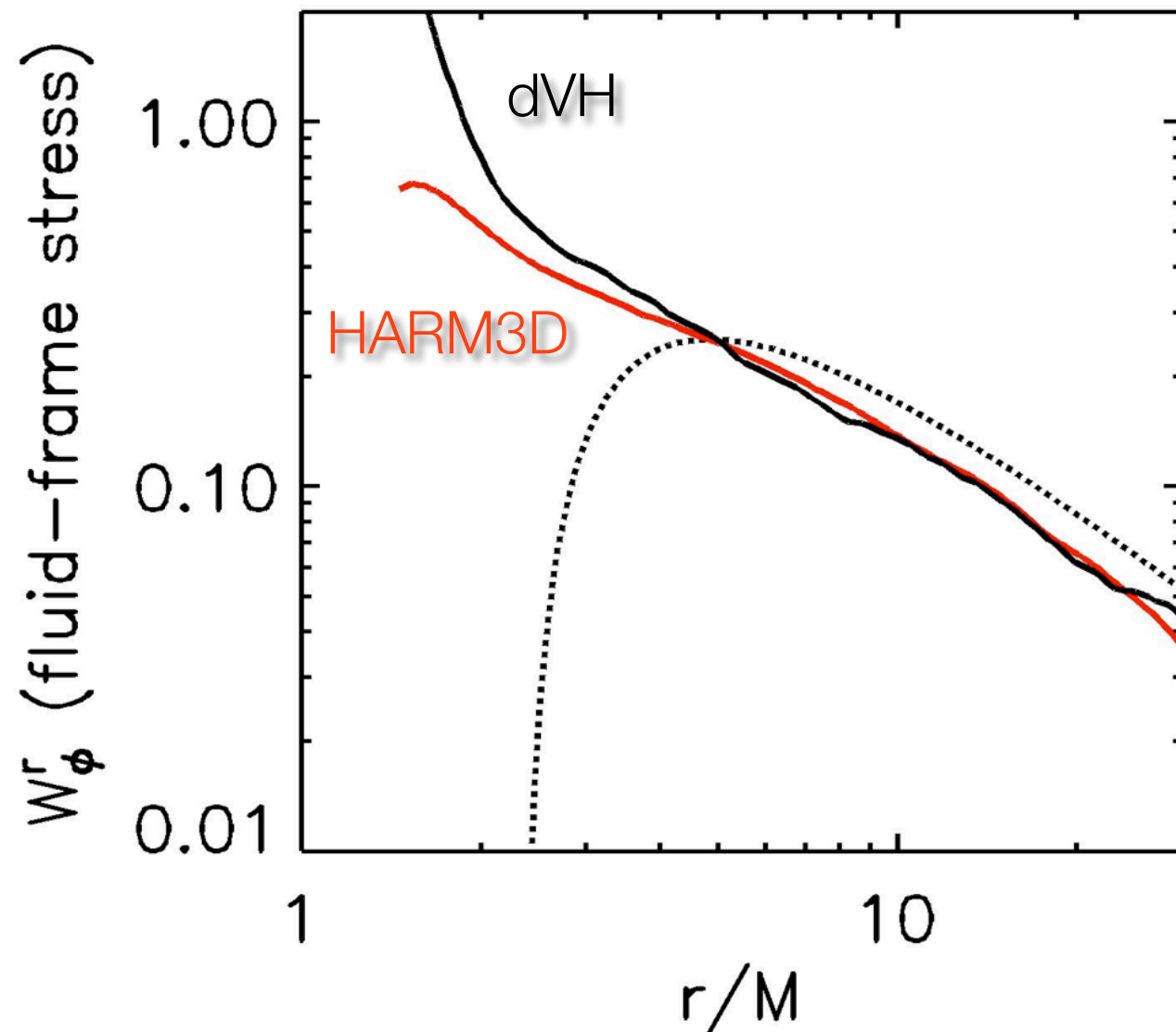


Mass Inflow Equilibrium



Thermal Equilibrium

Comparison to NT



- Retained Heat --> Stress Deficit
- Continuity through the ISCO

- Fits approx. to Agol & Krolik (2000)
 $\Delta\eta = 0.01$ $\Delta\eta/\eta = 7\%$
- ~5% flux deficit at all radii
 - Due to retained thermal and magnetic energy densities.

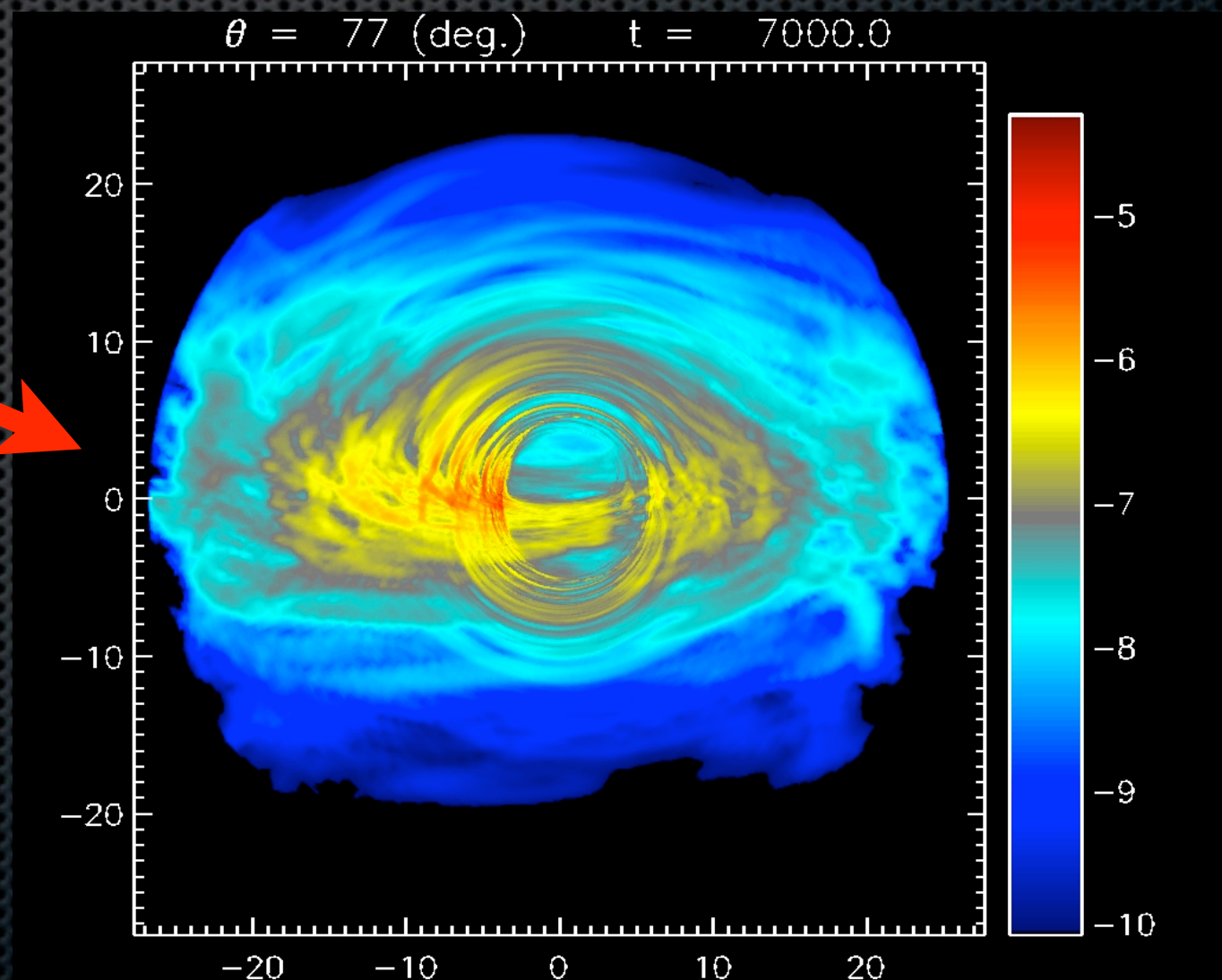
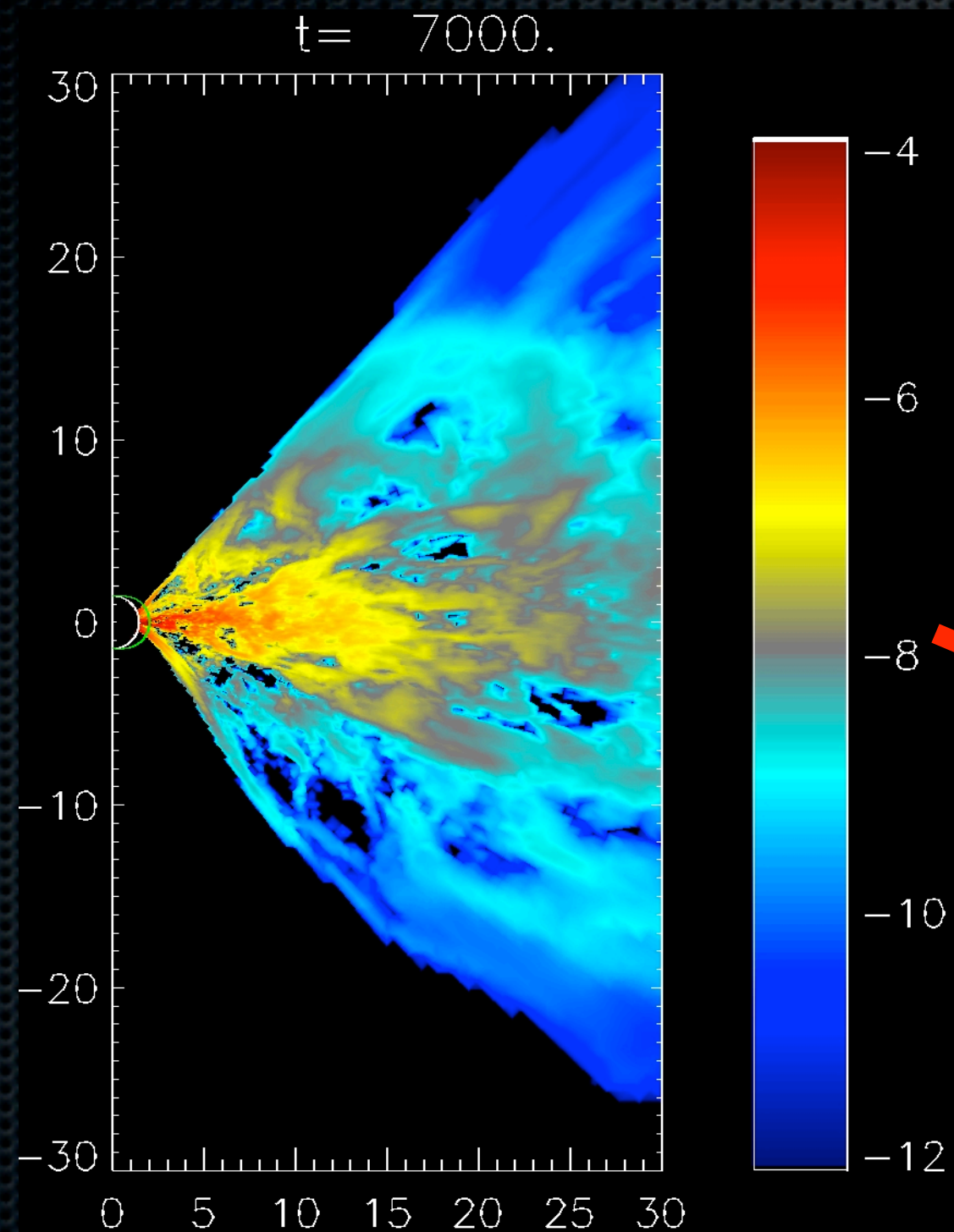
GR Radiative Transfer

$$\frac{d}{d\lambda} \left(I_\nu / \nu^3 \right) = j_\nu / \nu^2$$

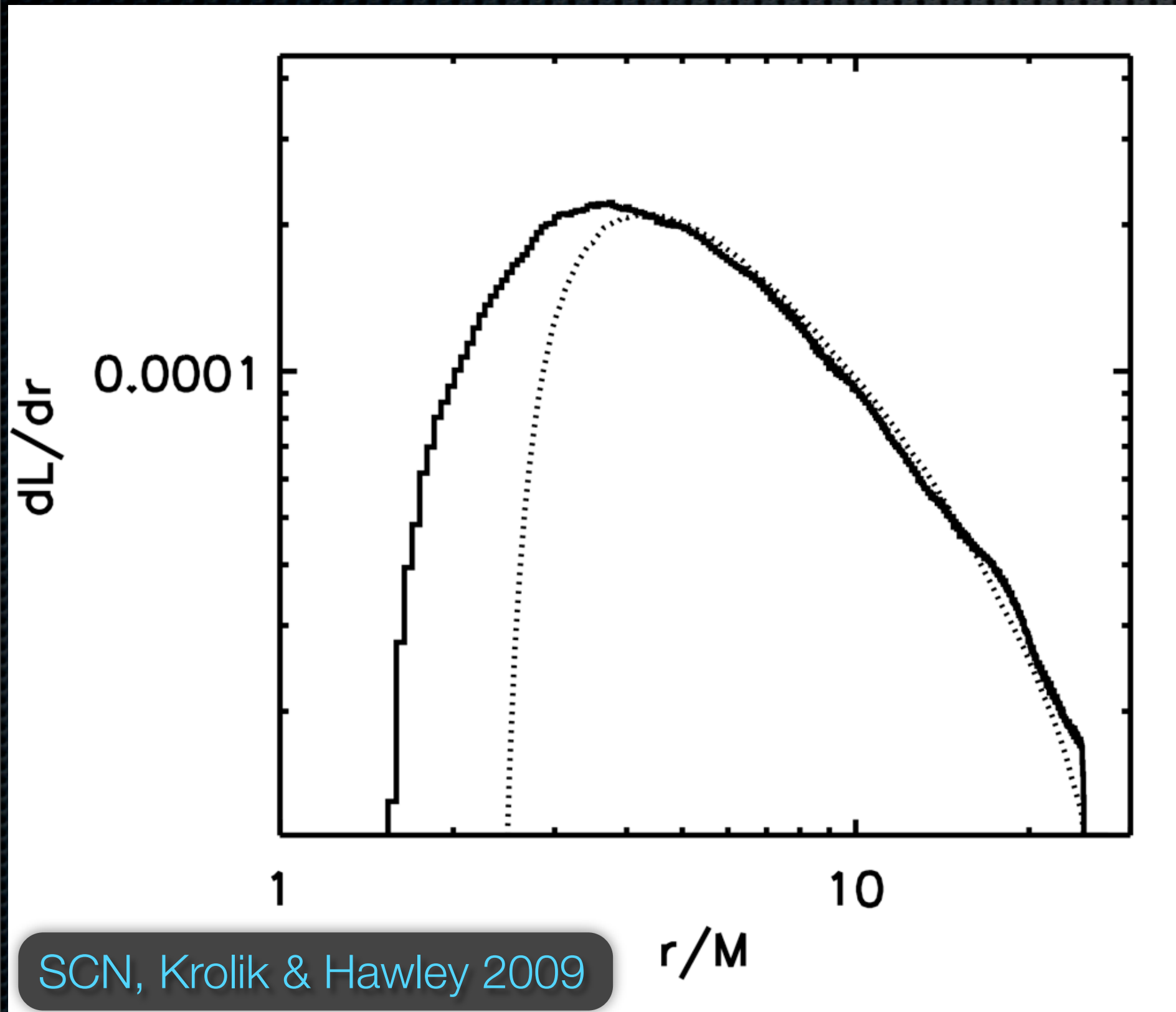
$$j_\nu = \mathcal{L} / 4\pi$$

- GR geodesic integration
- Doppler shift
- Gravitational redshift
- Relativistic beaming
- Interpolates simulation data in space & time

Allows us to explore dependence on time and disk orientation on the sky.



Angle & Time Average Bolometric Luminosity Profile



$$L = \eta \dot{M} c^2$$

$$\eta_{\text{NT}} = 0.143$$

$$\Delta\eta/\eta = 6\%$$

$$\Delta T_{\text{max}}/T_{\text{max}} = 7\%$$

$$\Delta R_{\text{in}}/R_{\text{in}} = 80\%$$

$$T \rightarrow 0 : \Delta\eta/\eta = 20\%$$

Suggests previous spectral fits
may overestimate spin.

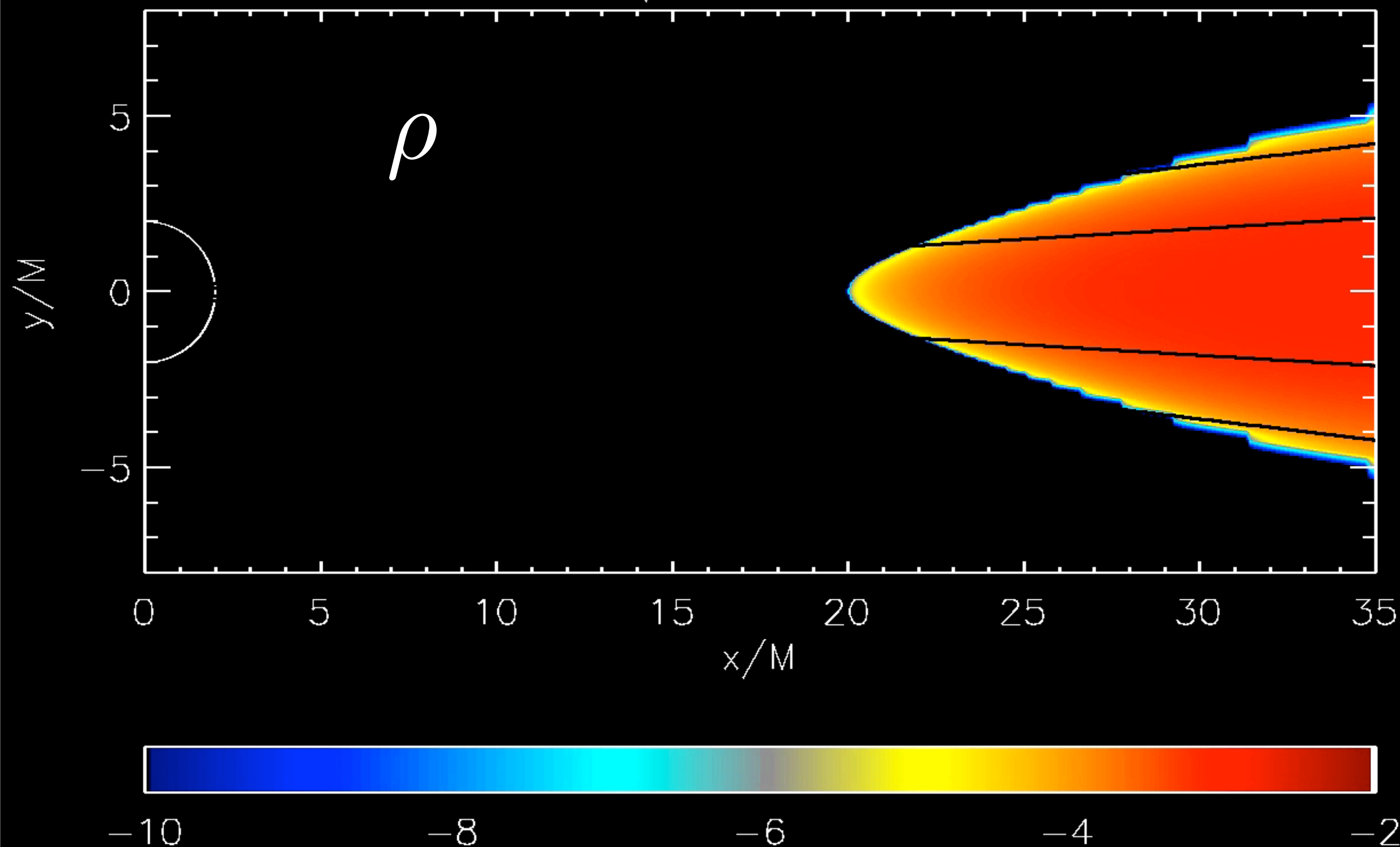
NT model may underestimate
luminosity in some disks.

