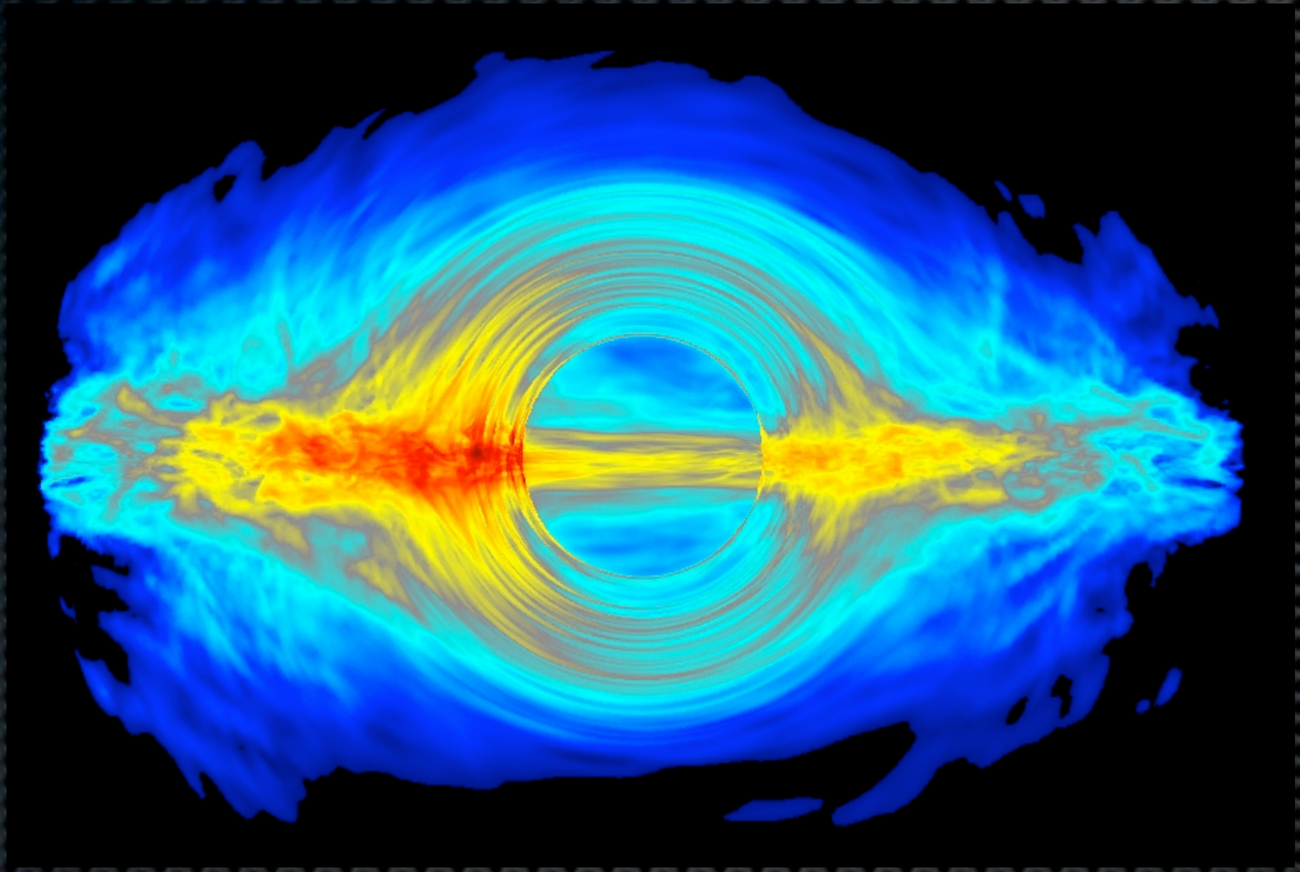
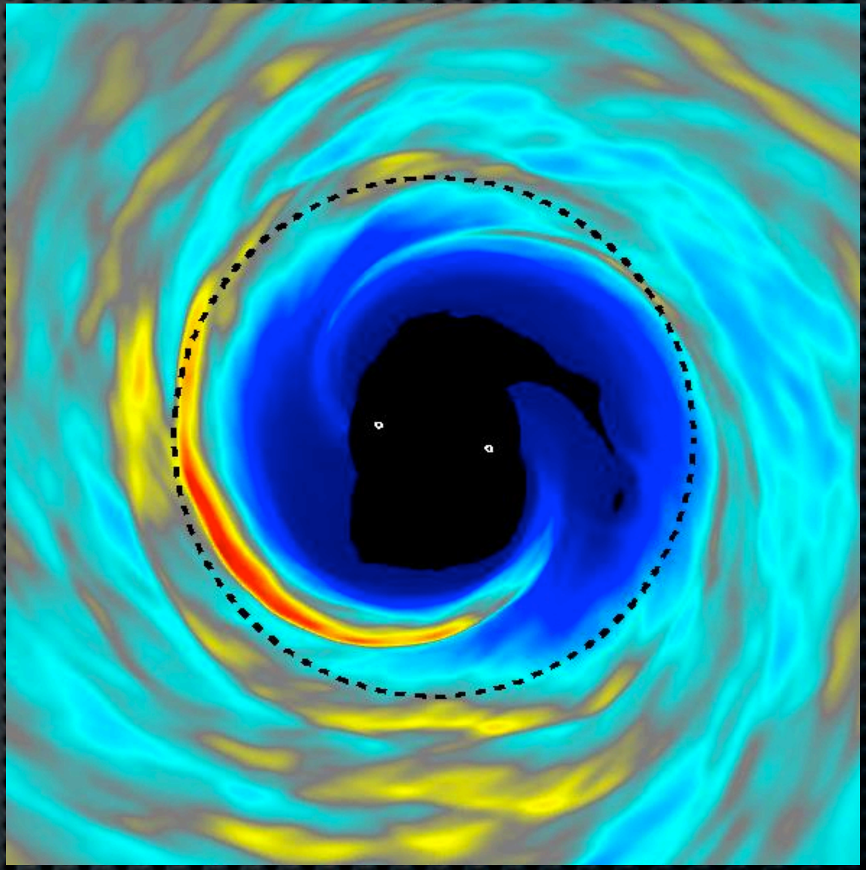
Frontiers in Computational Relativistic Magnetohydrodynamics Applied to Astrophysical Systems:
Predicting Light Signatures of Black Holes





