# Dynamically Perturbed Neutron Stars

Scott C. Noble

with

Matt W. Choptuik

September 27, 2001

# Motivation:

- Unstable Static (TOV) Solutions $\longrightarrow$ BH
  $\Rightarrow \exists$ Type-I Crit. Soln's;

- Type-II sol'ns observed for perfect fluids;

# Questions:

- $\text{Type} - \text{I} \overset{?}{\longrightarrow} \text{Type} - \text{II}$

- $\text{Type} - \text{I Soln}'\text{s} \overset{?}{=} \text{TOV Unstable Soln}'\text{s}$

- What are End States of Type-I Solutions?
  Dispersal & BH?

# Recent Studies:

Hawley and Choptuik (2000) :

- Boson Star + Massless Klein-Gordon;
- Type-I soln's = Unstable Boson Star soln's;
- Found Typical Lifetime Scaling;

Novak (2001) :

- Initial $v(r)$ induced NS collapse;
- Oscillatory$\rightarrow$Shock/Bounce/BH$\rightarrow$Direct BH;
- Found $Min(M_{BH}) \approx 10^{-2} M_{\odot}$;

Siebel, Font, Papadopoulos (2001) :

- NS + EMKG;
- Used Characteristic Formulation;
- Found end states to be Oscillatory and BH;
- Oscillations are star's fundamental radial mode and its overtones;

# The Equations:

## Minimally-Coupled System:

$$T_{ab} = T_{ab}^{fluid} + T_{ab}^{scalar}$$

## Polar-Areal Coordinates:

$$ds^2 = -\alpha\left(r,t\right)^2 dt^2 + a\left(r,t\right)^2 dr^2 + r^2 d\Omega^2$$

## Massless Klein-Gordon Field:

$$T_{ab}^{scalar} = \nabla_a\phi\nabla_b\phi - \tfrac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi$$

$$\nabla^a\nabla_a\phi = 0$$

$$\Xi = \partial_r\phi \quad , \quad \Upsilon = \tfrac{a}{\alpha}\partial_t\phi$$

$$\partial_t\Xi = \partial_r\left(X\Upsilon\right)$$

$$\partial_t\Upsilon = \tfrac{1}{r^2}\partial_r\left(r^2 X\Xi\right)$$

## Synge Gas Equations:

$$T_{ab}^{fluid} \;=\; (\rho + P)\, u_a u_b + P g_{ab}$$

$$P \;=\; (\Gamma - 1)\rho_\circ \epsilon$$

$$\partial_t \mathbf{q} + \tfrac{1}{r^2}\partial_r \left( r^2 X \mathbf{f} \right) \;=\; \boldsymbol{\psi}$$

$$\mathbf{q} = \begin{bmatrix} D \\ S \\ \tau \end{bmatrix} \;,\; \mathbf{f} = \begin{bmatrix} Dv \\ Sv + P \\ v\,(\tau + P) \end{bmatrix}$$

## Geometry Equations:

$$\tfrac{a'}{a} \;=\; a^2 \left[ 4\pi r\,(\tau + D) - \tfrac{M}{r^2} \right] + 2\pi r \left( \Xi^2 + \Upsilon^2 \right)$$

$$\tfrac{\alpha'}{\alpha} = a^2 \left[ 4\pi r\,(Sv + P) + \tfrac{M}{r^2} \right] + 2\pi r \left( \Xi^2 + \Upsilon^2 \right)$$

## Tolman-Oppenheimer-Volkoff (TOV) Soln's:

- Solutions to Einstein's Equation for static, spherical fluid w/ EOS $P = P(\rho)$;

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{(\rho+P)\left(m+4\pi r^3 P\right)}{r(r-2m)}$$

$$\frac{d\varphi}{dr} = -\frac{1}{\rho+P}\frac{dP}{dr}$$

$$a(r) = \sqrt{\frac{r}{r-2\,m(r)}}$$

$$\alpha(r) = e^{\varphi(r)}$$

EOS used: $P = K\rho_\circ^\Gamma$