ThinHR: $H/R = 0.06$ $912 \times 160 \times 64$ $a = 0M$

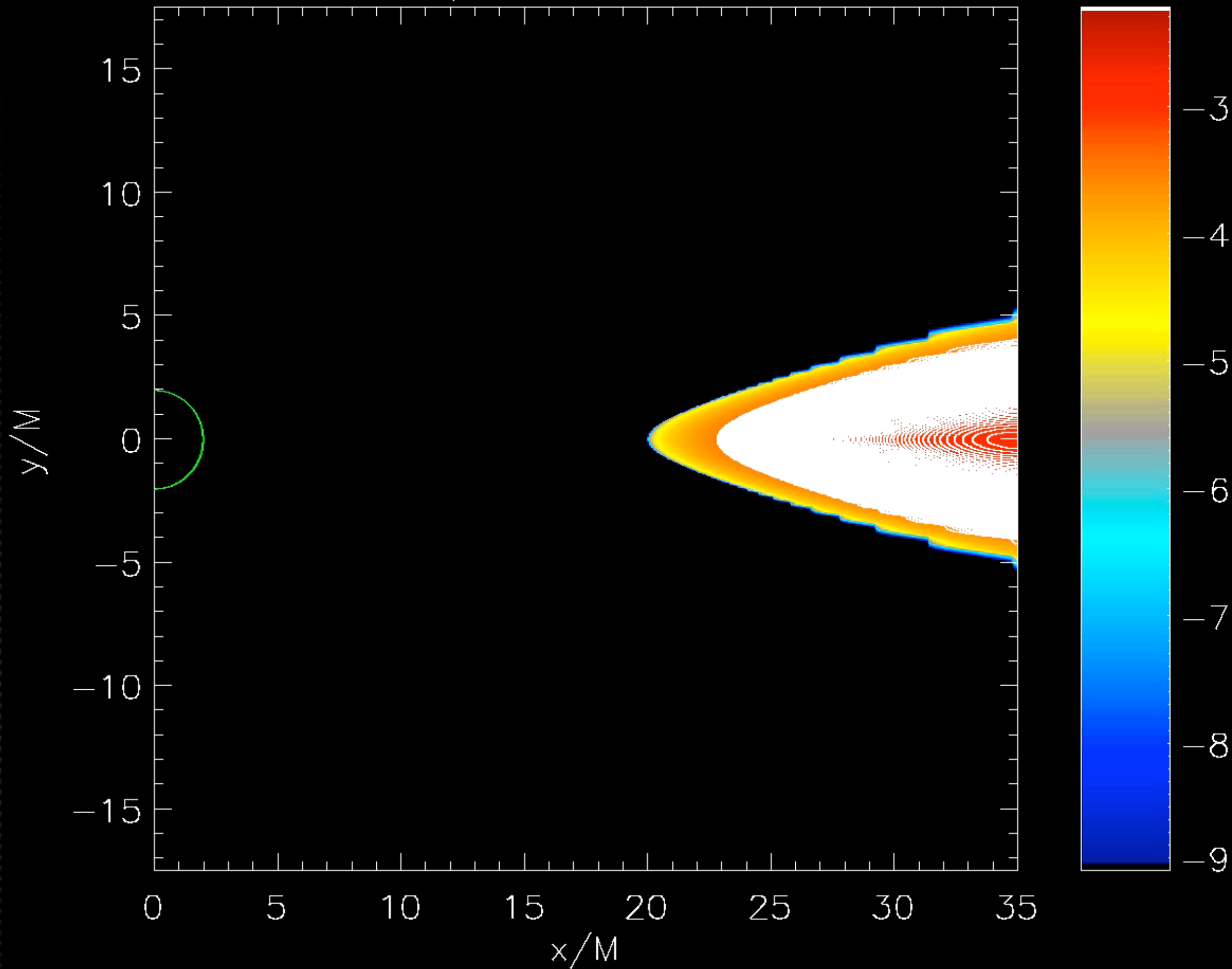
ρ

ThinHR: $H/R = 0.06$ $912 \times 160 \times 64$ $a = 0M$

$t/M = 0.$



$$t/M = 0$$



	Original	ThinLR	MediumLR	ThinHR	MediumHR	ThickHR
BH Spin	0.9M	0	0	0	0	0
Resolution $N_r \times N_\theta \times N_\phi$	192x192x64	192x192x64	192x192x64	912x160x64	512x160x64	348x160x64
Target H/R	0.1	0.06	0.08	0.06	0.08	0.16
Actual H/R	0.07-0.12	0.085	0.091	0.061	0.10	0.17
Init. Inner Edge	15M	15M	15M	20M	20M	20M
Init. Radius of P_{\max}	25M	25M	25M	35M	35M	35M
Start at Target H/R?	No	No	No	Yes	Yes	Yes
N_{cells} per H/R	15-30	60	35	81	103	74

Motivation:

- Explore H/R dependence;
- Resolve height with >60 cells ([Davis++ 2009](#)) ;
- Attempt at isotropic dissipation with nearly cubical cells;

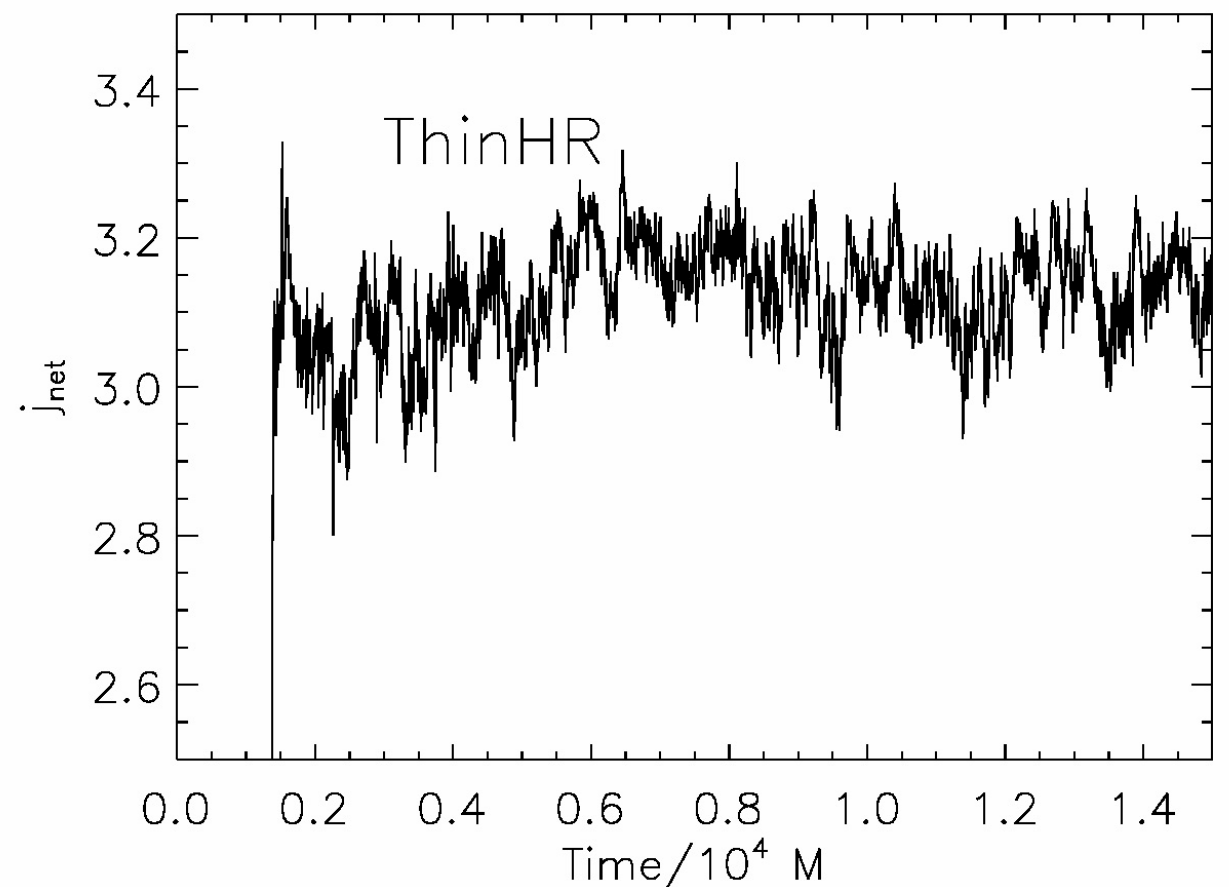
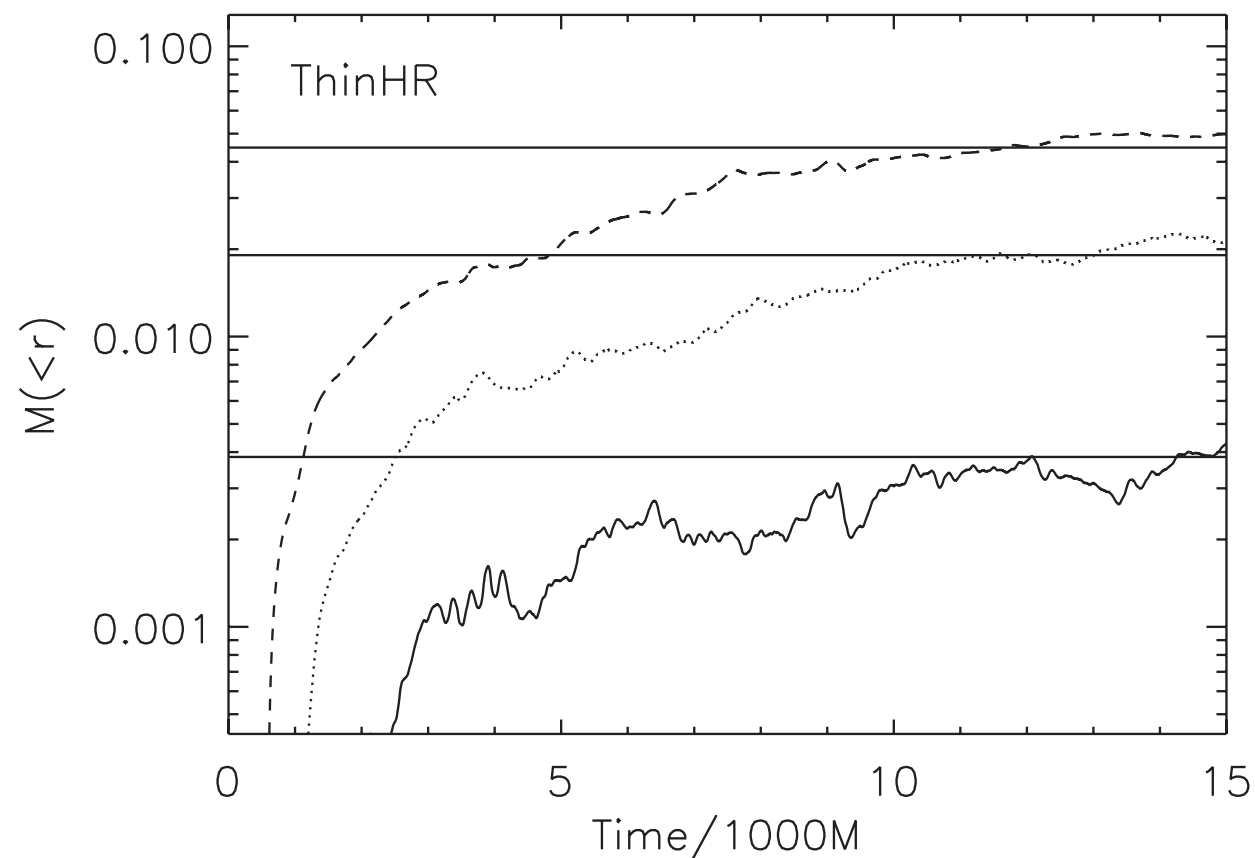
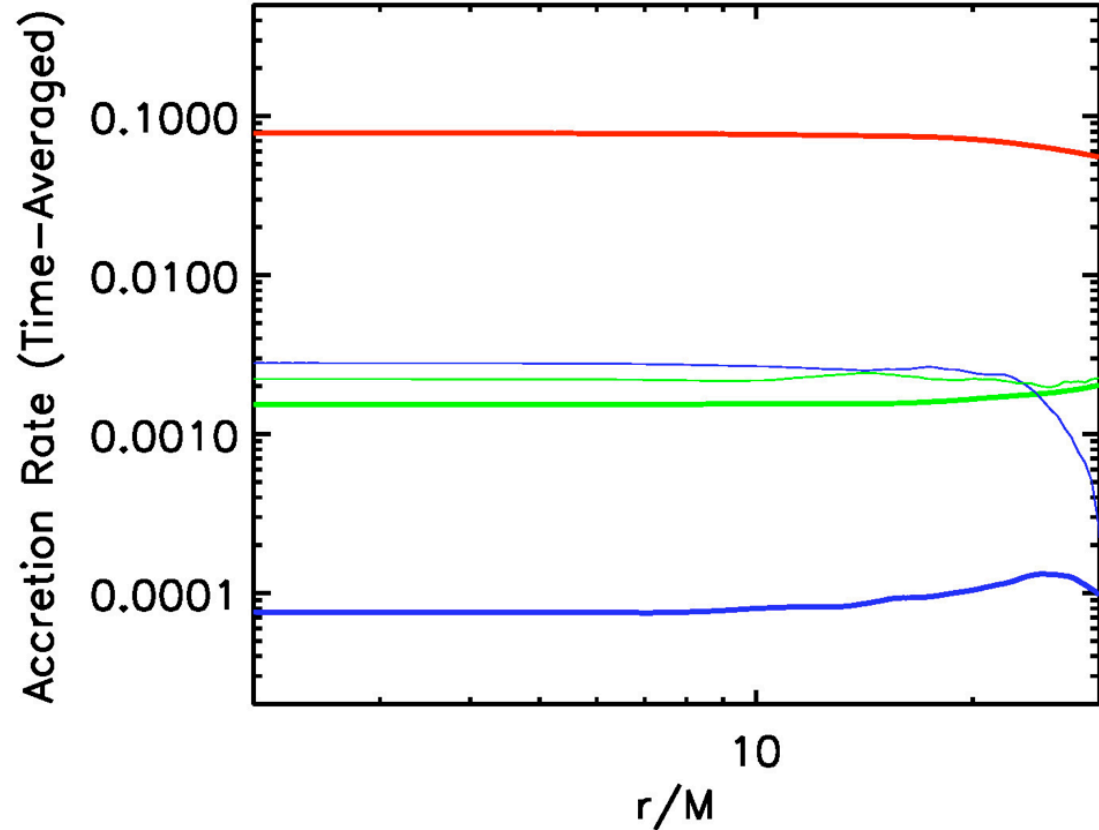
[SCN, Krolik, Hawley 2010](#)

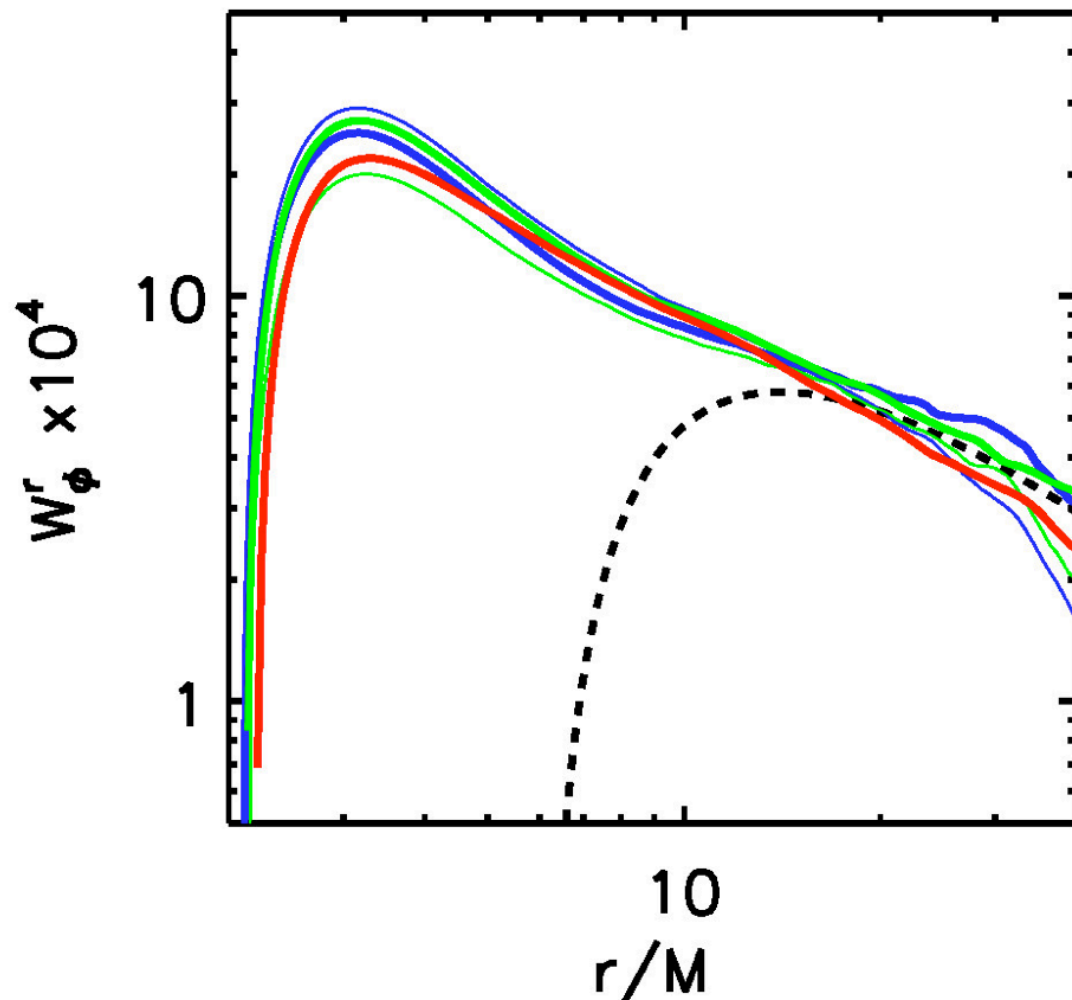
Inflow Equilibrium

Defined to be when:

- 1) Accreted specific angular momentum (j_{net}) is steady;
- 2) Mass flux shows no trends in time over radius;