Scott Noble

M. Campanelli, J. Krolik, B. Mundim, H. Nakano,
B. Vazquez, J. Schnittman, N. Yunes, Y. Zlochower

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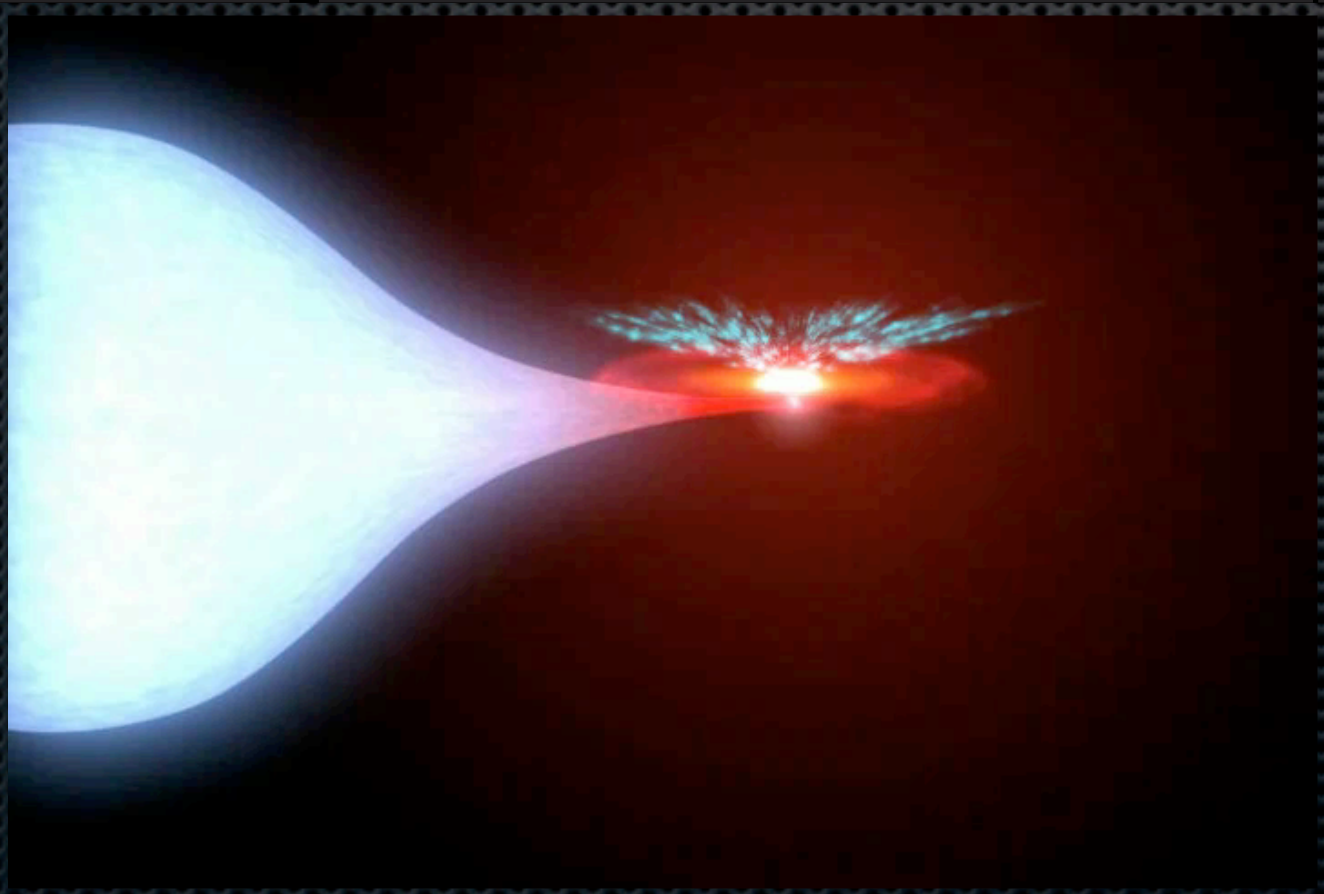
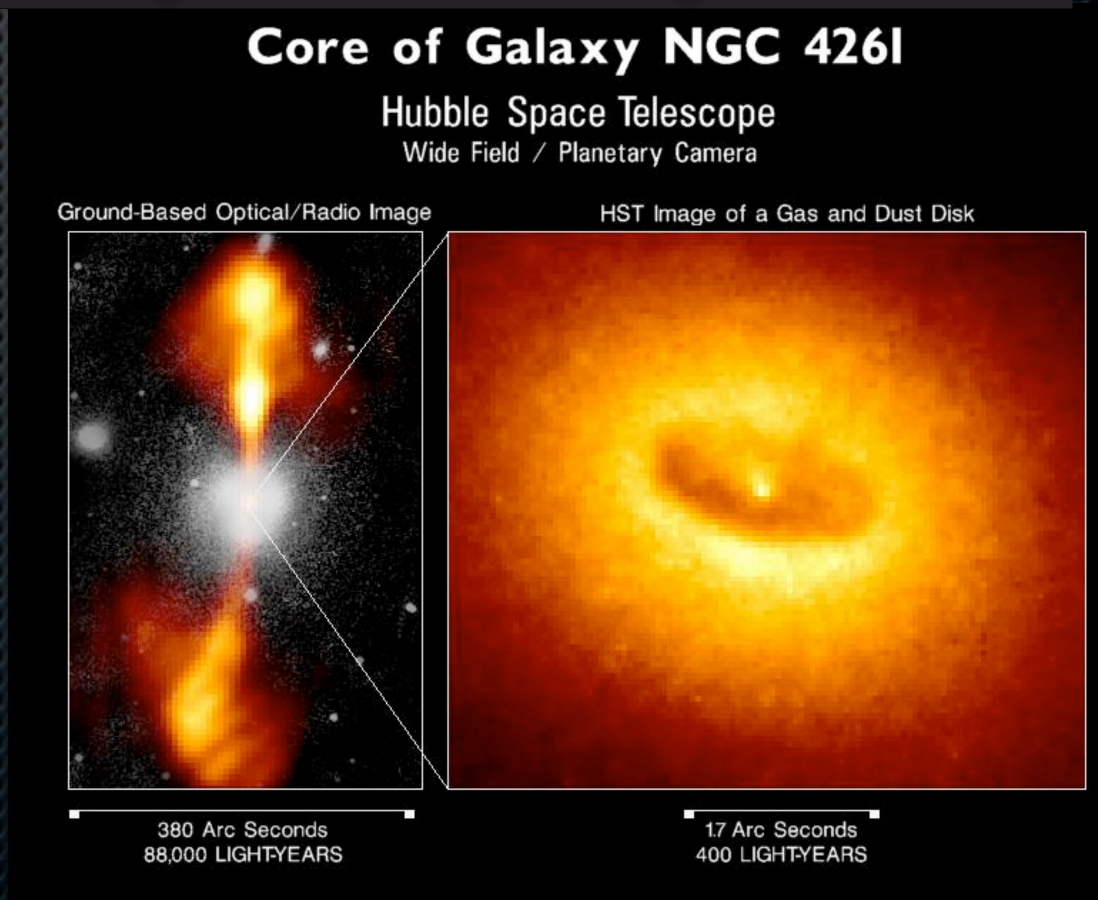
Johns Hopkins
Montana State U.
NASA Goddard

CCP 2012 * Kobe, Japan * October 18, 2012

The Exciting World of Black Hole Accretion!

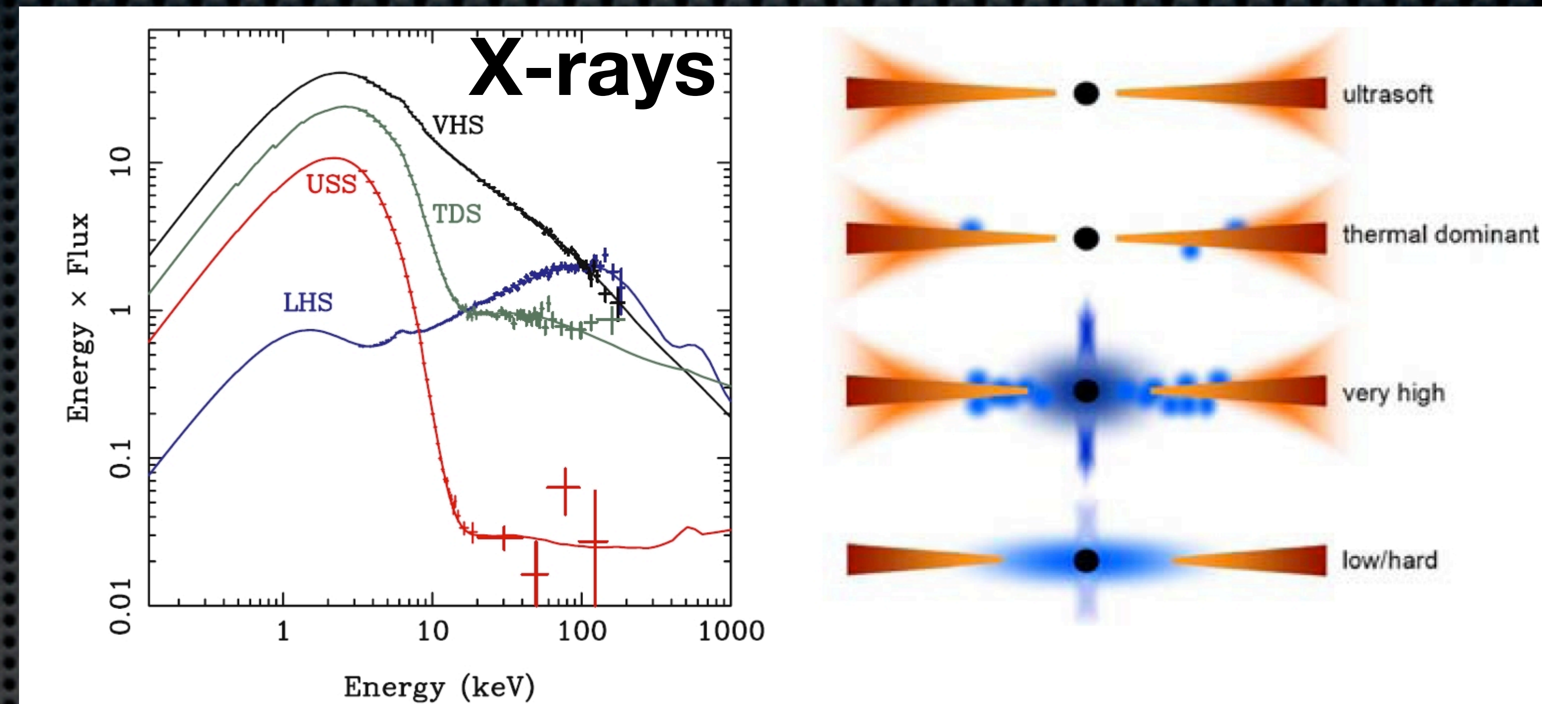
Name	Small Black Holes	Big Black Holes
Aliases	Stellar Mass Black Holes Galactic Black Holes	Supermassive Black Holes Active Galactic Nuclei (AGN)
Masses	~3 - 100 M_{sun}	10^5 - 10^{10} M_{sun}
Locale	Our Galaxy (those we can seen)	Centers of Galaxies
Typical Method of Observation	Radio, X-rays	Radio, X-rays
Jets?	Yes	Yes
Greater Relevance	Stelllar Mass Distribution, Grav. Wave source populations	Star formation rates, interstellar/ intergalactice medium, galactic evol.

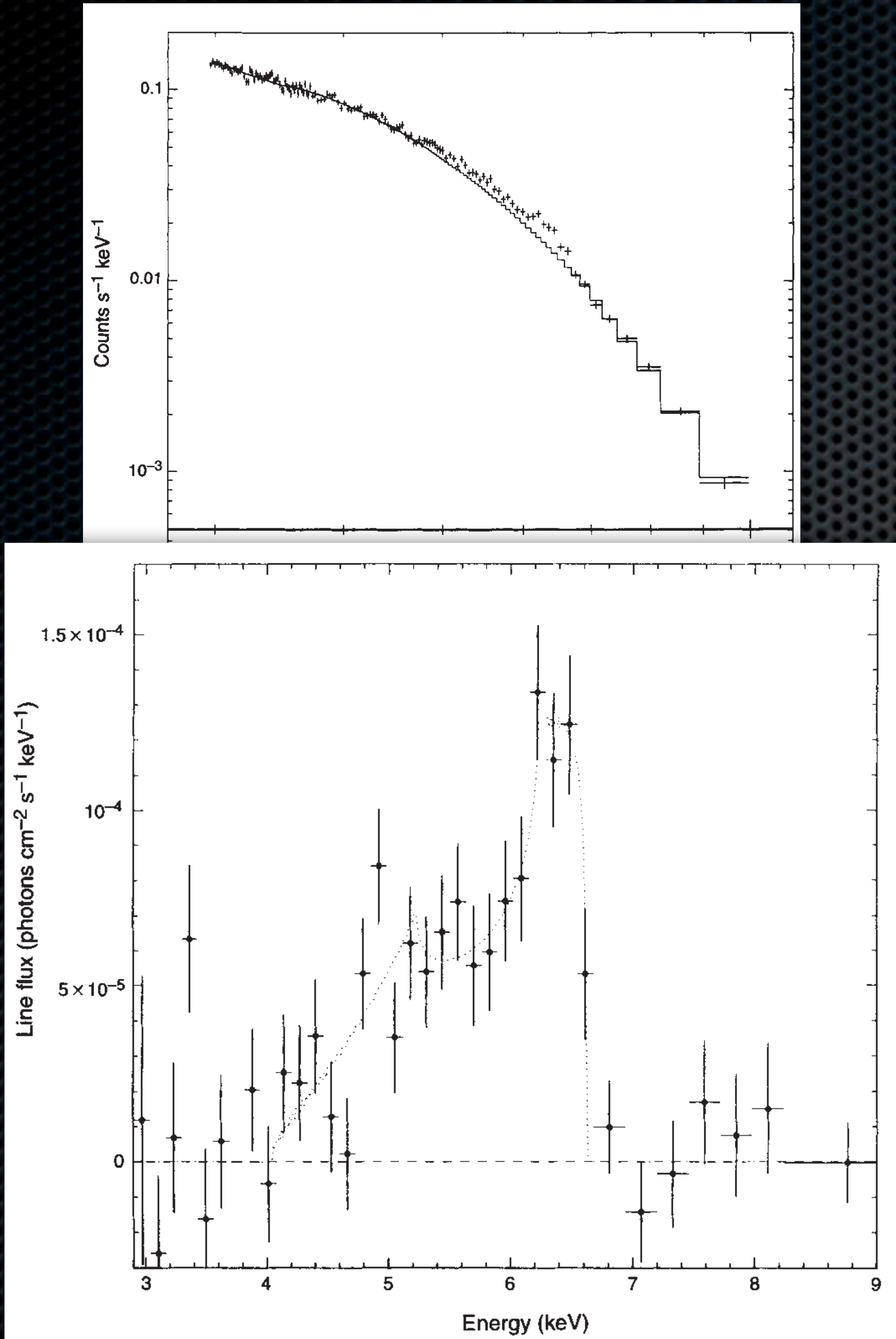




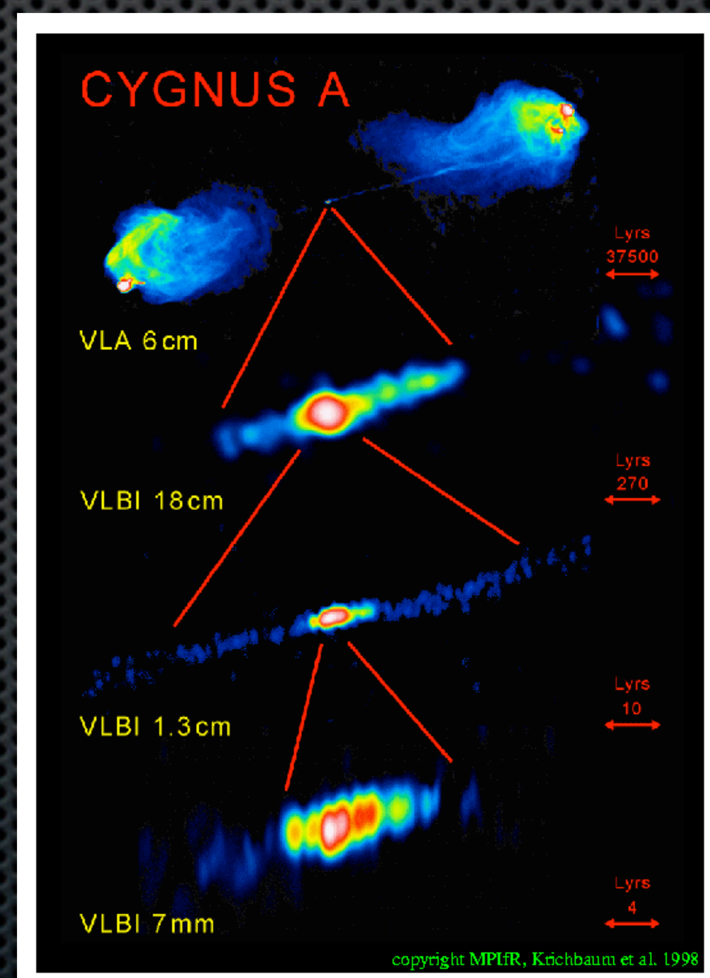
Closeup Views of Black Holes

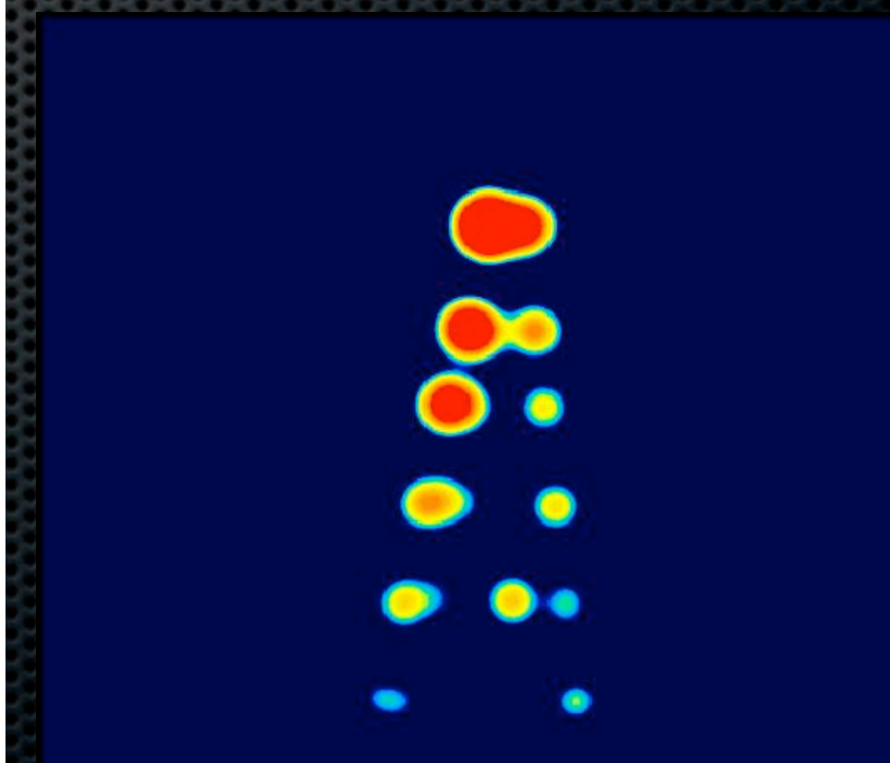
Done et al 2007





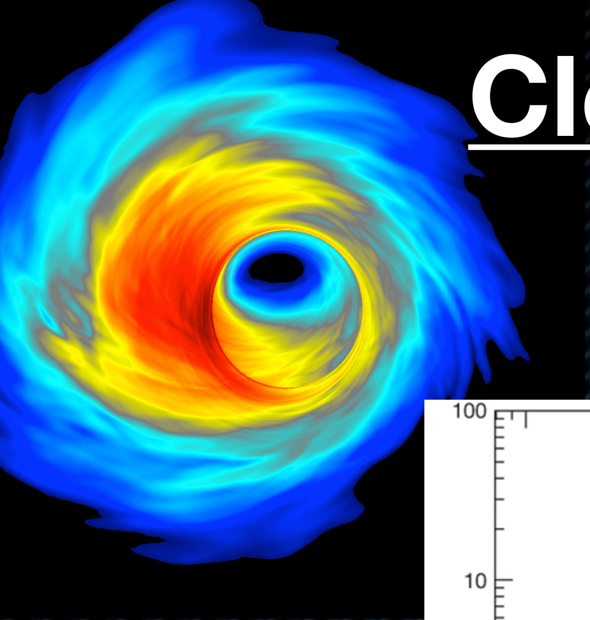
Tanaka++1995
ASCA Observation



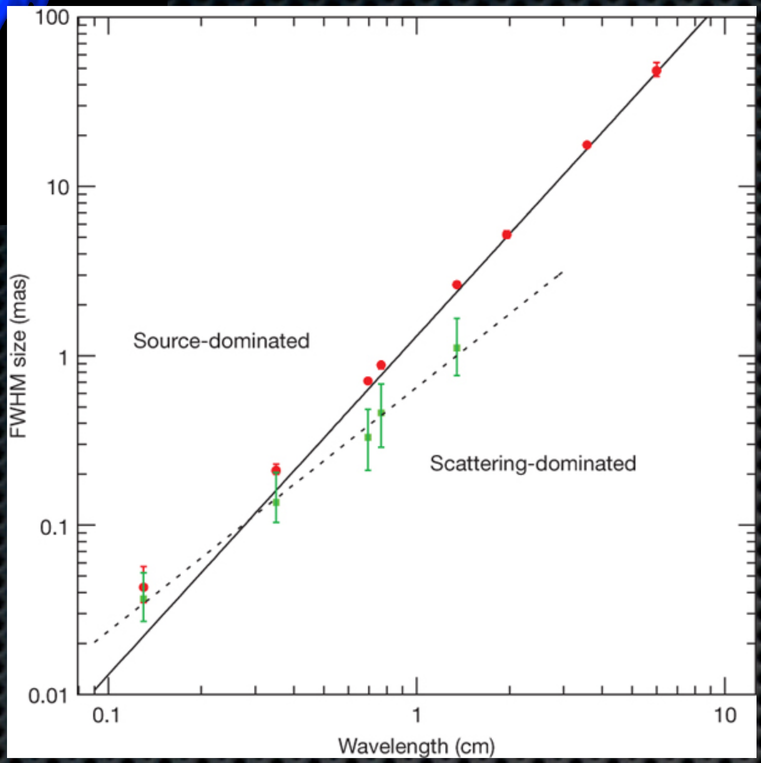


GRS 1915+105
Mirabel & Rodriguez 1994 / VLA

Closeup Views of Black Holes



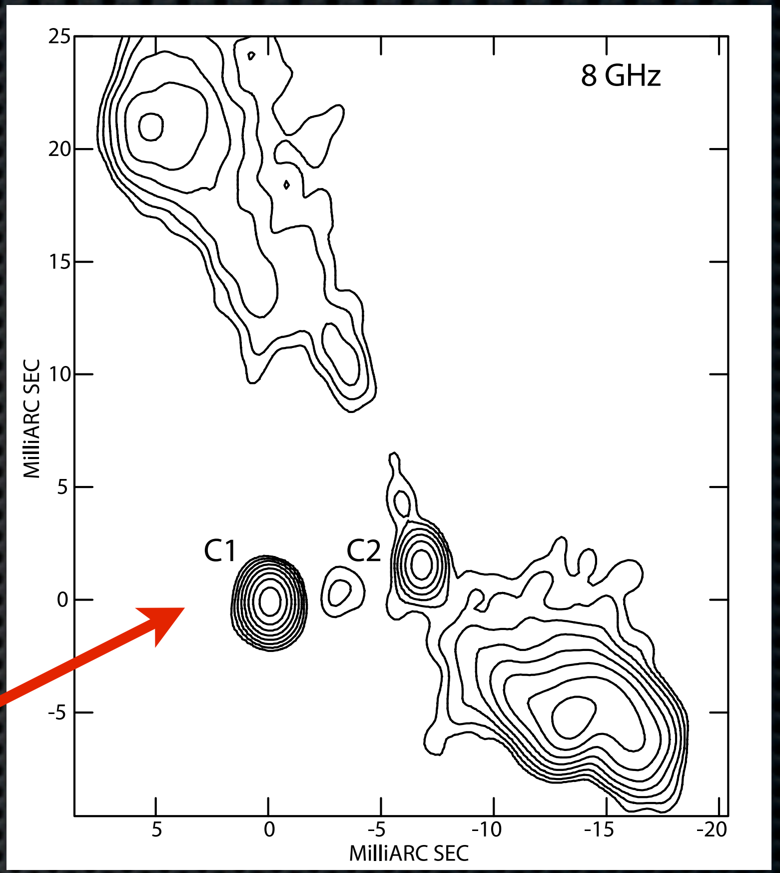
Sgr A*



0402+379
d = 5 pc

Xu et al. 1994
Maness et al. 2004
Rodriguez et al. 2006

Weakly Emitting Gravitational Waves



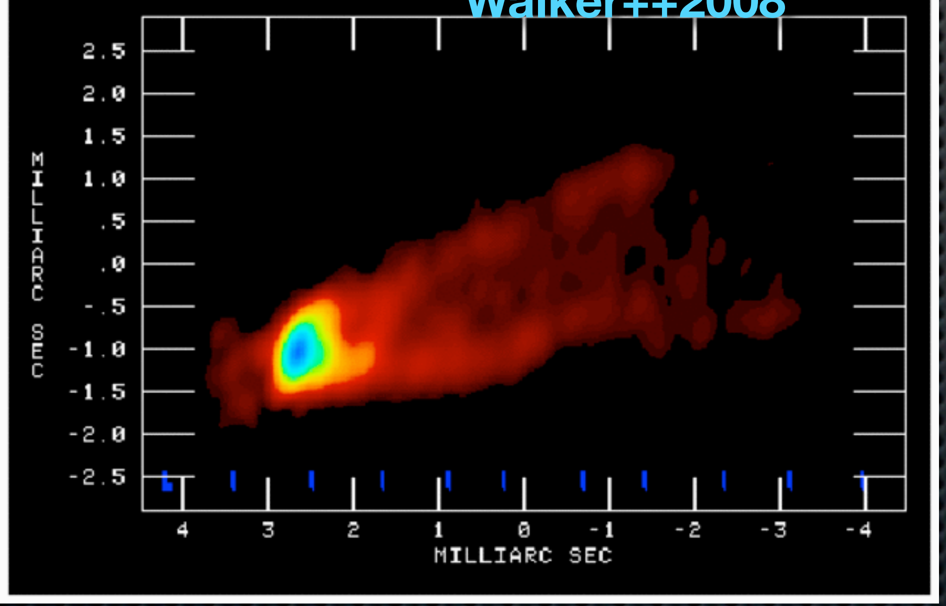
SDSS J153636.22+044127.0
d = 0.1pc

Lauer & Boroson 2009

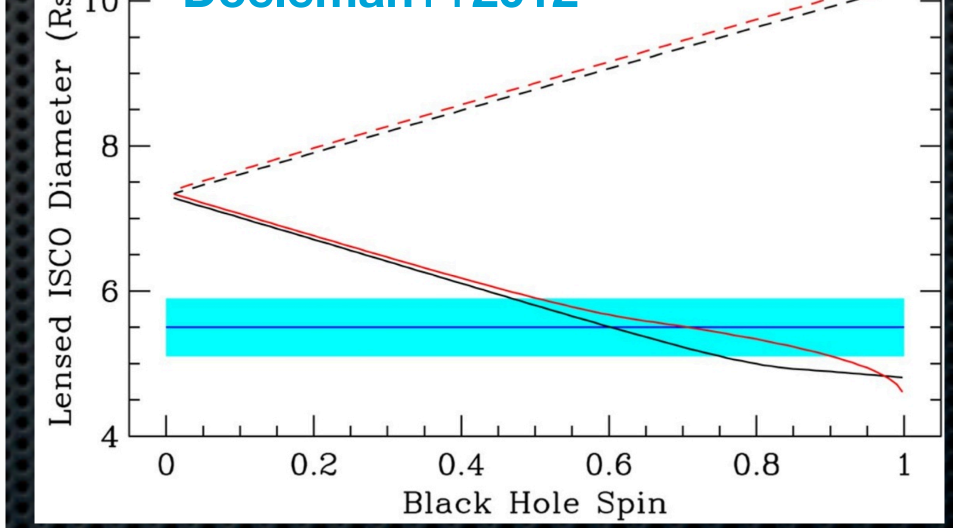
Radio and sub-mm

M87

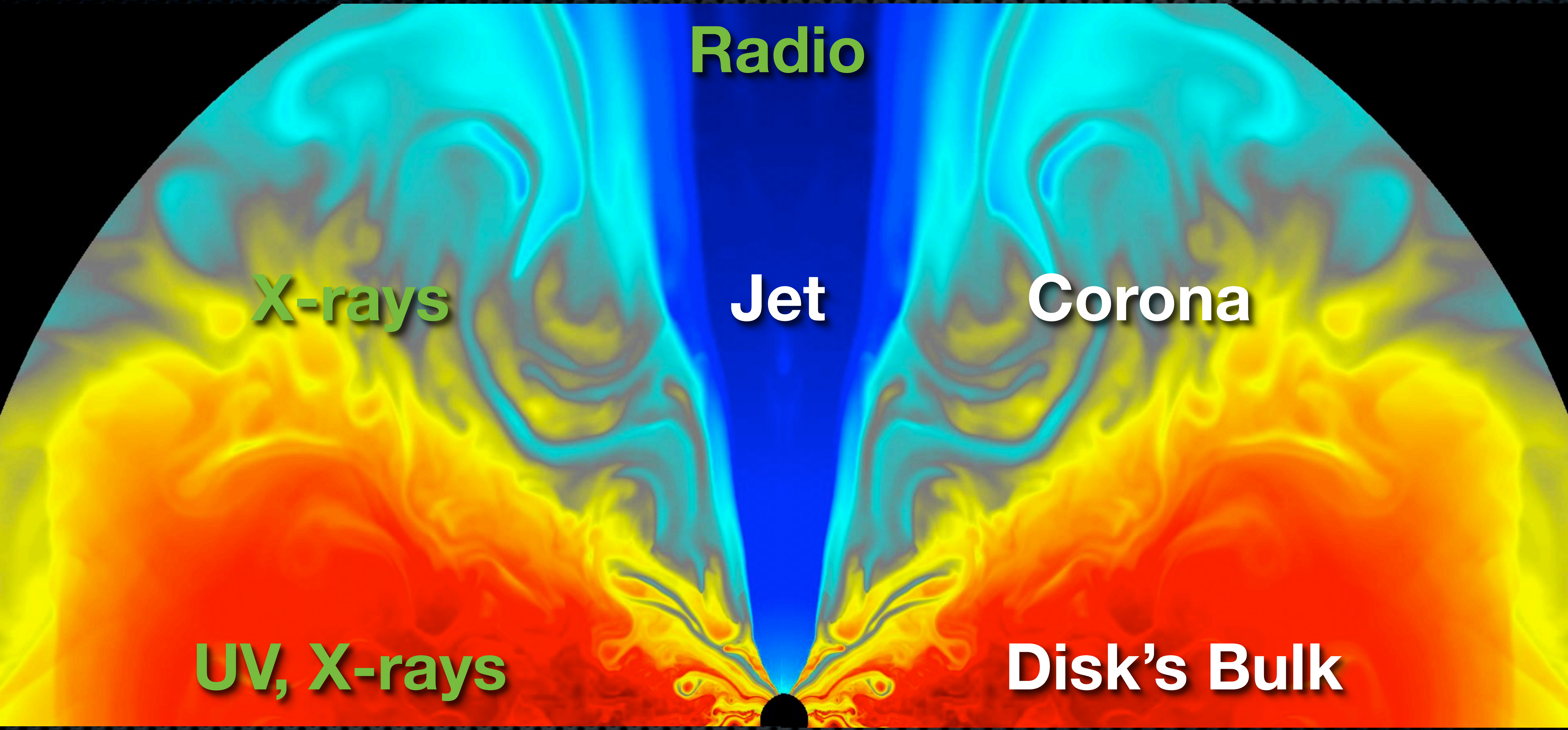
Walker++2008



Doeleman++2012

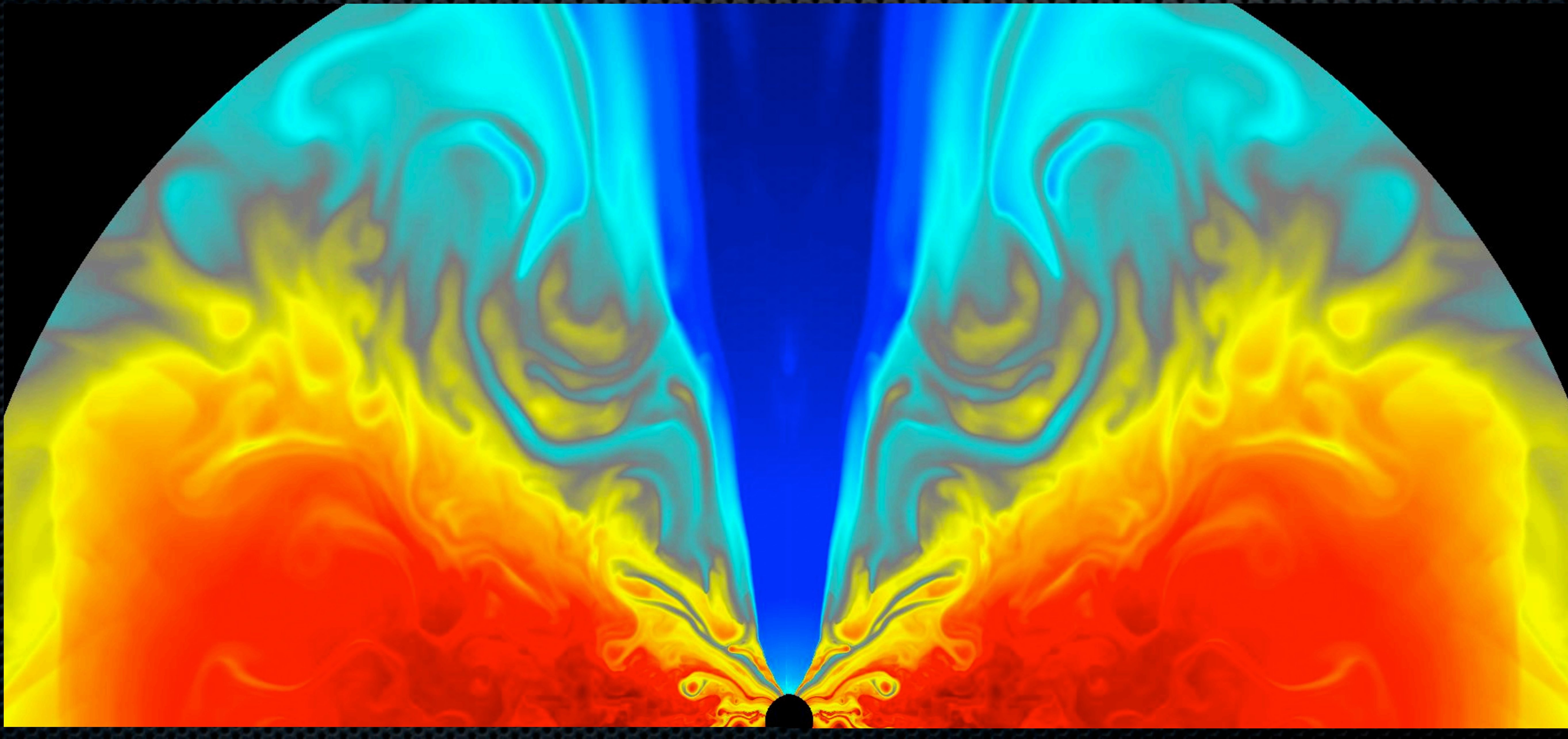


Black Hole Accretion Anatomy



Black Hole Accretion Anatomy

- Ideal MHD = Magnetohydrodynamics
- Radiative Transfer, Ray-tracing
- Multi-species thermodynamics
- GR = General Relativity



General Relativistic Magnetohydrodynamics

$$\frac{\partial}{\partial t}\mathbf{U}\left(\mathbf{P}\right) + \frac{\partial}{\partial x^i}\mathbf{F}^i\left(\mathbf{P}\right) = \mathbf{S}\left(\mathbf{P}\right)$$