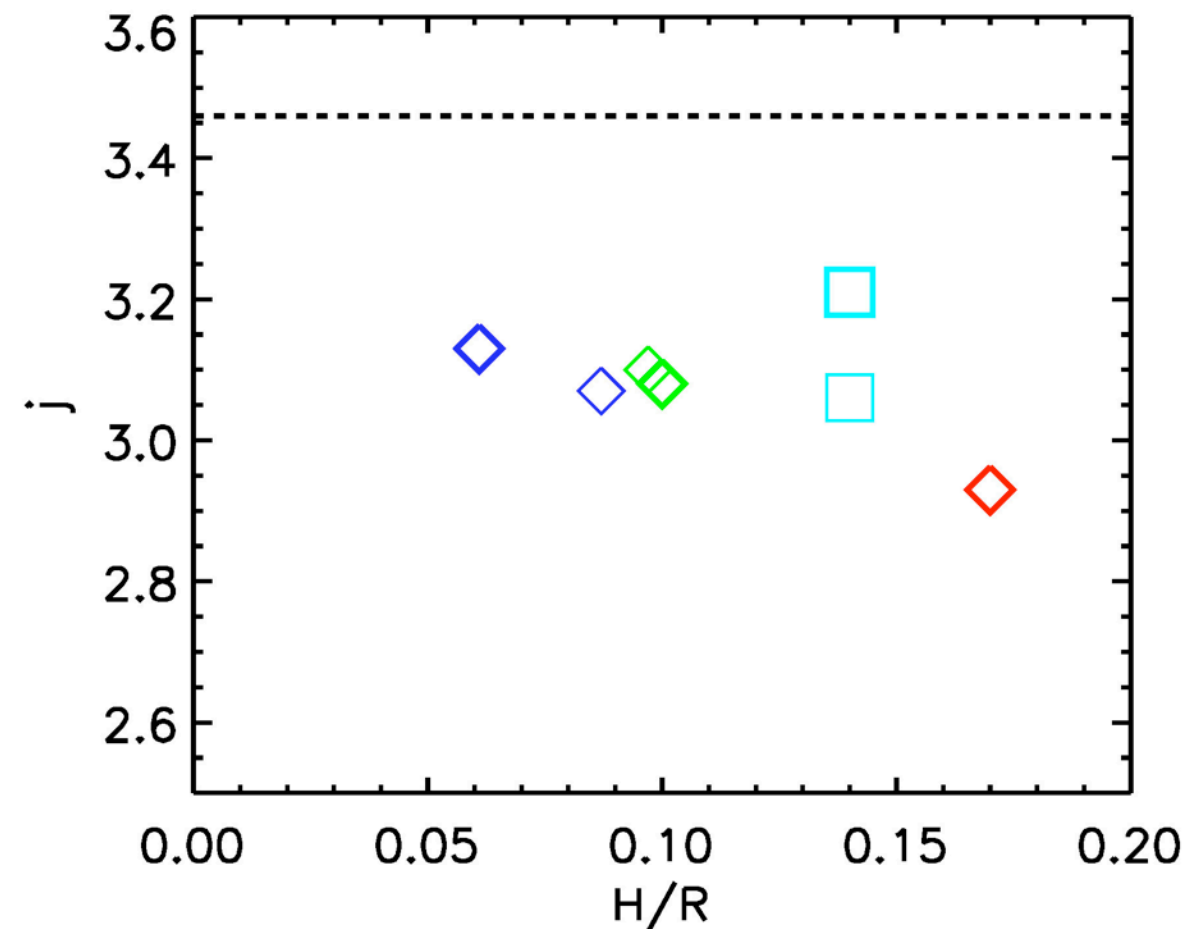
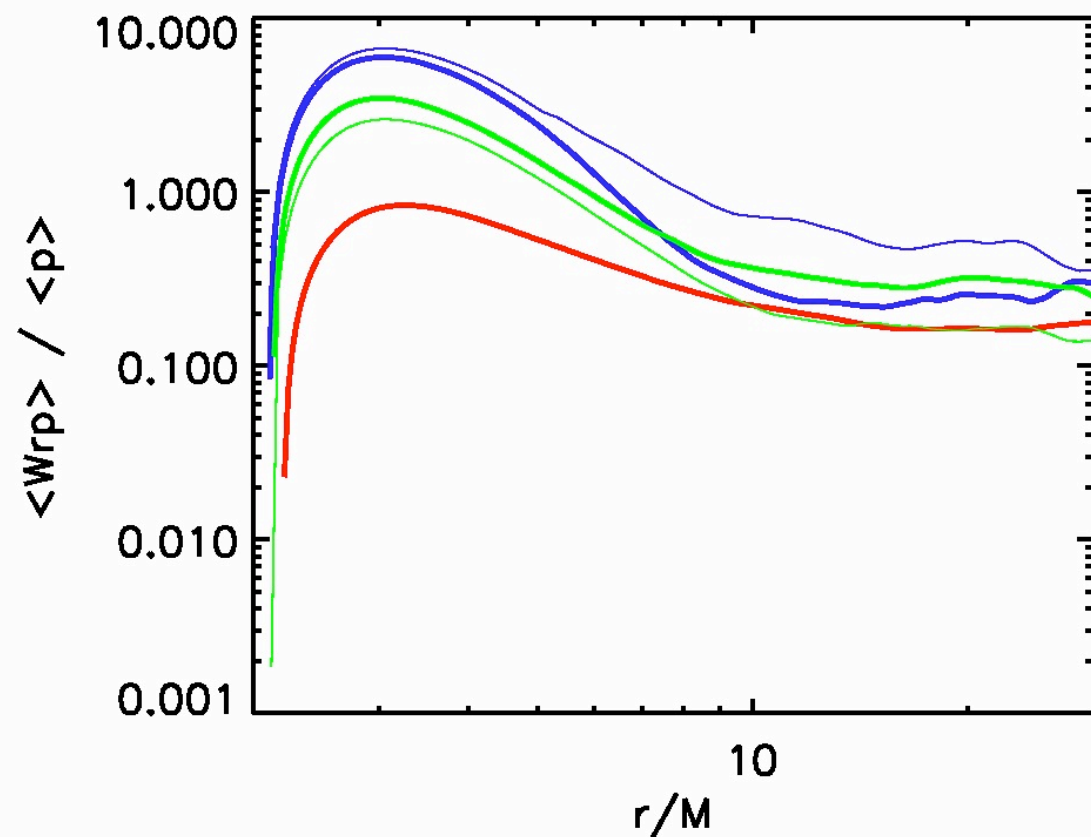
Remember these are turbulent MHD flows---they need not reach any kind of steady-state!





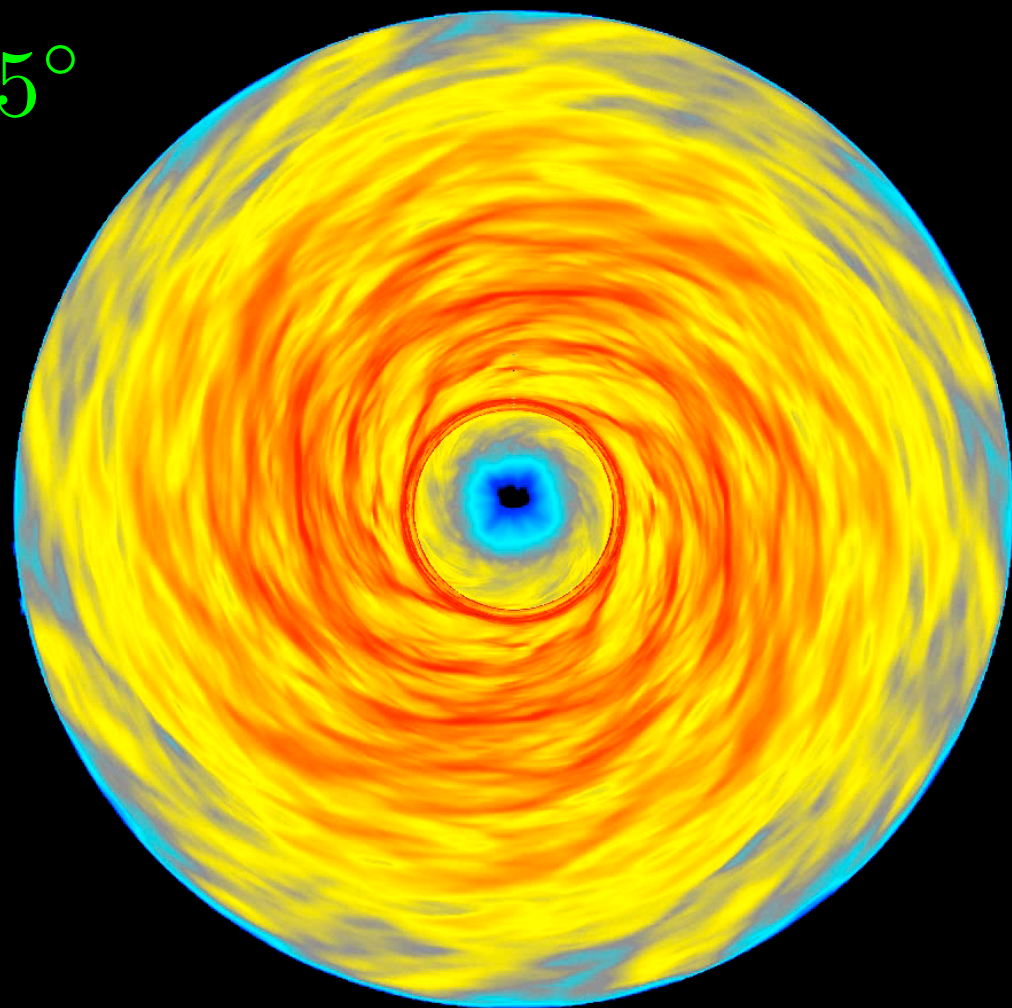
- No trend seen in Maxwell Stress
- Minor “sqrt” trend seen in spec. ang. mom.
- Due to additional Reynolds stress for thicker disks

- De Villiers & Hawley code
- Vertical field with De Villiers & Hawley code

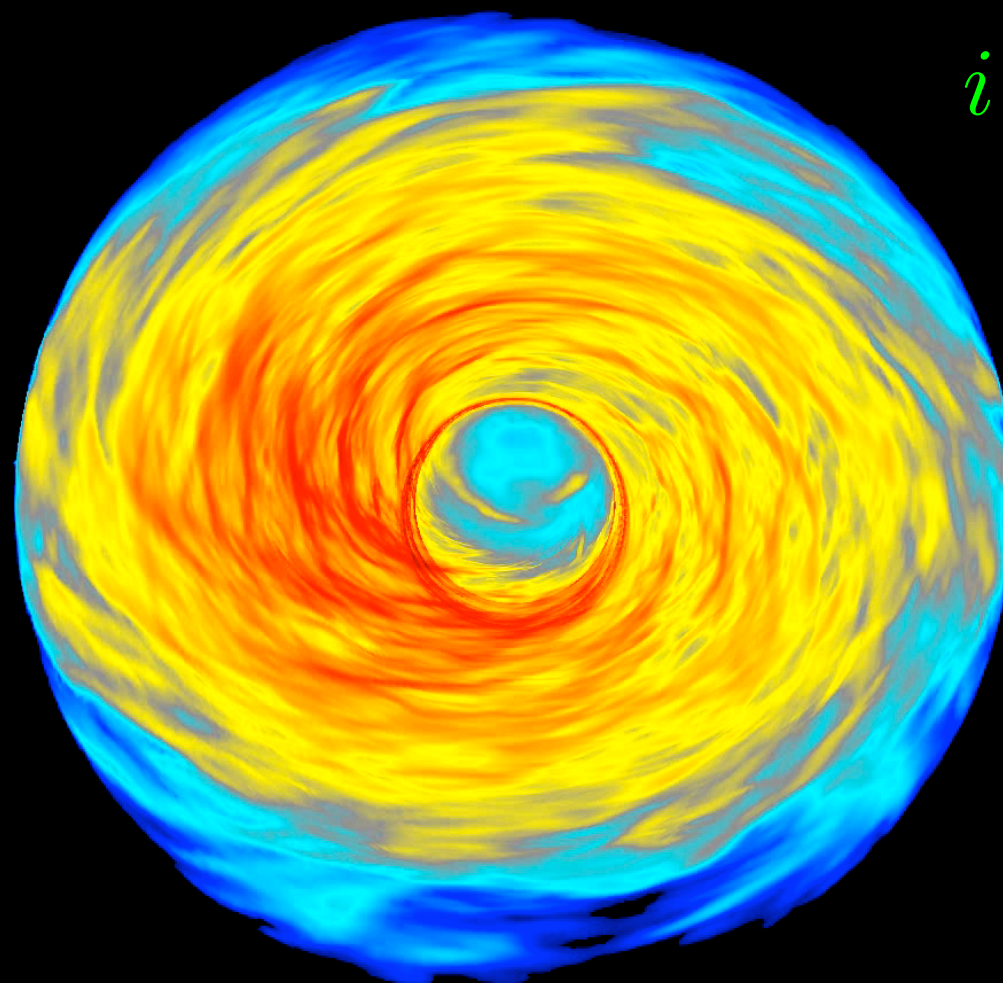


Preliminary Results!!!