$$\frac{\partial}{\partial t}\sqrt{-g}\begin{bmatrix} \rho u^t \\ T^t{}_t + \rho u^t \\ T^t{}_j \\ B^k \end{bmatrix} + \frac{\partial}{\partial x^i}\sqrt{-g}\begin{bmatrix} \rho u^i \\ T^i{}_t + \rho u^i \\ T^i{}_j \\ \left(b^i u^k - b^k u^i\right) \end{bmatrix} = \sqrt{-g}\begin{bmatrix} 0 \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{t\kappa} - \mathcal{F}_t \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{j\kappa} - \mathcal{F}_j \\ 0 \end{bmatrix}$$

Radiative Energy & Momentum Loss

$$T_{\mu\nu} = \left(\rho + u + p + 2p_m\right) u_\mu u_\nu + \left(p + p_m\right) g_{\mu\nu} - b_\mu b_\nu$$

Mass Density

Internal Energy Density

Gas Pressure

Fluid's 4-velocity

Magnetic Pressure

Magnetic 4-vector

Spacetime Metric: $$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

General Relativistic Magnetohydrodynamics

$$\frac{\partial}{\partial t}\sqrt{-g}\left[\begin{array}{c} \rho u^t \\ T^t{}_t + \rho u^t \\ T^t{}_j \\ B^k \end{array}\right] + \frac{\partial}{\partial x^i}\sqrt{-g}\left[\begin{array}{c} \rho u^i \\ T^i{}_t + \rho u^i \\ T^i{}_j \\ \left(b^i u^k - b^k u^i\right) \end{array}\right] = \sqrt{-g}\left[\begin{array}{c} 