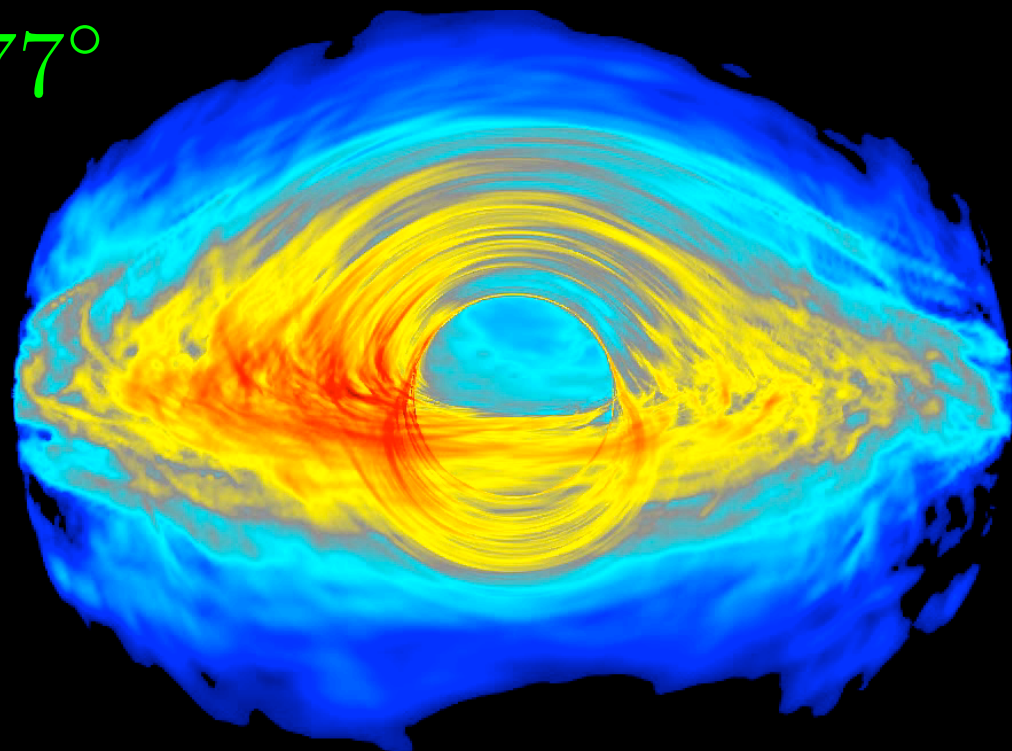
$i = 5^\circ$



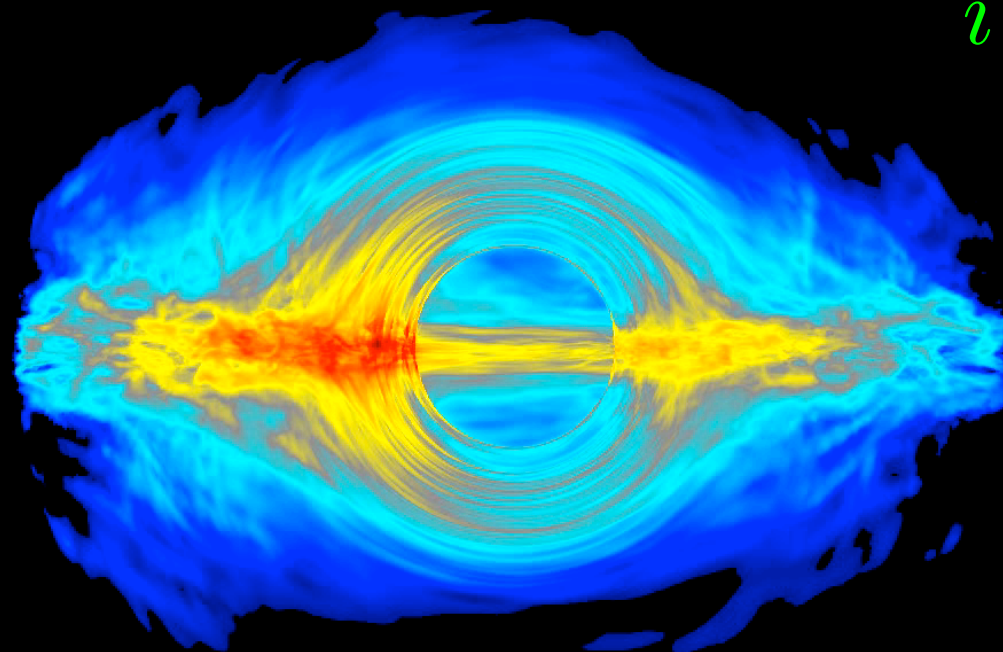
$i = 41^\circ$



$i = 77^\circ$

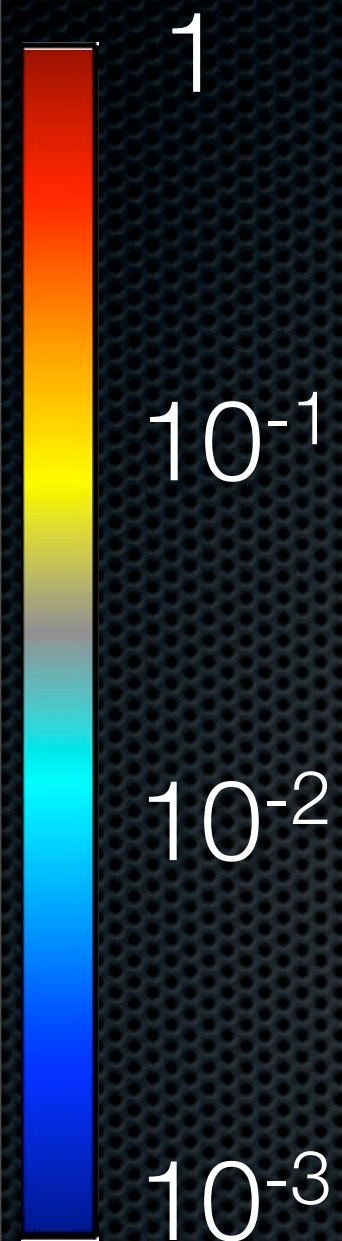


$i = 89^\circ$



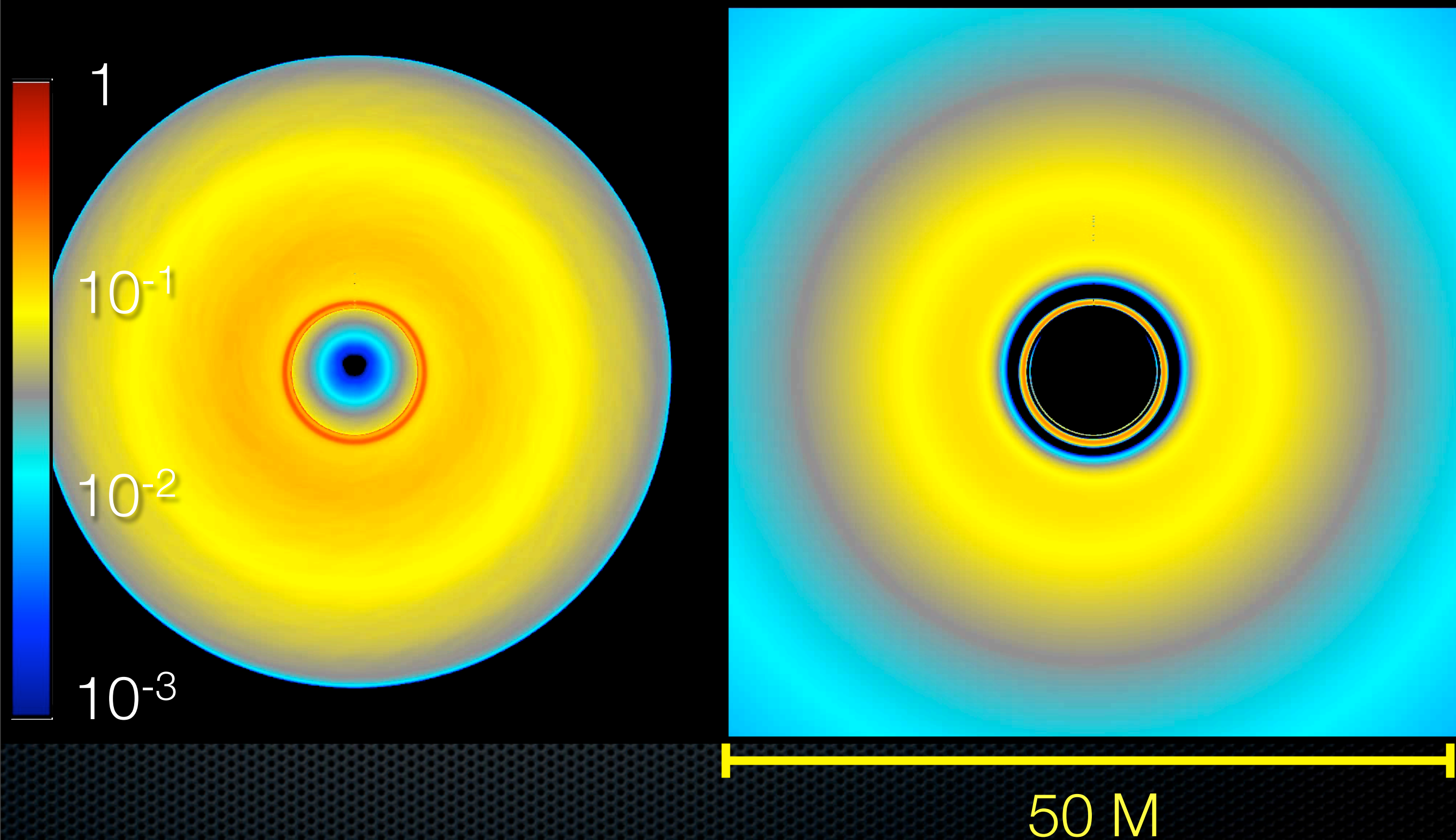
Time-averaged ThinHR

NT



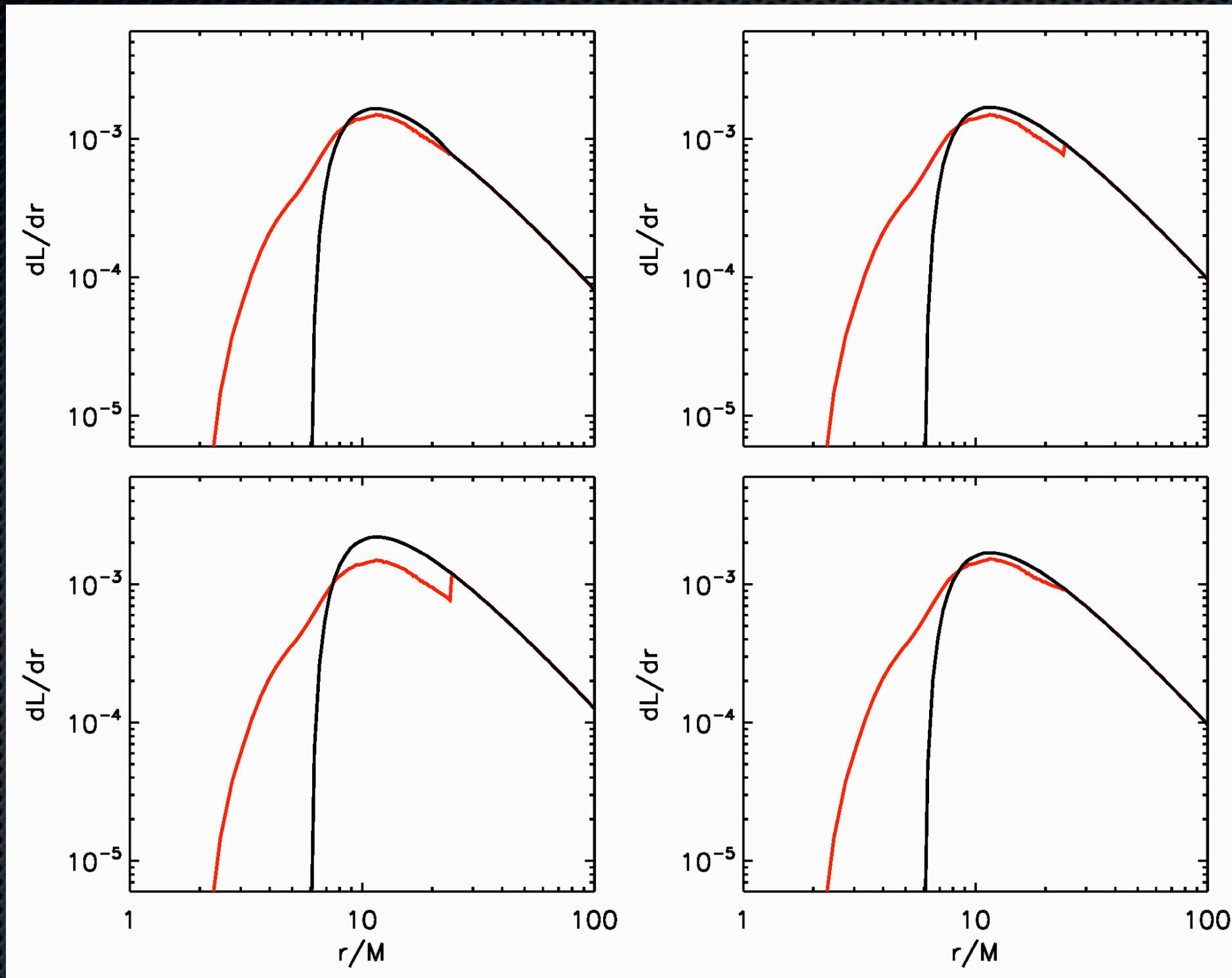
Time-averaged ThinHR

NT



ThickHR

$\frac{\Delta\eta}{\eta}$
-1%



$\frac{\Delta\eta}{\eta}$
-3%

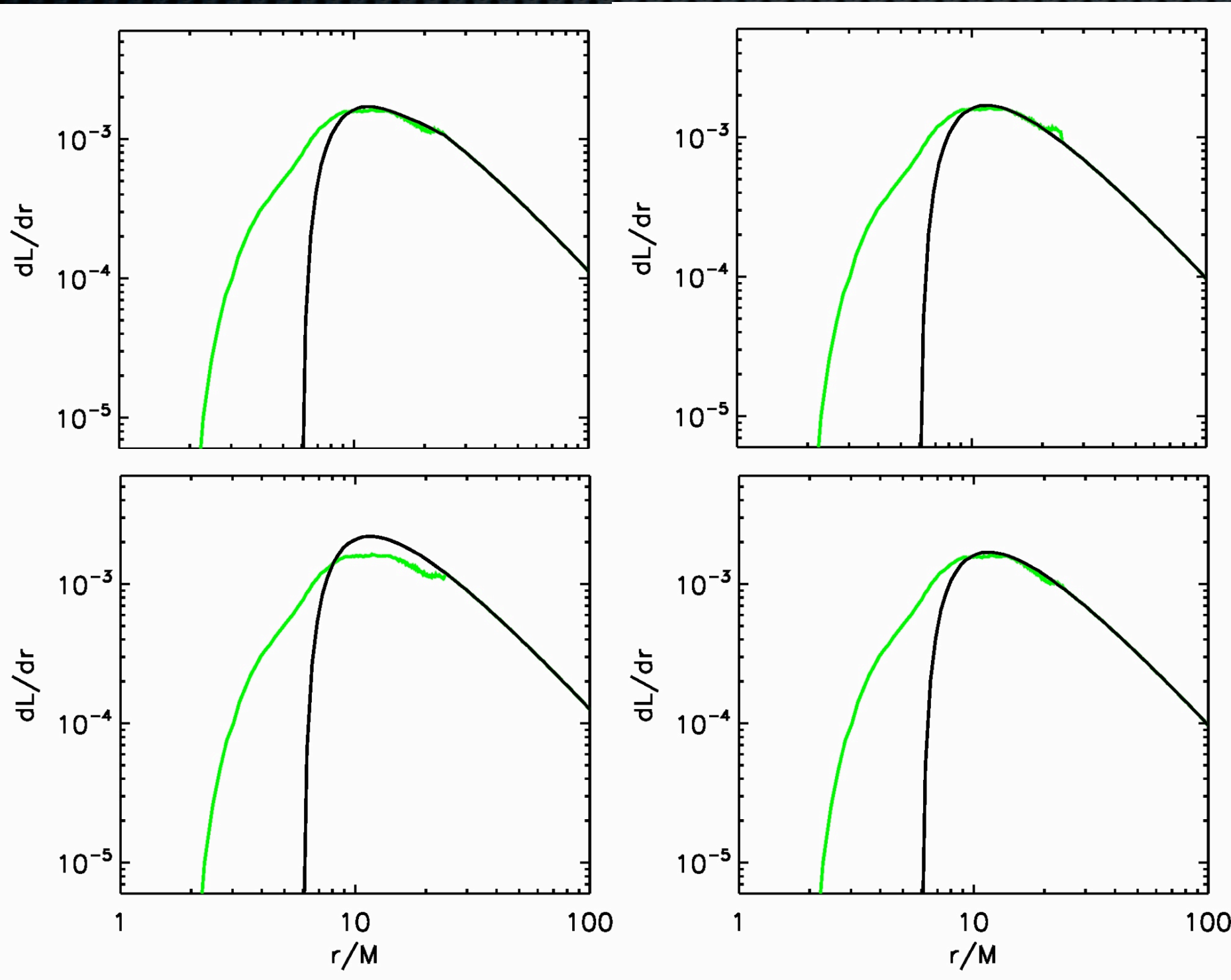
-1%

MediumHR

$$\frac{\Delta\eta}{\eta}$$

$$\eta$$

+3%



$$\frac{\Delta\eta}{\eta}$$

$$\eta$$

+5%

+6%

+4%

ThinHR

$$\frac{\Delta\eta}{\eta}$$

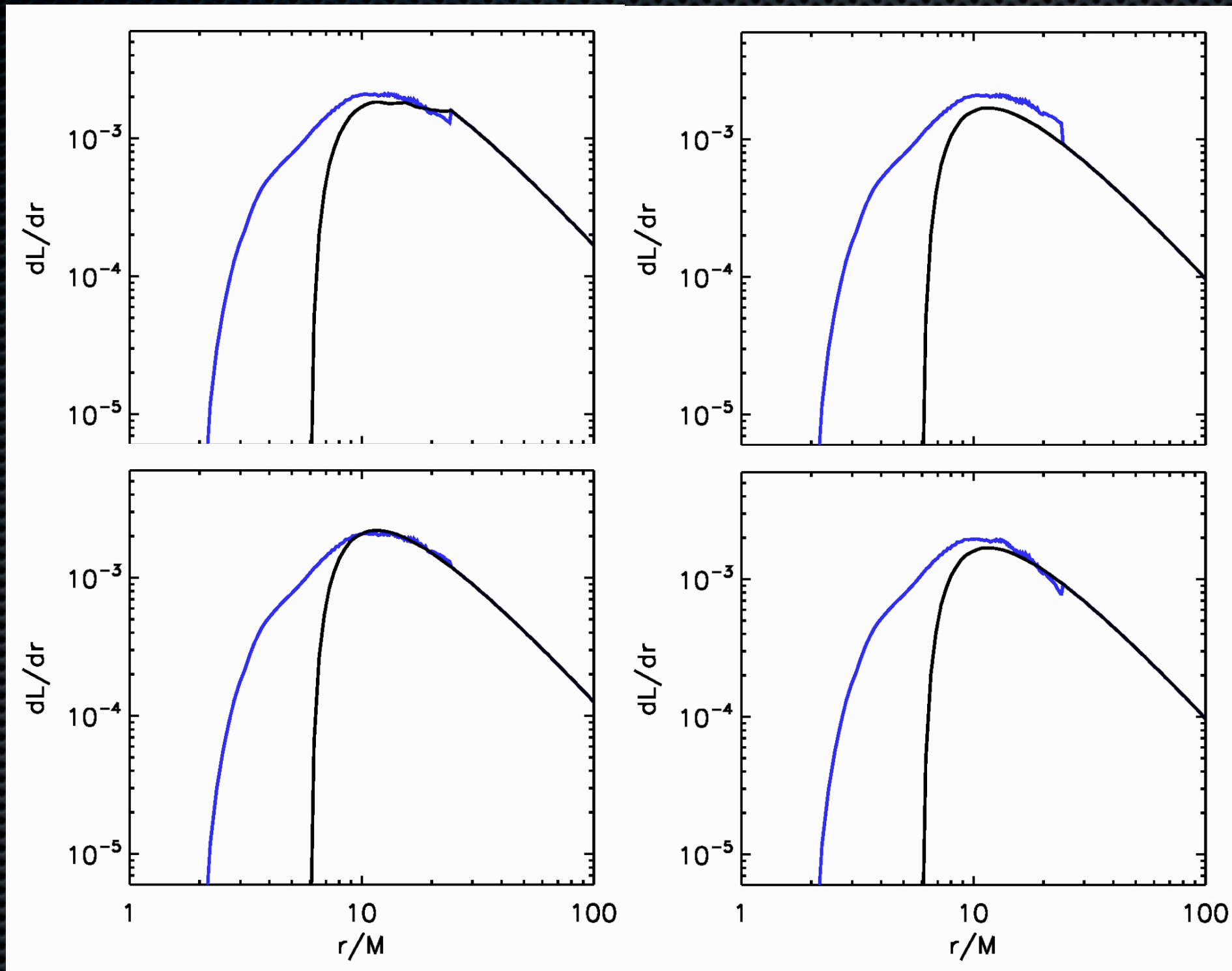
+7%

$$\frac{\Delta\eta}{\eta}$$

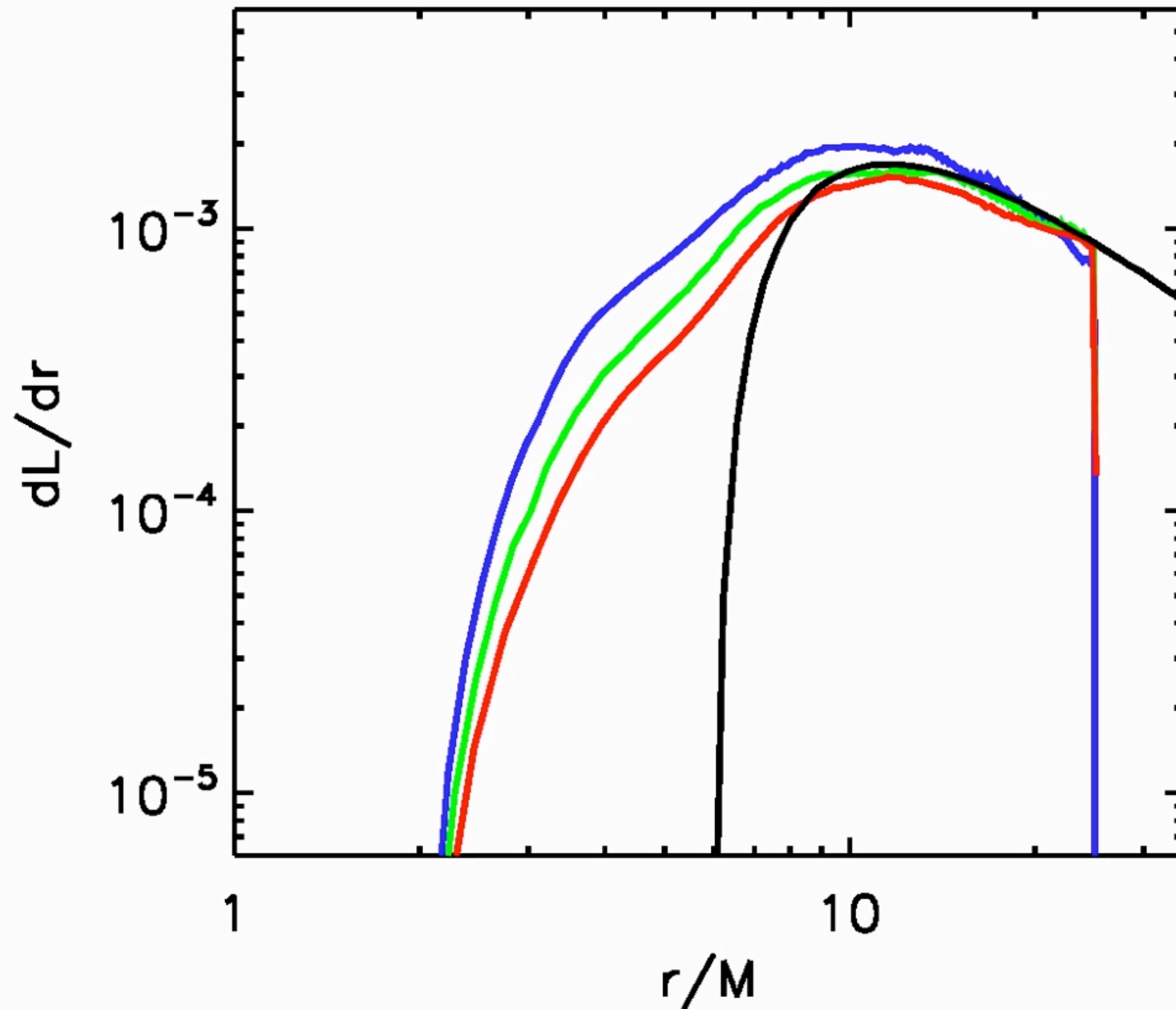
+18%

$$+5\%$$

$$+10\%$$



Efficiency Trend with Scaleheight



$$R_{\text{NT}} = 11.4$$

$$R_{\text{ThinHR}} = 10.3$$

$$\Delta T_{\text{max}}/T_{\text{max}} = 8\%$$

$$\Delta R_{\text{in}}/R_{\text{in}} = 11\%$$

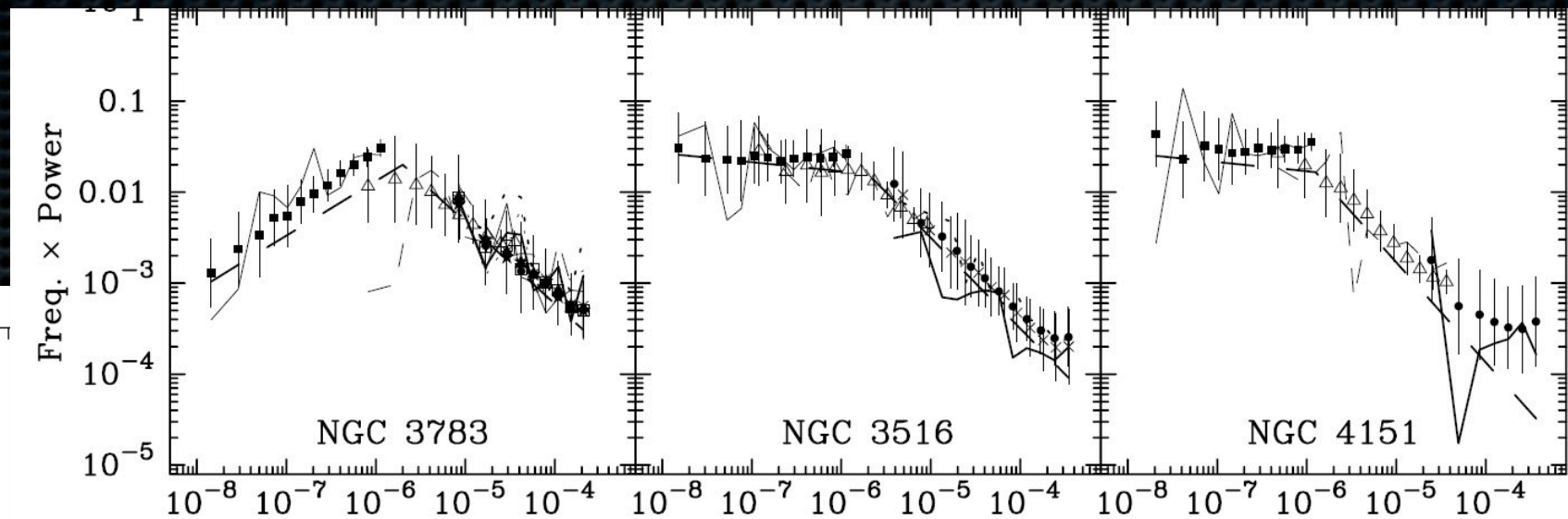
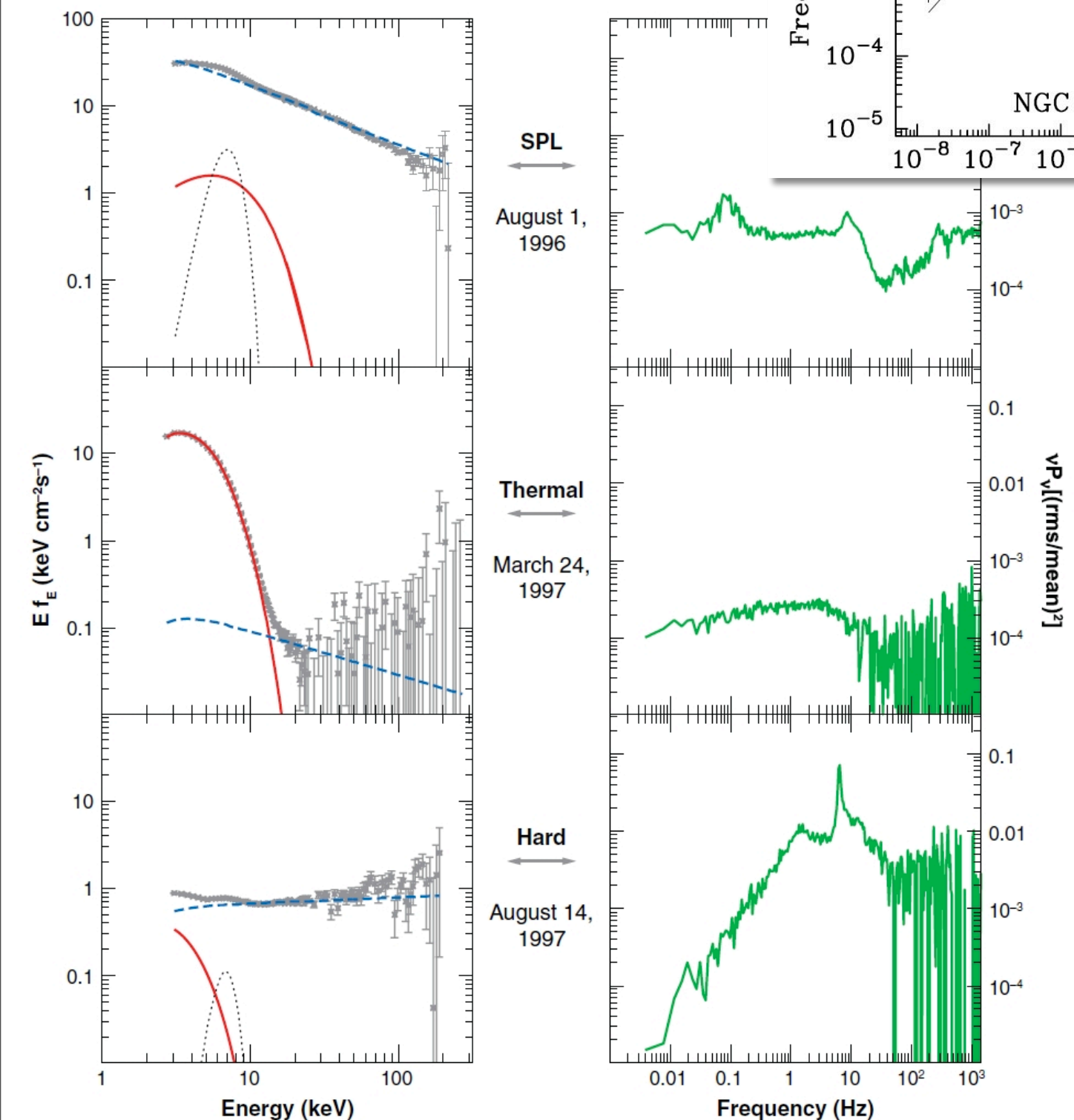
$$\Delta \eta/\eta = 10\%$$

Possibly, more light
can be generated
from retained heat
and magnetic field.

Bonus Material:

Variability

Coronal X-ray Variability



AGN Markowitz et al 2003

X-ray variability:

- is always dominated by corona;
- is dependent on spectral state;

$$P \sim \nu^\alpha$$

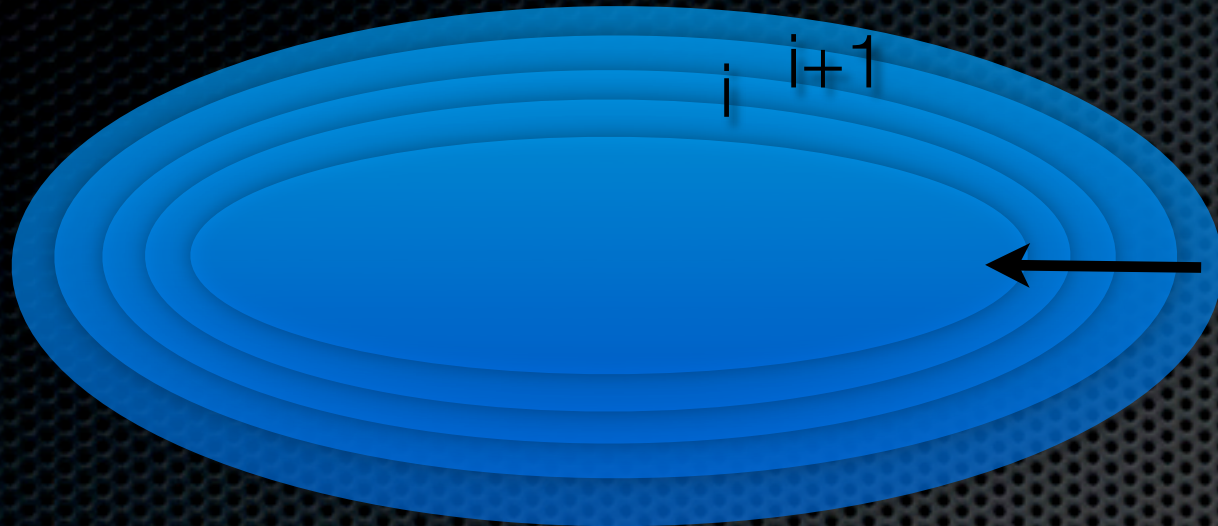
$$-3 < \alpha < -1$$

X-ray Binaries

Remillard & McClintock 2006

Variability Models

$$P \sim \nu^\alpha$$



Lyubarskii et al 1997

- Total variability is a superposition of independent variability from larger radii modulating interior annuli on inflow (viscous) times scales

Churazov et al 2001

- Outer radius of corona may be cause of (temporal) spectral slope

$$\tau_a = \left[\alpha \left(\frac{H}{r} \right)^2 \Omega_K \right]^{-1}$$

- Accretion rate modulation modeled as variability of α (disk parameter)
- Predicts phase coherence at frequencies longer than inverse of inflow timescale

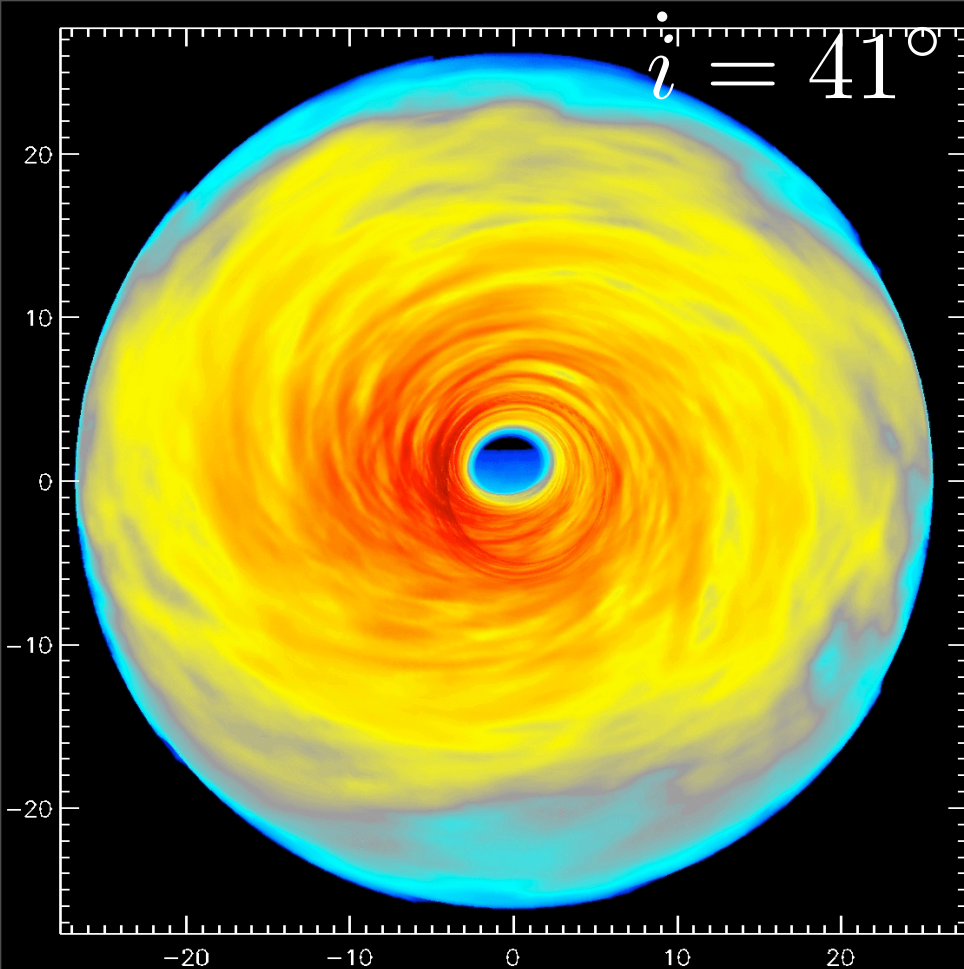
Armitage & Reynolds 2003

Machida & Matsumoto 2004

Schnittman et al 2006

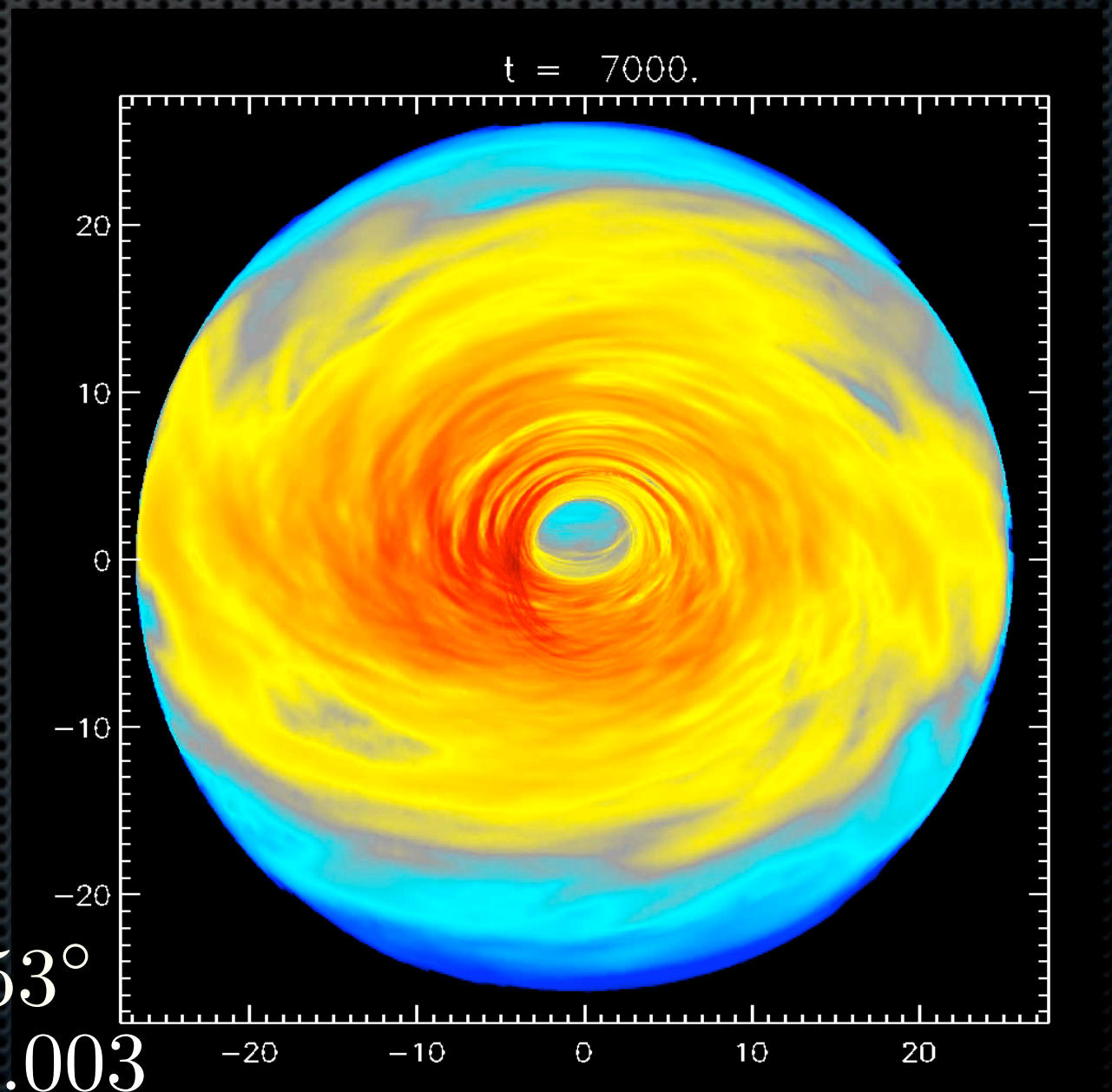
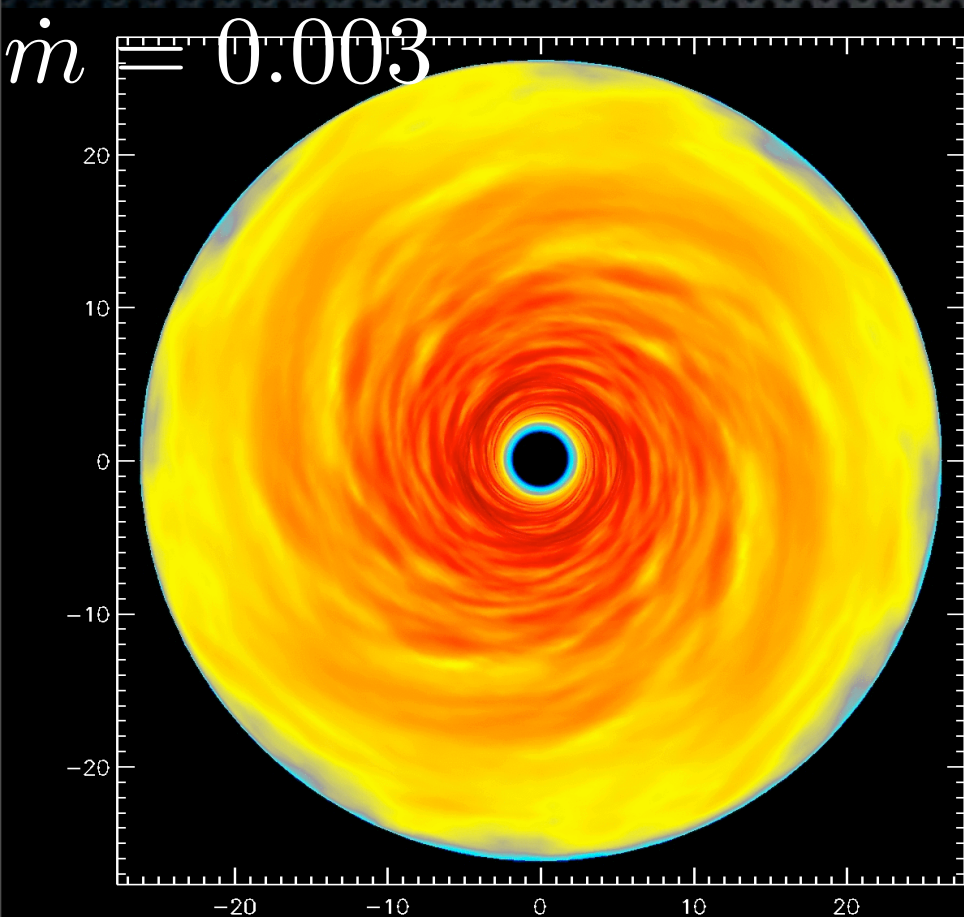
Reynolds & Miller 2009

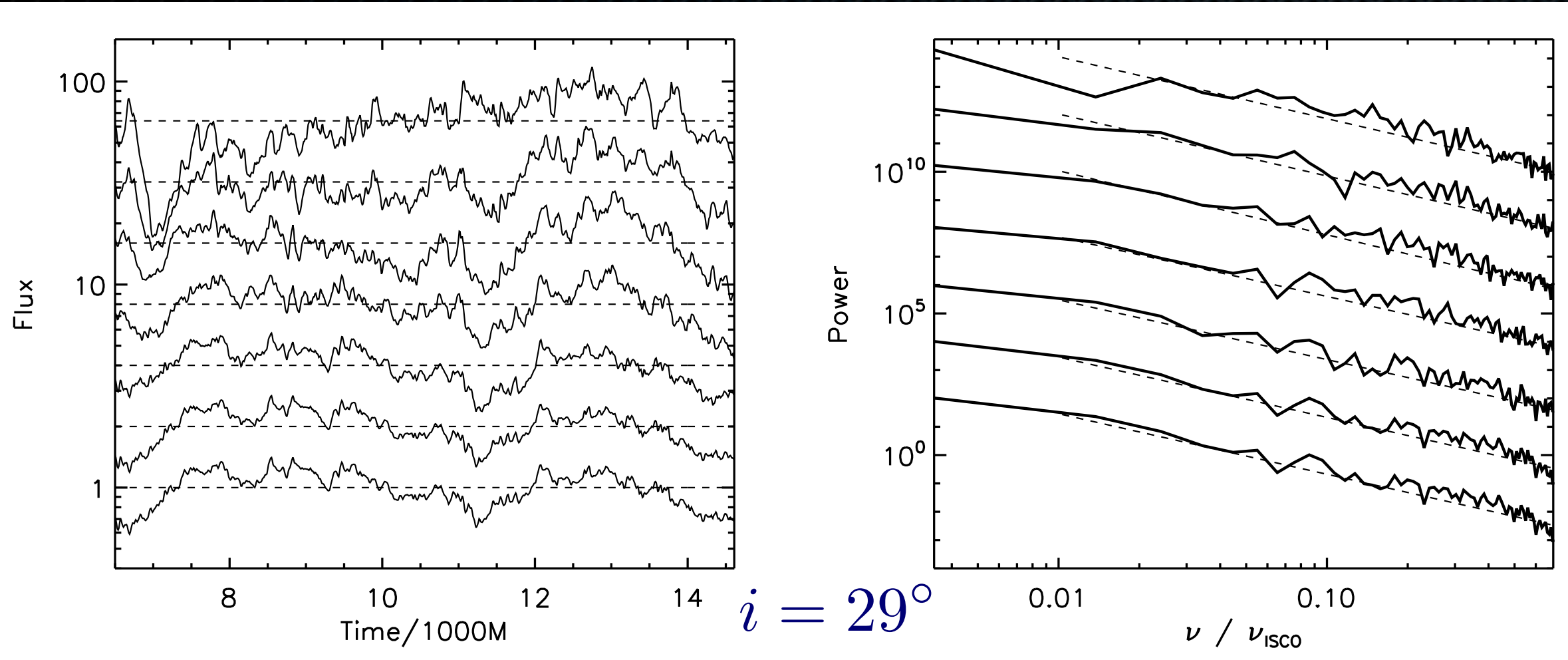
- Used accretion rate or stress as dissipation proxies
- PLD breaks at local orbital frequency per annulus
- Composite PLD $\rightarrow \alpha \simeq -2$