0 \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{t\kappa} - \mathcal{F}_t \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{j\kappa} - \mathcal{F}_j \\ 0 \end{array}\right]$$

Spacetime Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

E.g., Schwarzschild Metric:

$$ds^2 = -\left(1 - 2M/r\right) dt^2 + \left(1 - 2M/r\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Christoffel Symbols:

$$\Gamma^\mu{}_{\nu\kappa} = \frac{1}{2} g^{\mu\sigma}\left(\frac{\partial}{\partial x^\nu} g_{\kappa\sigma} + \frac{\partial}{\partial x^\kappa} g_{\nu\sigma} - \frac{\partial}{\partial x^\sigma} g_{\nu\kappa}\right)$$

$$\begin{aligned} x^{0'} &= t &&= x^0 \\ x^{1'} &= r &&= M e^{x^1} \\ x^{2'} &= \theta &&= \theta(x^2) = \frac{\pi}{2}\left[1 + (1-\xi)\left(2x^2 - 1\right) + \left(\xi - \frac{2\theta_c}{\pi}\right)\left(2x^2 - 1\right)^n\right] \\ x^{3'} &= \phi &&= x^3 \end{aligned}$$

$$g_{\mu\nu} = \frac{\partial x^{\mu'}}{\partial x^\mu}\frac{\partial x^{\nu'}}{\partial x^\nu} g_{\mu'\nu'}$$

GRMHD Numerical Method (Harm3D)

Geometry and Coordinates:

- Harm3d written largely independent of chosen coordinate system (covariance)
  - GRMHD code Noble++2009