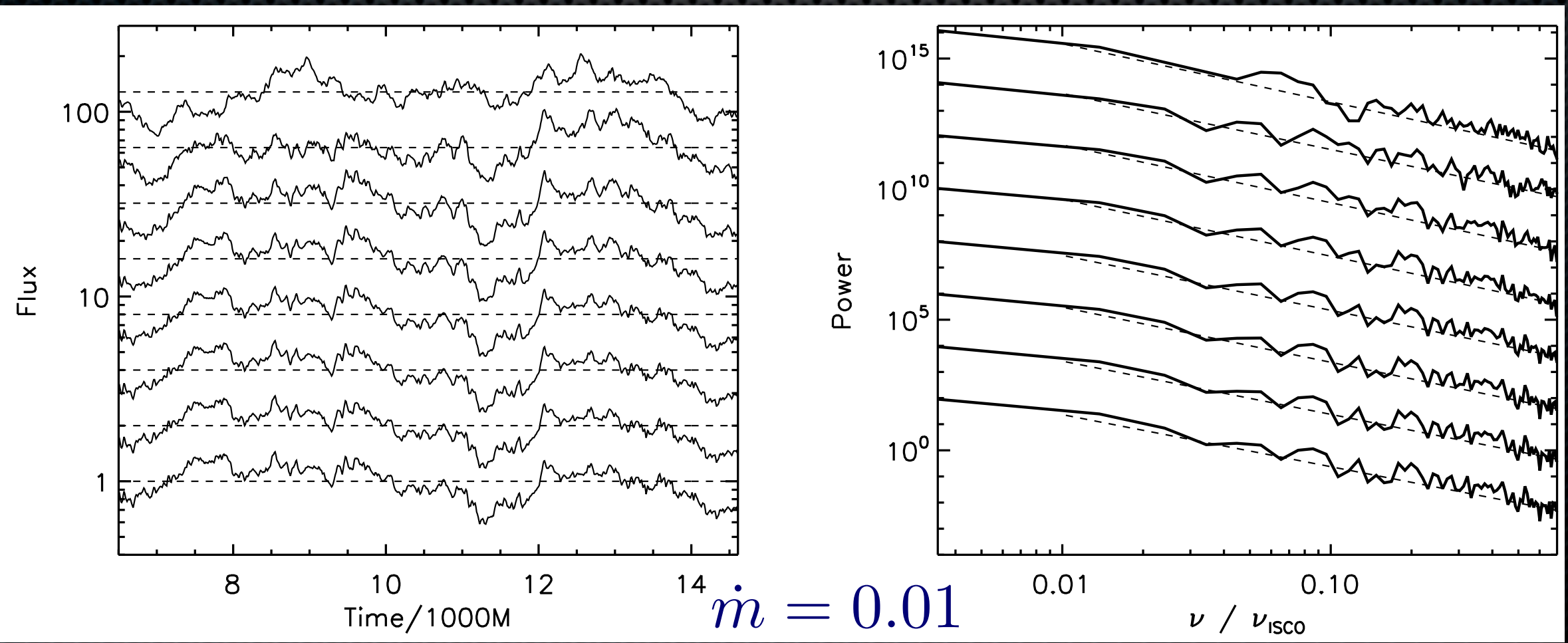
SCN & Krolik 2009

- Use “thin disk” cooling rate in corona as emissivity
- Thomson Opacity model (e- scattering)
- Integrate to photosphere ($\tau = 1$)
- Include finite light speed effect
- Parameterized by accretion rate and inclination

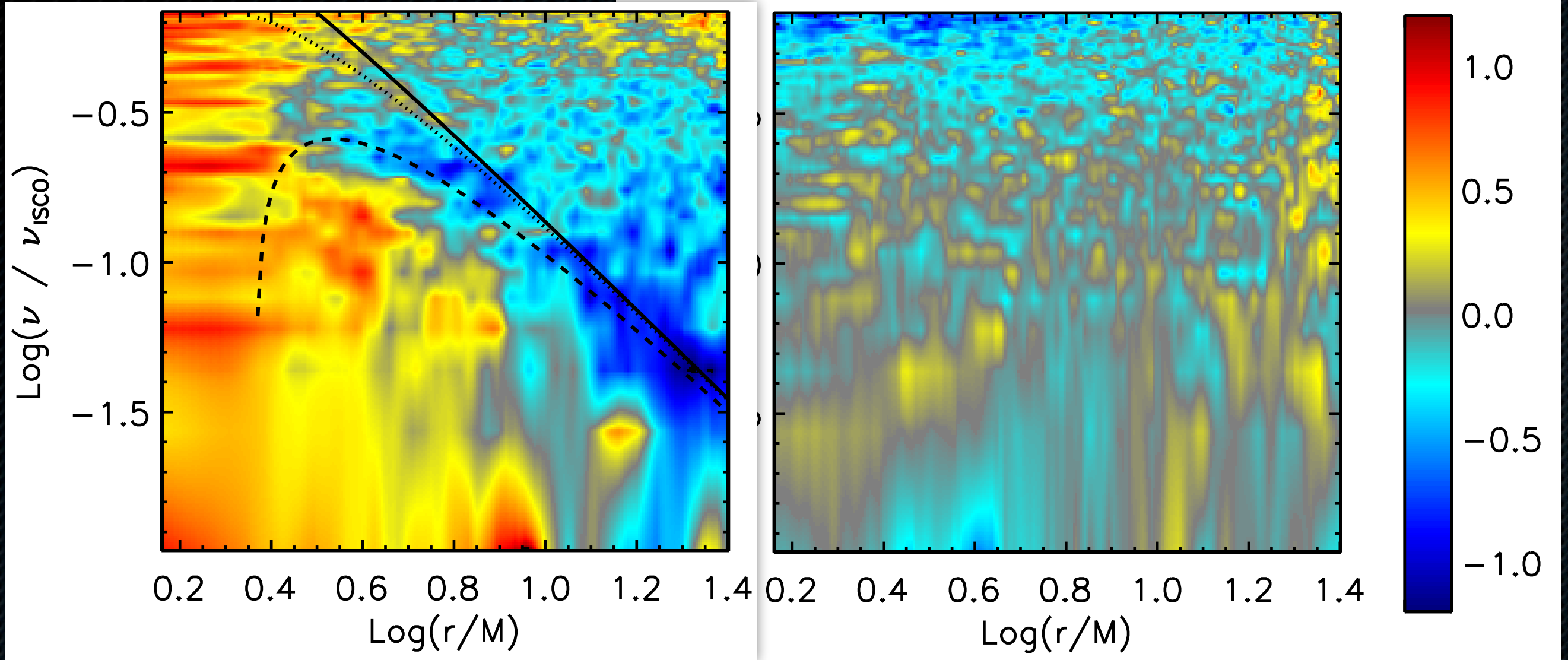




↑
 \dot{m}



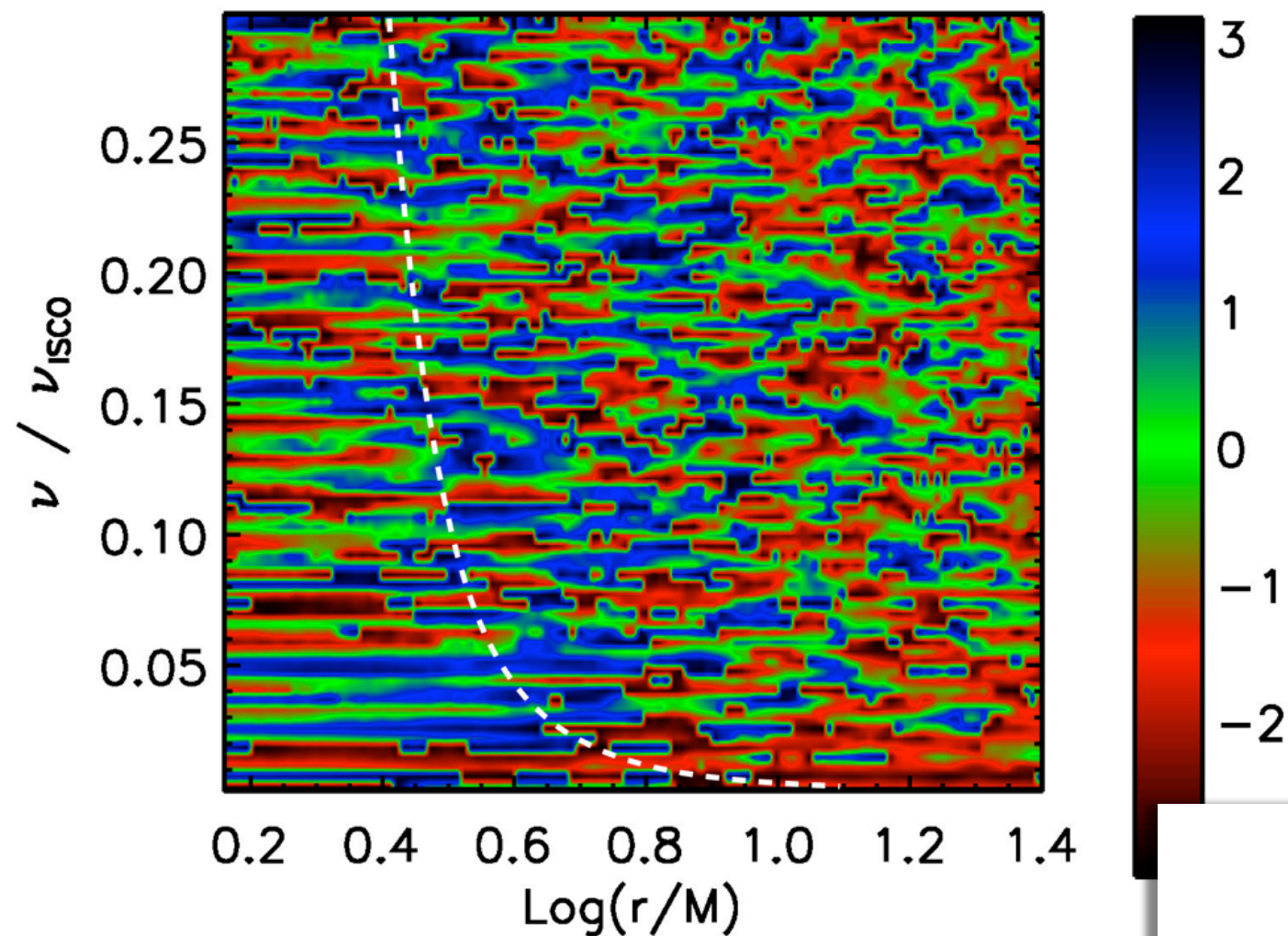
↑
 i



$$\log \frac{P_{\text{diss}}(\nu, r)}{P_{\dot{M}}(\nu, r)}$$

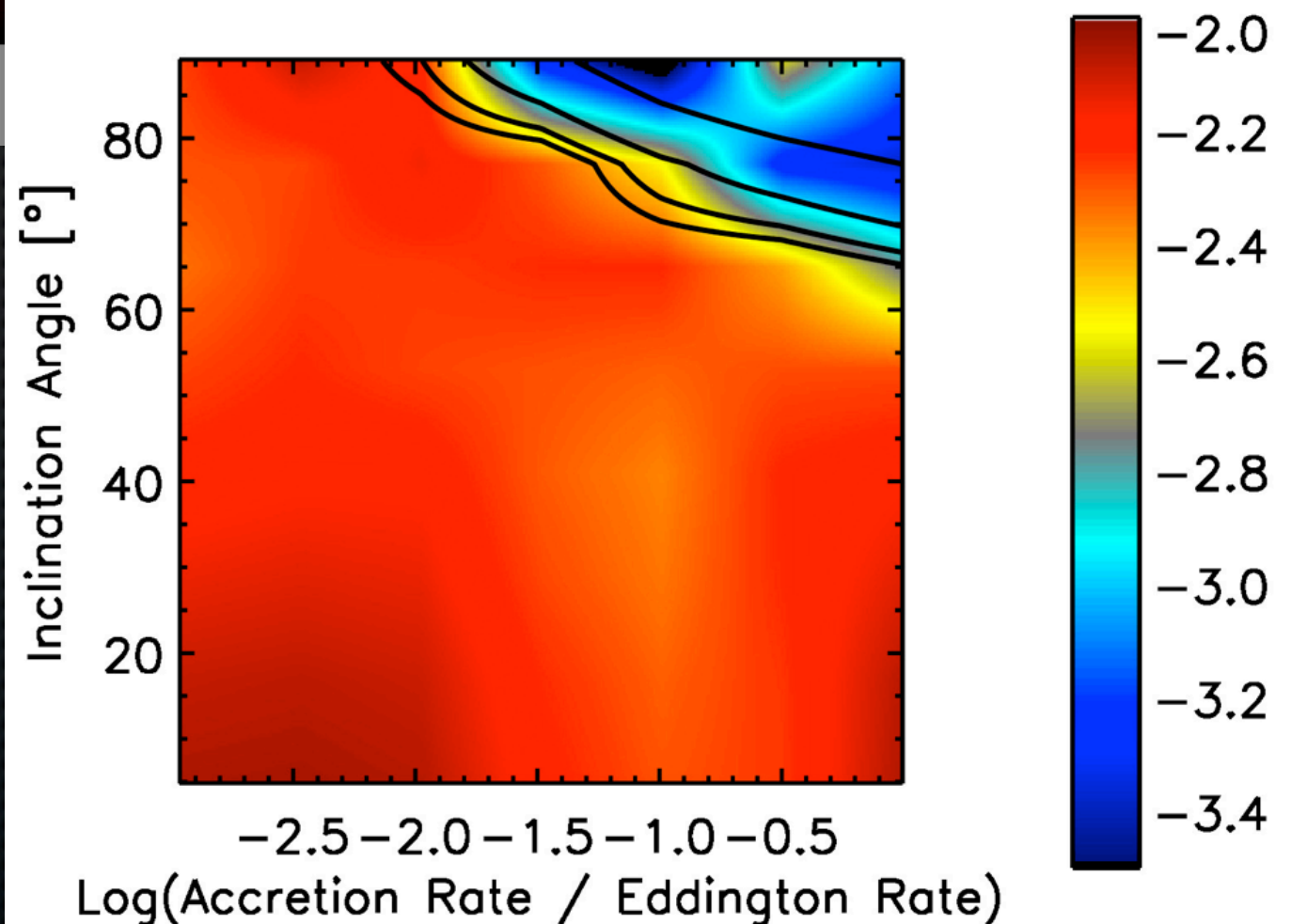
$$\log \frac{P_I(\nu, r)}{P_{\text{diss}}(\nu, r)}$$

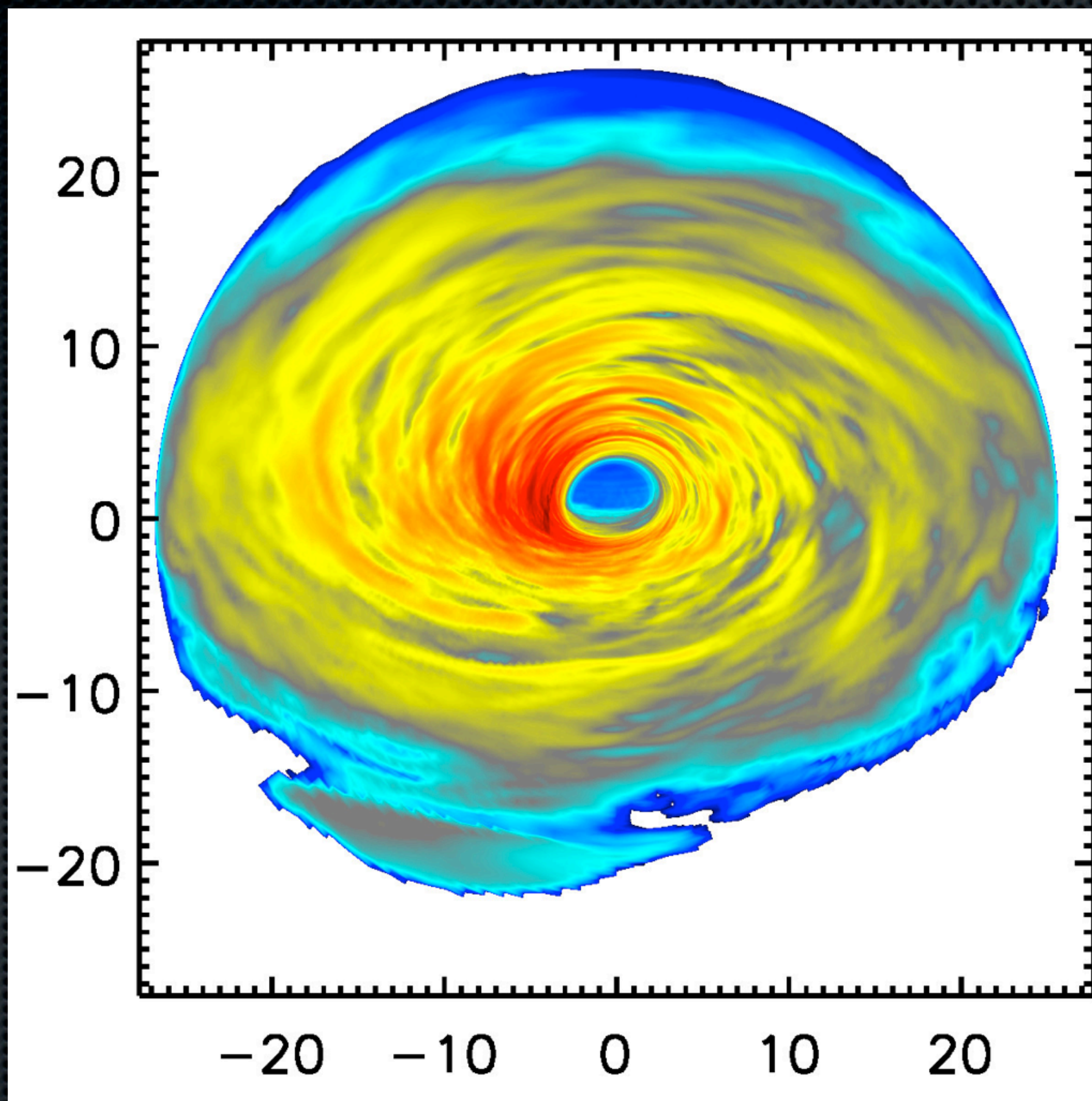
- Dissipation approximately follows accretion rate
- Not all accretion rate modes are dissipated
- Variability at infinity follows local dissipation var.



- Mostly incoherent between adjacent radii and frequencies;
- Possible coherence at $\nu < 1/T_{\text{inflow}}(r)$
- Need longer runs to verify;

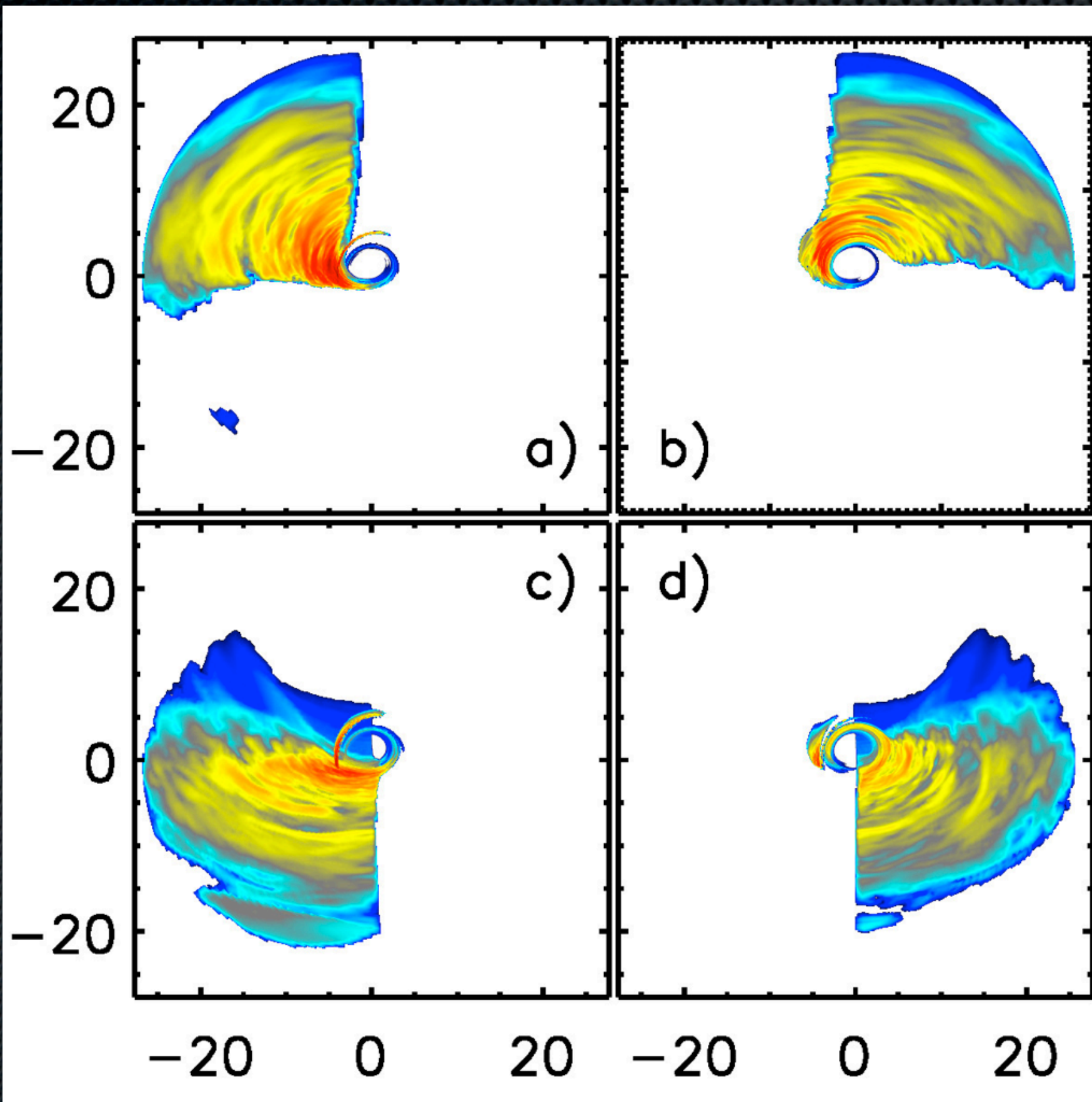
- Degenerate Result;
- No inclination angle effect;
- Consistent w/ observed power-law exponents
- See no QPOs, though we lie between LFQPO and HFQPO range





Degeneracy Explanation

$$\alpha_a > -2$$



$$\alpha_b > -2$$

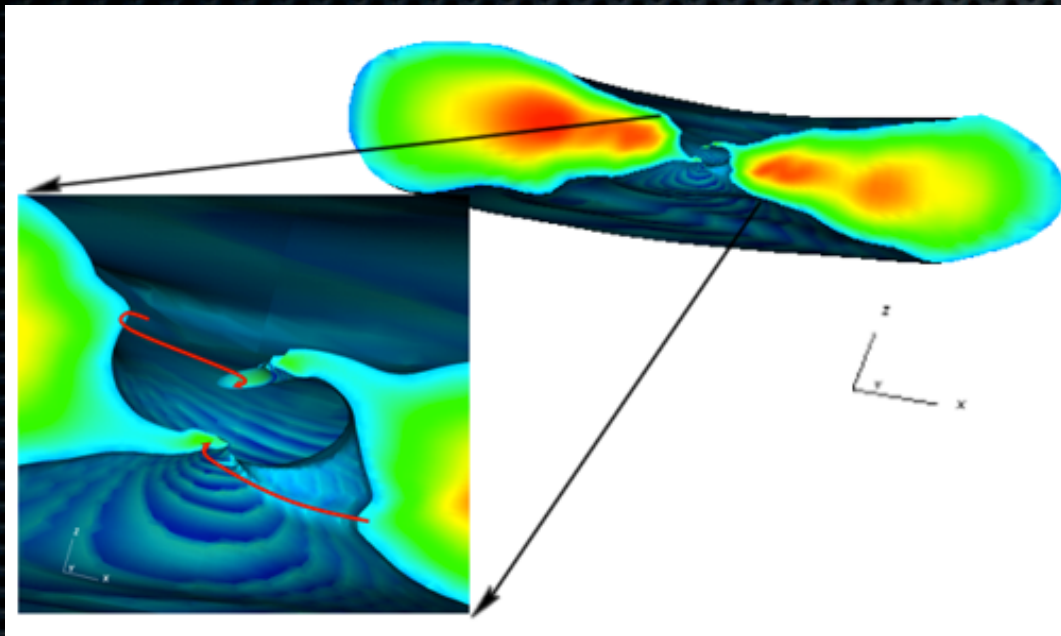
$$\alpha_c < -2$$

$$\alpha_d < -2$$

$$i \sim 0^\circ$$
$$\alpha_i \simeq -2$$

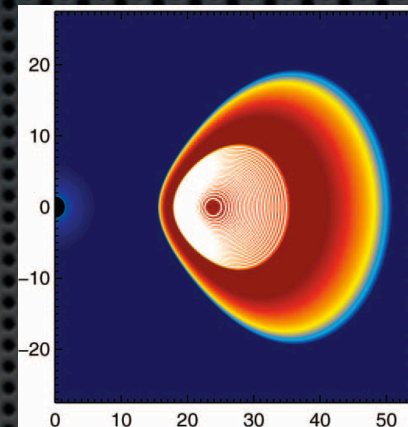
Out-standing Issues in black hole accretion

Warped Disks Fragile et al. 2007-2009



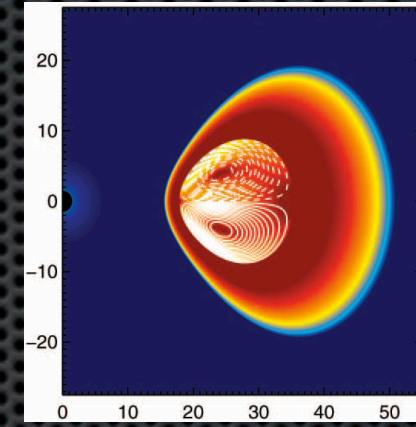
Initial Field Topology

Beckwith et al. 2008



Poloidal

Jet



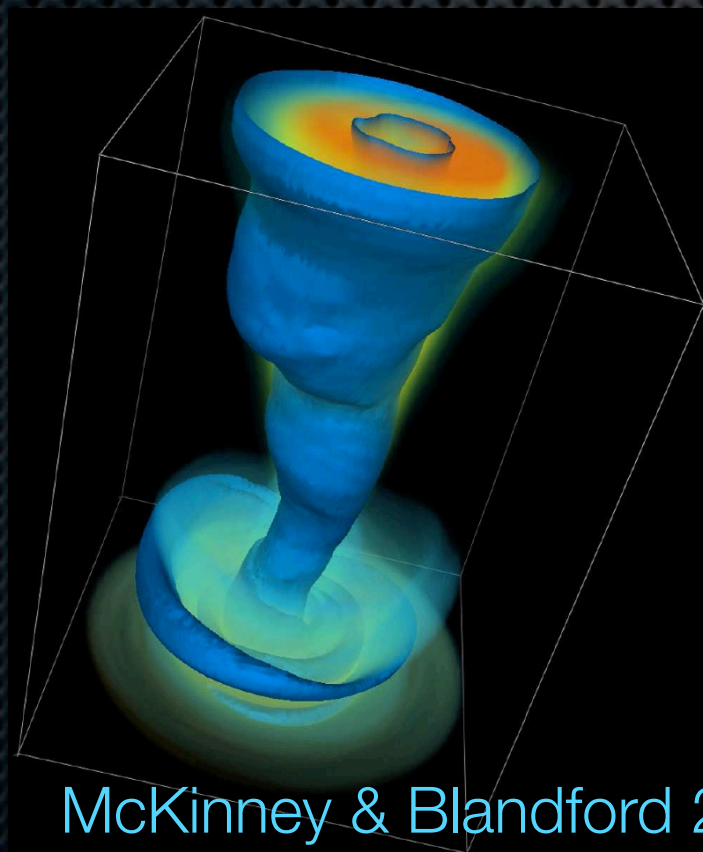
Quadrupolar

Jet



Toroidal

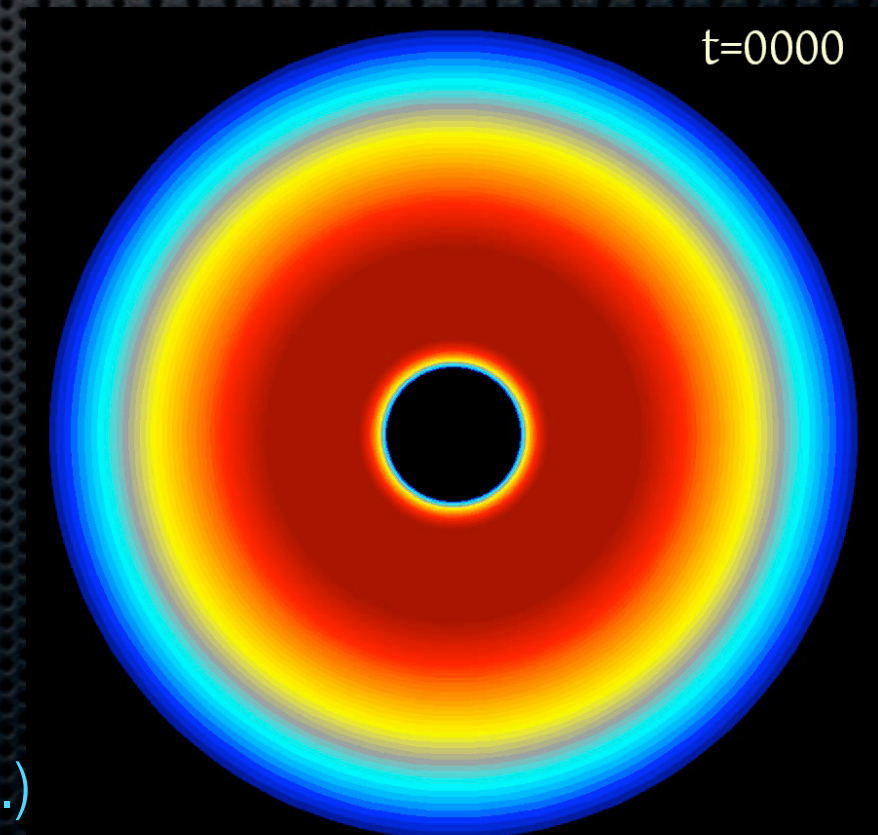
"No" Jet



Full 2π Evolutions
 $m=1$ mode dominance

McKinney & Blandford 2009

Gammie et al (unpub.)



$t=0000$

Summary & Conclusion:

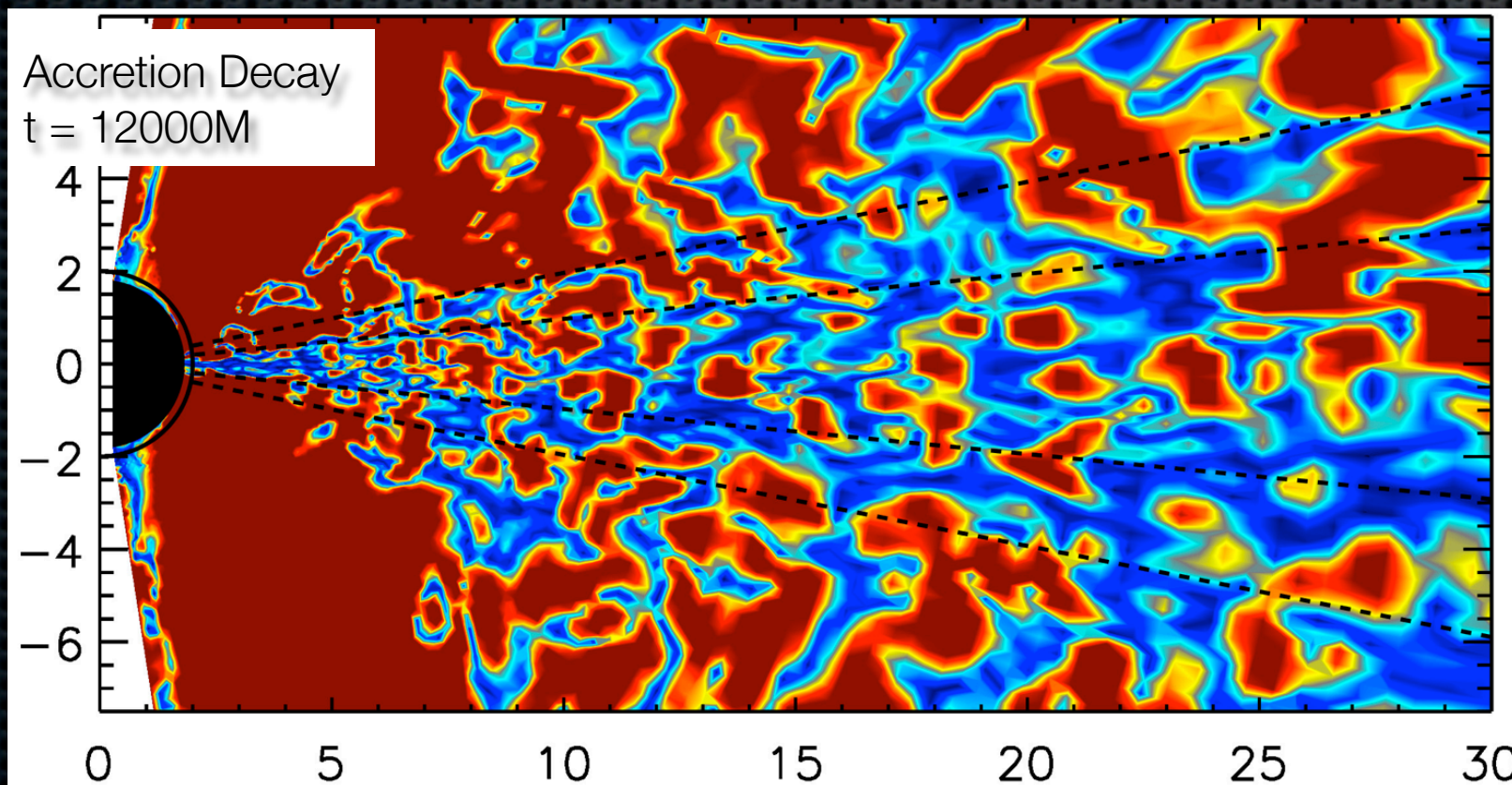
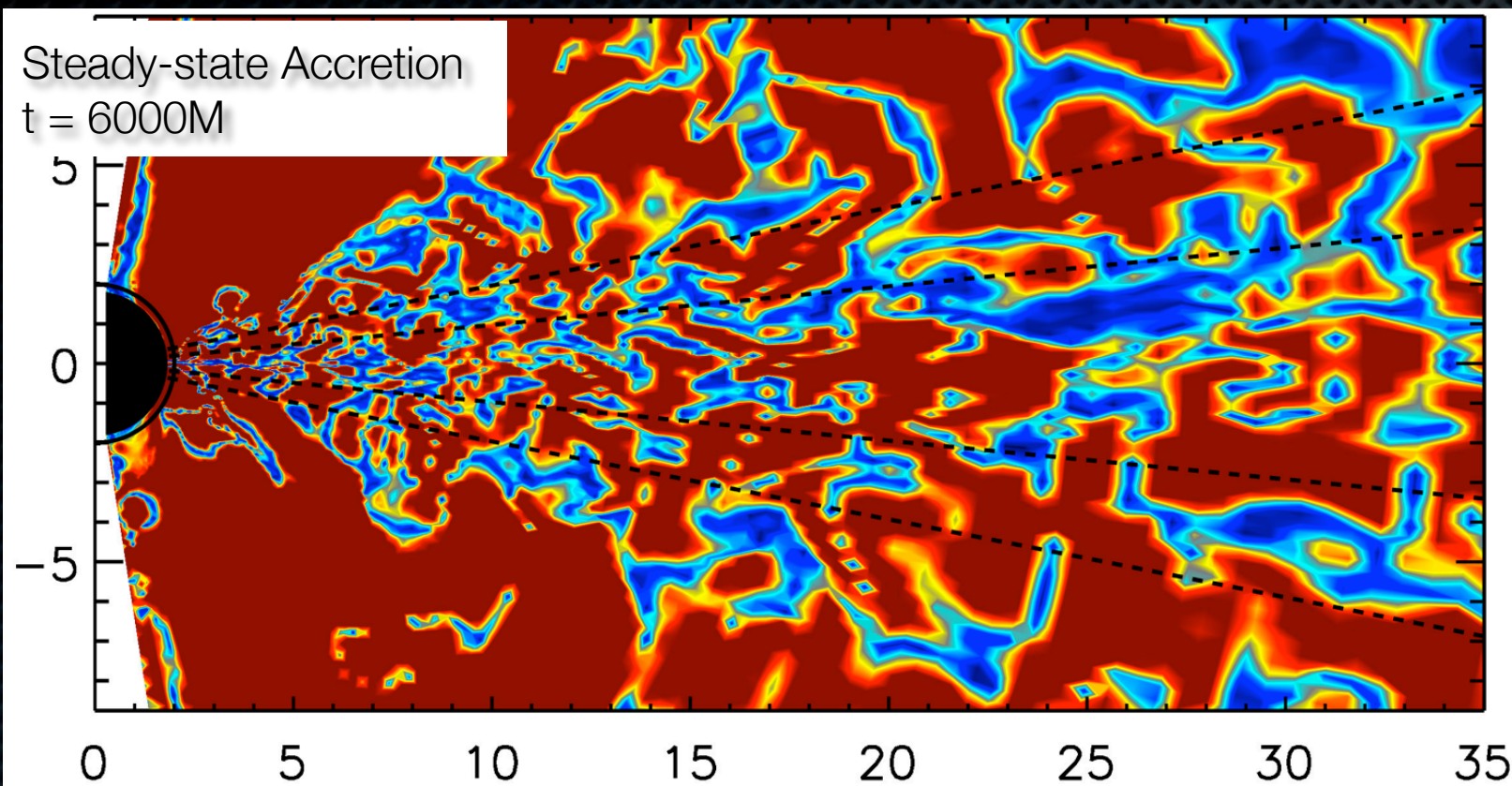
- ✦ Moving towards fully self-consistent accretion models;
- ✦ Building the analytical tools to evaluate disks' statistical steady-state;
- ✦ Magnetic fields can change the “thin disk” picture within the ISCO;
- ✦ MRI turbulence can explain the high frequency X-ray variability in AGN and low/hard state of galactic black holes;
 - ✦ Emissivity is not trivially dependent on accretion rate;

Future Work:

- ✦ Fill in H/R vs. spin parameter space;
- ✦ Further magnetic field topology studies;
- ✦ What are “natural” initial disk conditions?
- ✦ Does variability depend on disk thickness?
- ✦ How does Unary Black Hole accretion physics carry over to Binary Black Holes?