  - Now able to handle “arbitrary” spacetimes, though one must be specified;
  - Equations solved on a uniform discretized domain in system of coordinates tailored to the problem;
  - Efficiency through simple uniform domain decomposition:
    - Adaptivity pushed to the warped system of coordinates;
- Prefer to use coordinates similar to spherical coordinates to accurately evolve disks with significant azimuthal component;
  - Minimizes dissipation;
  - Allows us to better track transport of angular momentum --> essential for understanding disks;
- Grid must resolve dominant modes of the magnetorotational instability, responsible for ang. mom. transport;

Sano++2004 Noble++2010 Hawley++2011

Finite Volume Method:

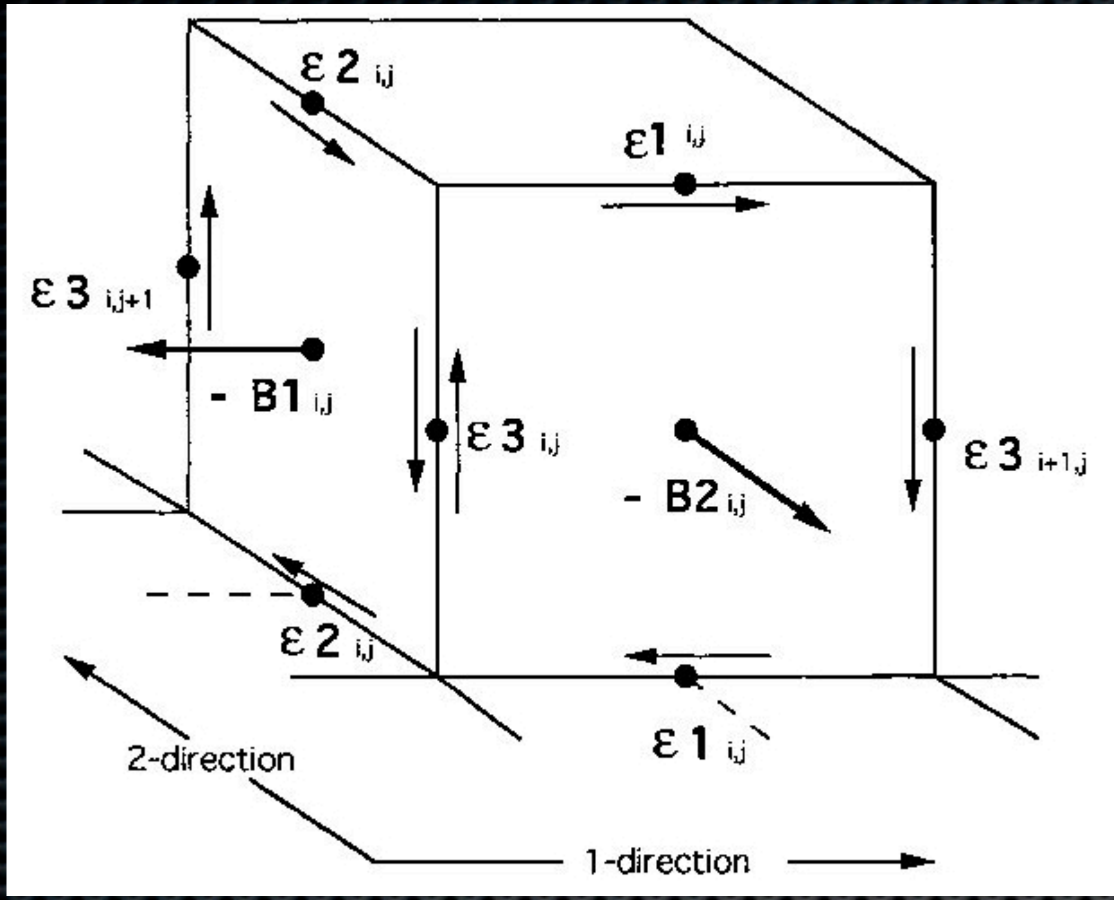
- High-resolution Shock-capturing techniques;
- Reconstruction of Primitive var’s (density, pressure, velocities) at cell interfaces:
  - Piecwise parabolic (PPM)
- Approximate Riemann solver:
  - Lax-Friedrichs (HLL available)
- Conserved variables are advanced in time using Method of Lines with 2nd-order Runge-Kutta;
- Primitives are recovered from Conserved var’s using “2D” and “1DW” algorithms from Noble++2006

Solenoidal Constraint Enforcement:

- $\partial_i B^i \neq 0$ leads to :
  - Non-perpendicular Lorentz forces to B^i
  - Inconsistency with MHD;
  - Sometimes instabilities and artifacts;
- 3d, modified version of Flux-CT of Toth 2000

$$\mathcal{E}^z = v^x B^y - v^y B^x = f^x$$

GRMHD Numerical Method (Harm3D)



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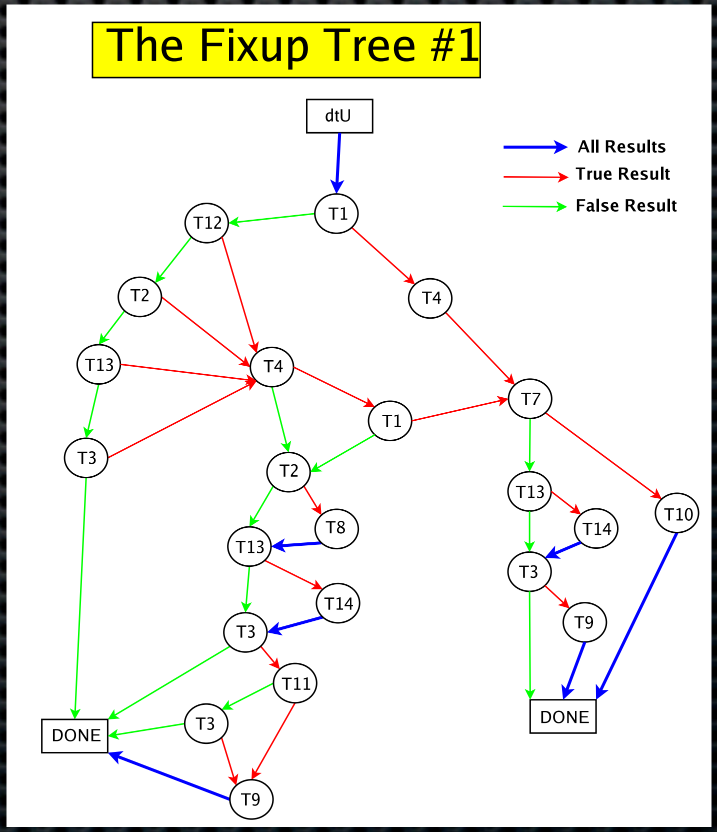
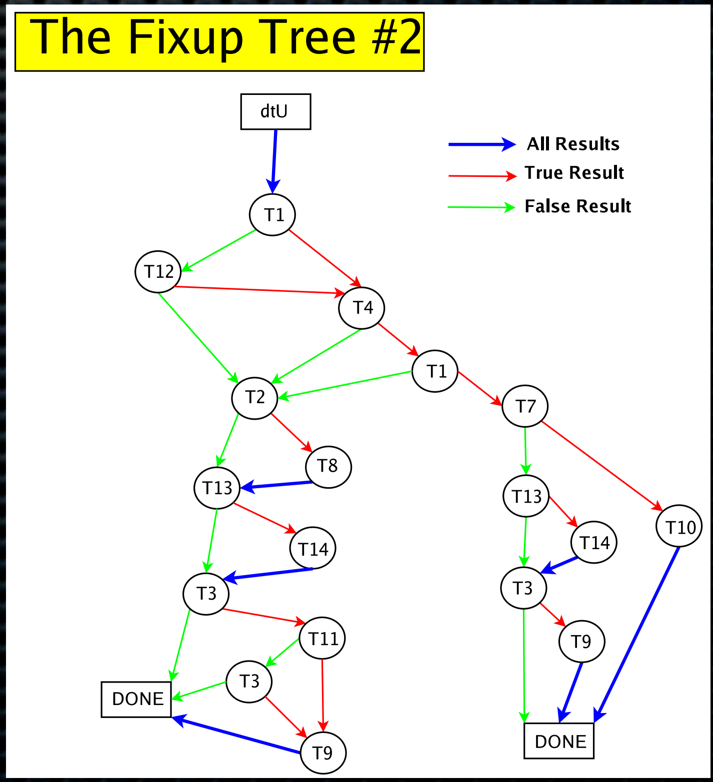
$$\mathcal{E}^z = v^x B^y - v^y B^x = f^x$$

Failure Recovery:

- "Con2Prim" or Primitive var. calculation can often lead to an unphysical state;
- Large scale simulations demand robust schemes that handle instabilities on the fly;
- Use a variety of methods:
  - Alternate Con2Prim routines;
  - Simpler state equations;
  - Interpolation...





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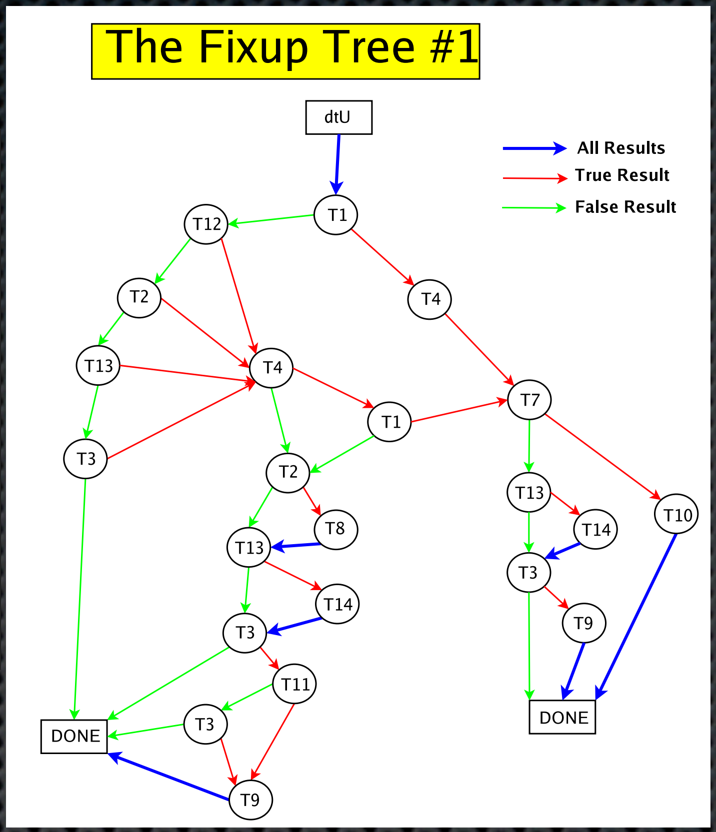
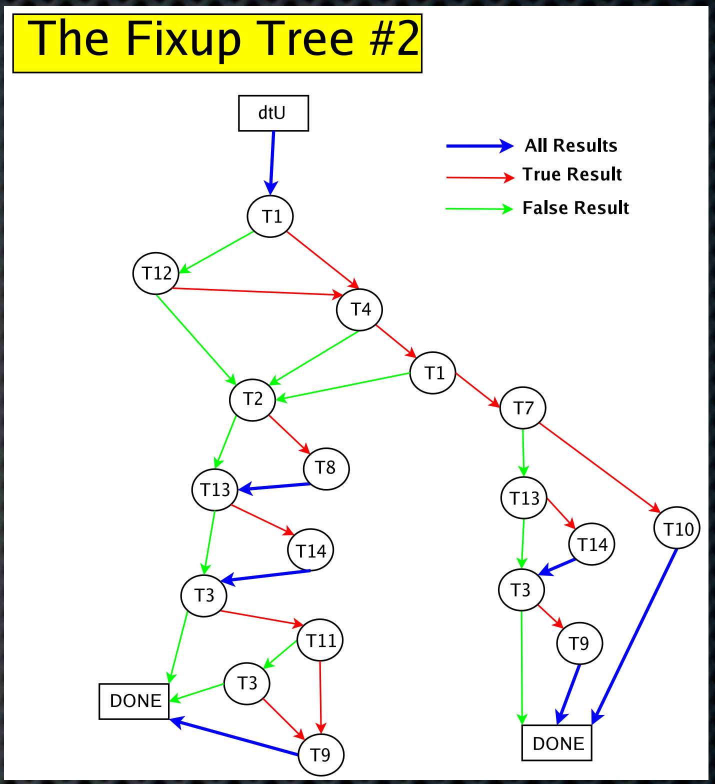