Extra Slides

Track MRI Resolution for all time!



Suggestions from local shearing box simulations:

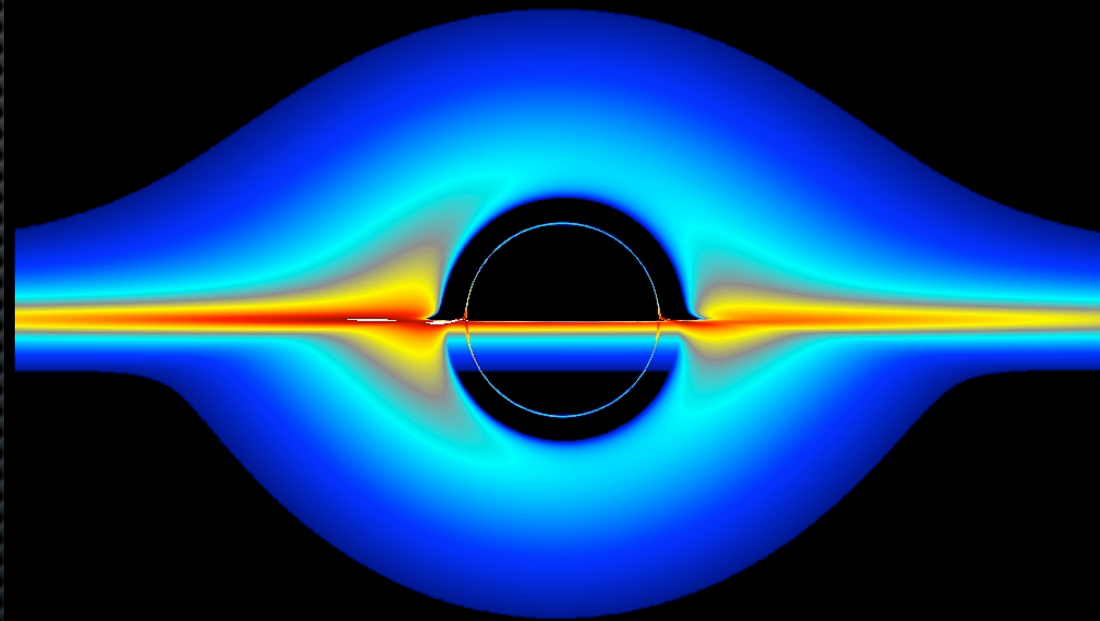
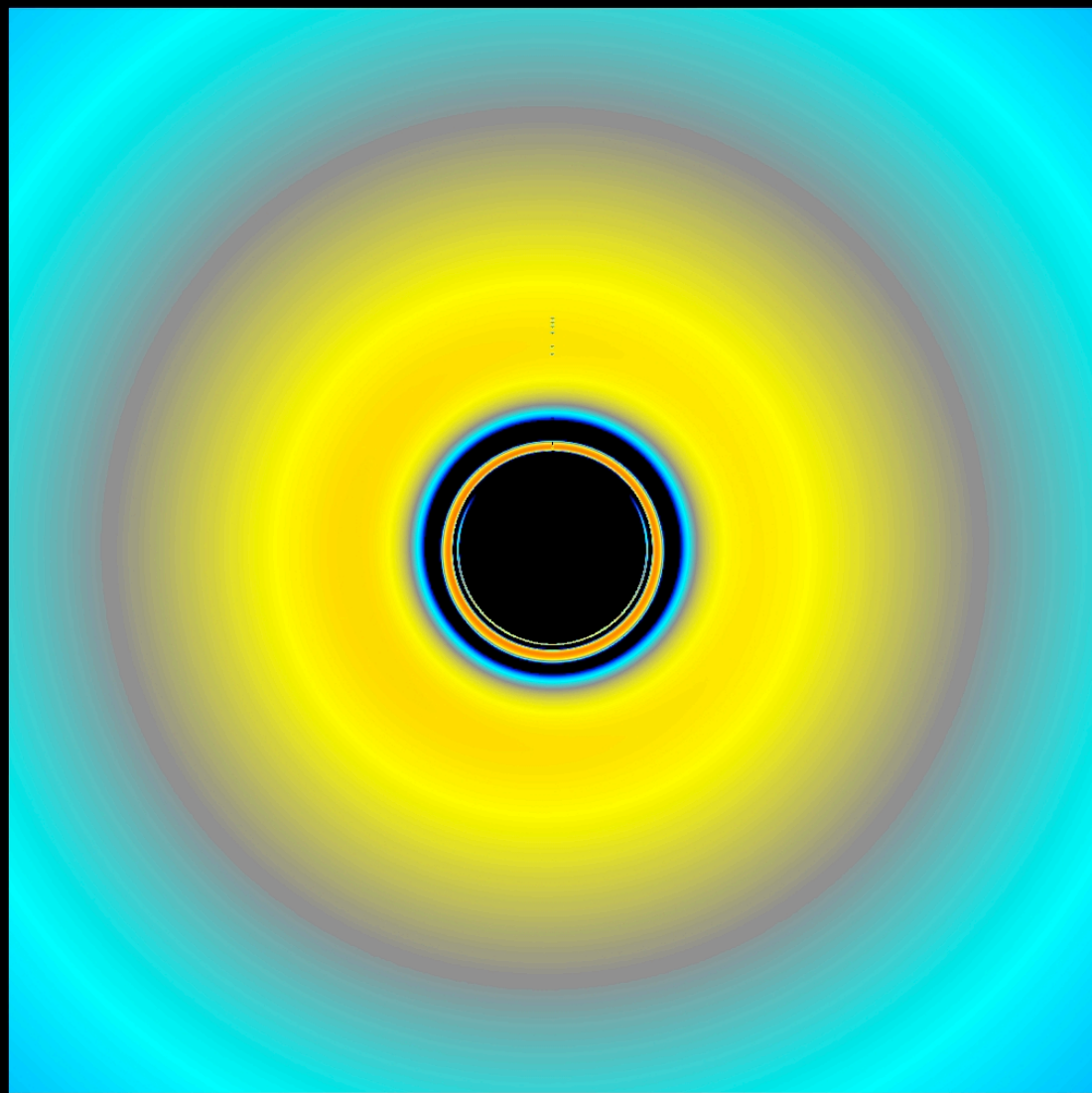
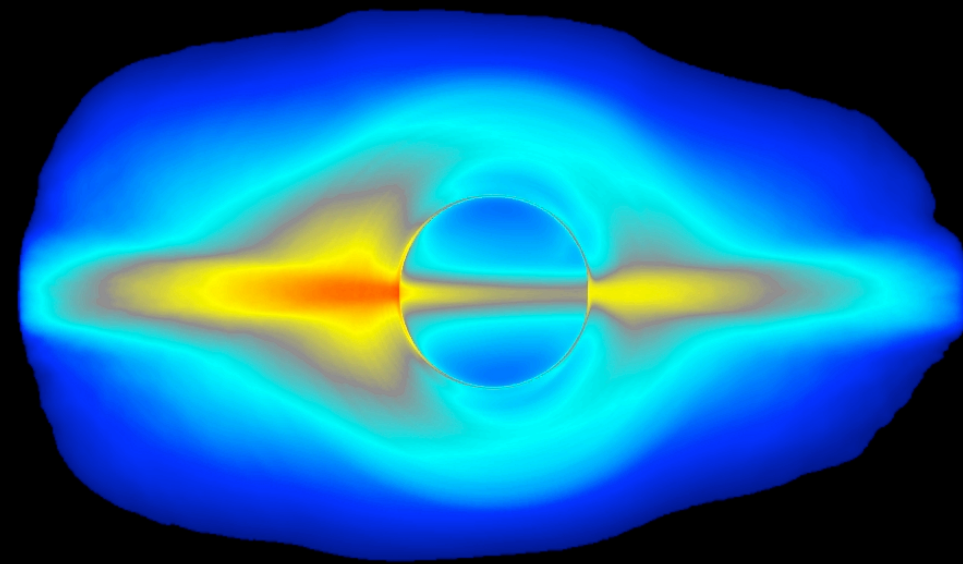
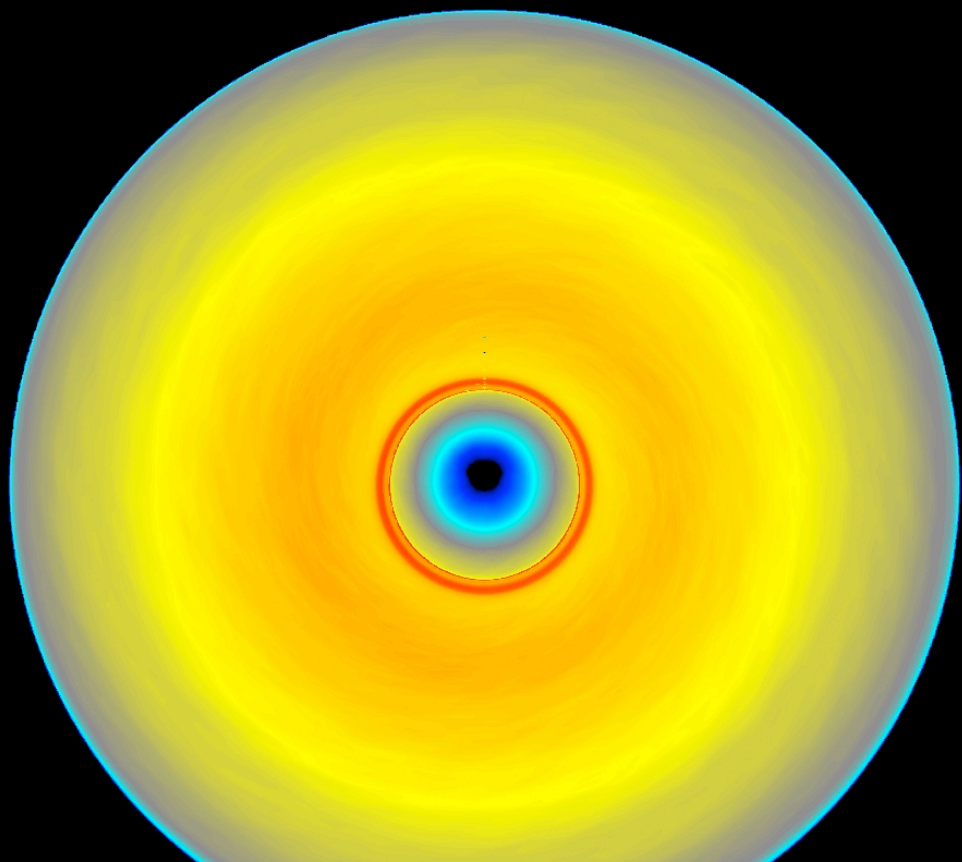
Sano et al. 2004

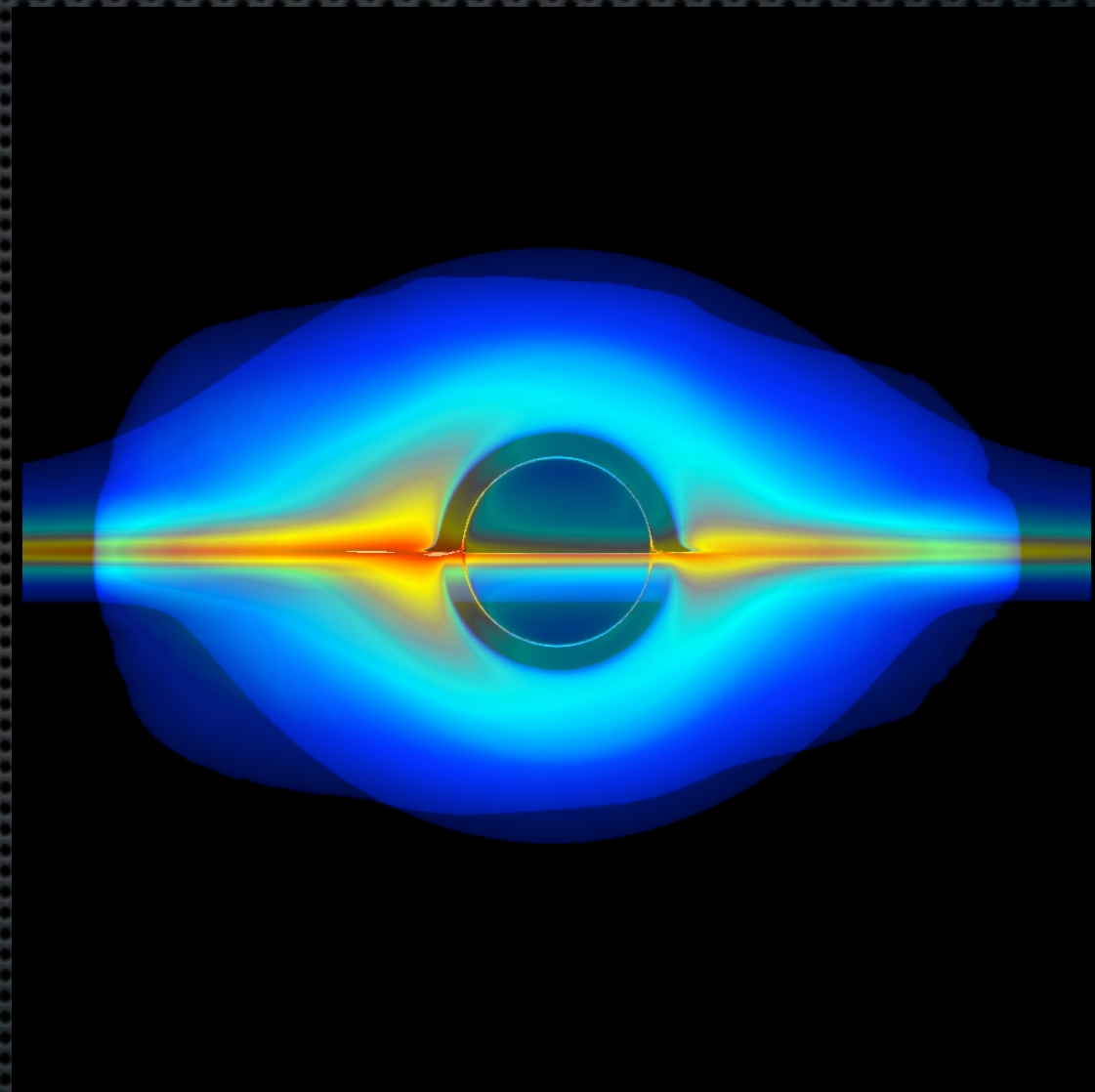
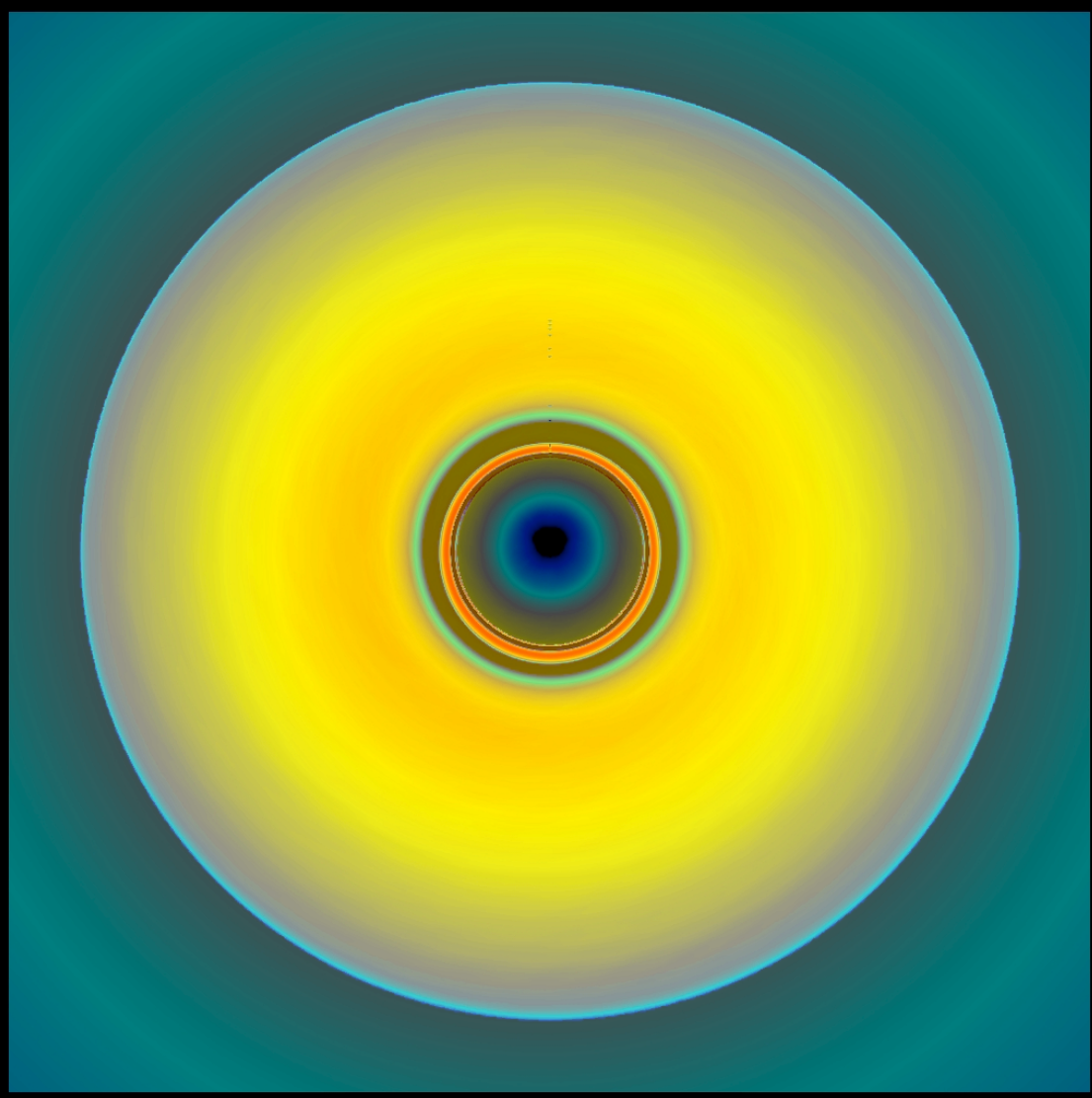
$$\lambda_{\text{MRI}} \equiv \frac{1}{\sqrt{4\pi\rho\Omega(R)}} b_\mu \hat{e}^\mu_{(\theta)}$$

$$\frac{\lambda_{\text{MRI}}}{\Delta z} > 6$$

Davis, Stone, & Pessah 2009

$$\frac{H}{\Delta z} > 60$$







Spin Over-estimation