Name	Description	True Condition
T1	$P = P_i(U)$	All $P_i(U)$ return true or $\gamma > \gamma_{\max 2}$
T2	$\mathtt{check\_floor}(P)$	$\rho < \rho_f$ or $u < u_f$
T3	$\mathtt{check\_gamma}(P)$	$\gamma > \gamma_{\max 1}$
T4	$P_{ee} = P_{ee}(U)$	If P_{ee} returns true
T7	$\mathtt{interp}_P$	If there are too few good nearest neighbors
T8	fixup_floor	Never
T9	fixup_gamma	Never
T10	$P = P_{\rm old}$	Never
T11	$\mathtt{interp}_v$	If there are too few good nearest neighbors
T12	$\mathtt{check\_entropy\_eq}(P)$	If $u < \beta_{\min} b^2$
T13	check_Tmax	$u > T_{\max}\rho$
T14	fixup_Tmax	Never

Typical Production Run:

- ~ 1 Millions SUs;
- 2000-4000 cores;
  - 80% efficiency going from 2,400 --> 16,000 cores
- 2e7 cells, 1e6 - 1e7 time steps;
- Single BH: 33,000 zone-cycles/sec/core;
- Binary BH: 11,000 zone-cycles/sec/core;
- Clusters typically used: Ranger, Kraken

General Relativistic Radiative Transfer

Geodesic Calculation:

- 8 coupled ODEs per ray;
- Burlisch-Stoer Method:
  - Adaptive stepsize
  - Richardson Extrapolation;
- Special stepsize control near black holes
- Integrations start at camera and go through source to guarantee desired image resolution:
  - Rays point forward in time;
  - Rays are integrated backward in time;

$$u^\mu = \frac{\partial x^\mu}{\partial \lambda}$$

$$\frac{\partial u^\mu}{\partial \lambda} = -\Gamma^\mu{}_{\nu\kappa} u^\nu u^\kappa$$

$$\Gamma^\mu{}_{\nu\kappa} = \frac{1}{2} g^{\mu\sigma} \left(\frac{\partial}{\partial x^\nu} g_{\kappa\sigma} + \frac{\partial}{\partial x^\kappa} g_{\nu\sigma} - \frac{\partial}{\partial x^\sigma} g_{\nu\kappa} \right)$$

$$N_{\mathrm{rays}} = N_t N_\theta N_\theta N_i N_j N_\nu N_M N_\rho$$

$$N_{\mathrm{rays}} = 10^9 N_\nu N_M N_\rho$$

$$N_{\mathrm{rays}} \sim N_{x^0} N_{x^1} N_{x^2} N_{x^3}$$

Radiative Transfer:

- 1 ODE per ray
- Same intergrator as that used by geodesics;
- Neglects scattering;
- Difficulty is in accurate and fast emissivity and absorption function;
- Emissivity models:
  - Synchrotron;
  - Bremsstrahlung;
  - Black body;
  - Bolometric model; (see Noble++2009)

$$\frac{\partial I}{\partial \lambda} = j - \alpha I$$

$$\alpha = \alpha(\rho, p, u^\mu, B^i, \nu)$$

$$j = j(\rho, p, u^\mu, B^i, \nu)$$

Monte Carlo Radiative Transfer:

- Schnittman & Krolik 2009
- Rays shot from source, collected at distance observer;
- All other emissivity models plus:
  - Inverse Compton Scattering;
  - Reflection emission (e.g., Fe lines);

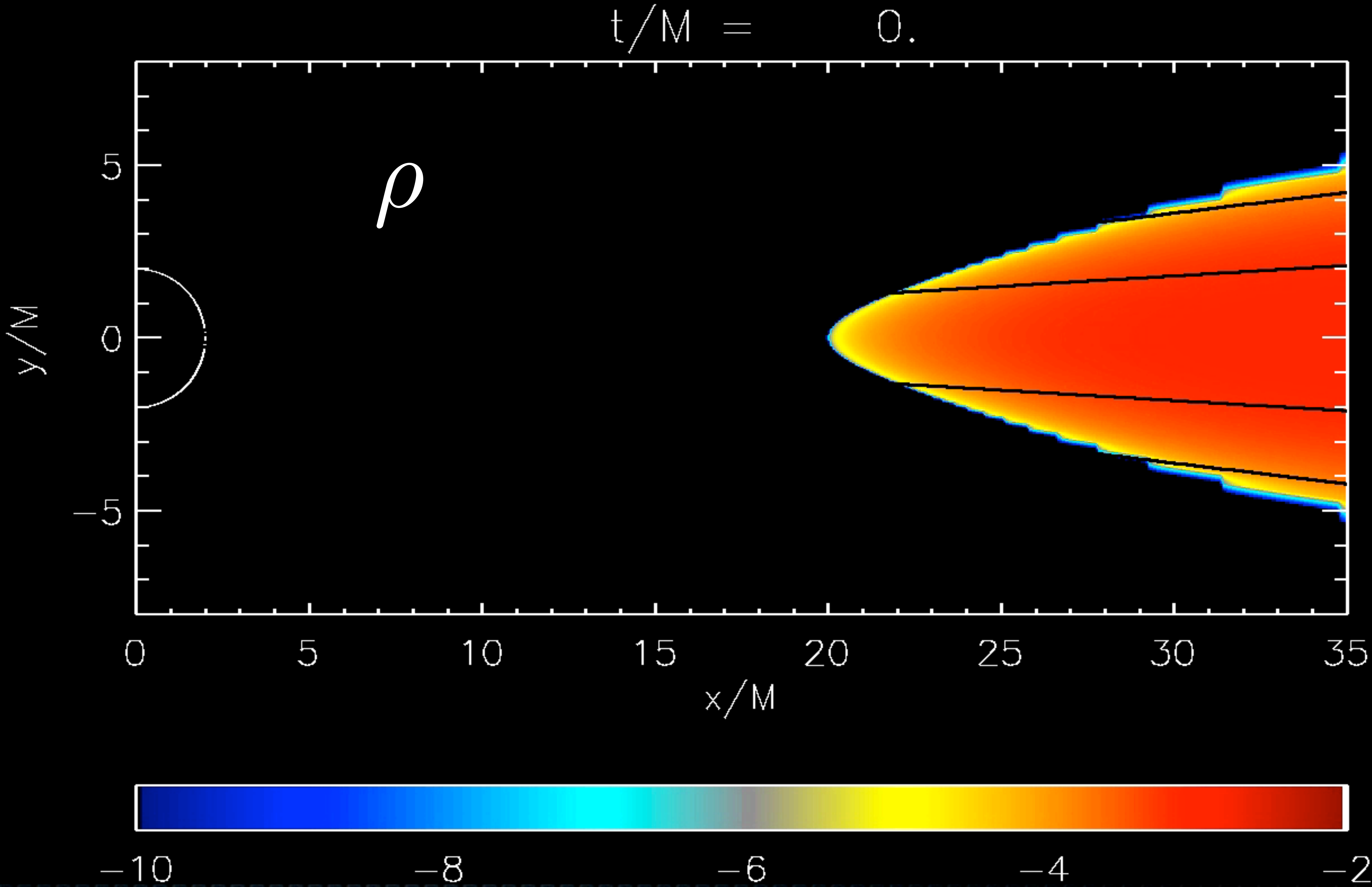
SCN, Krolik, Hawley 2010

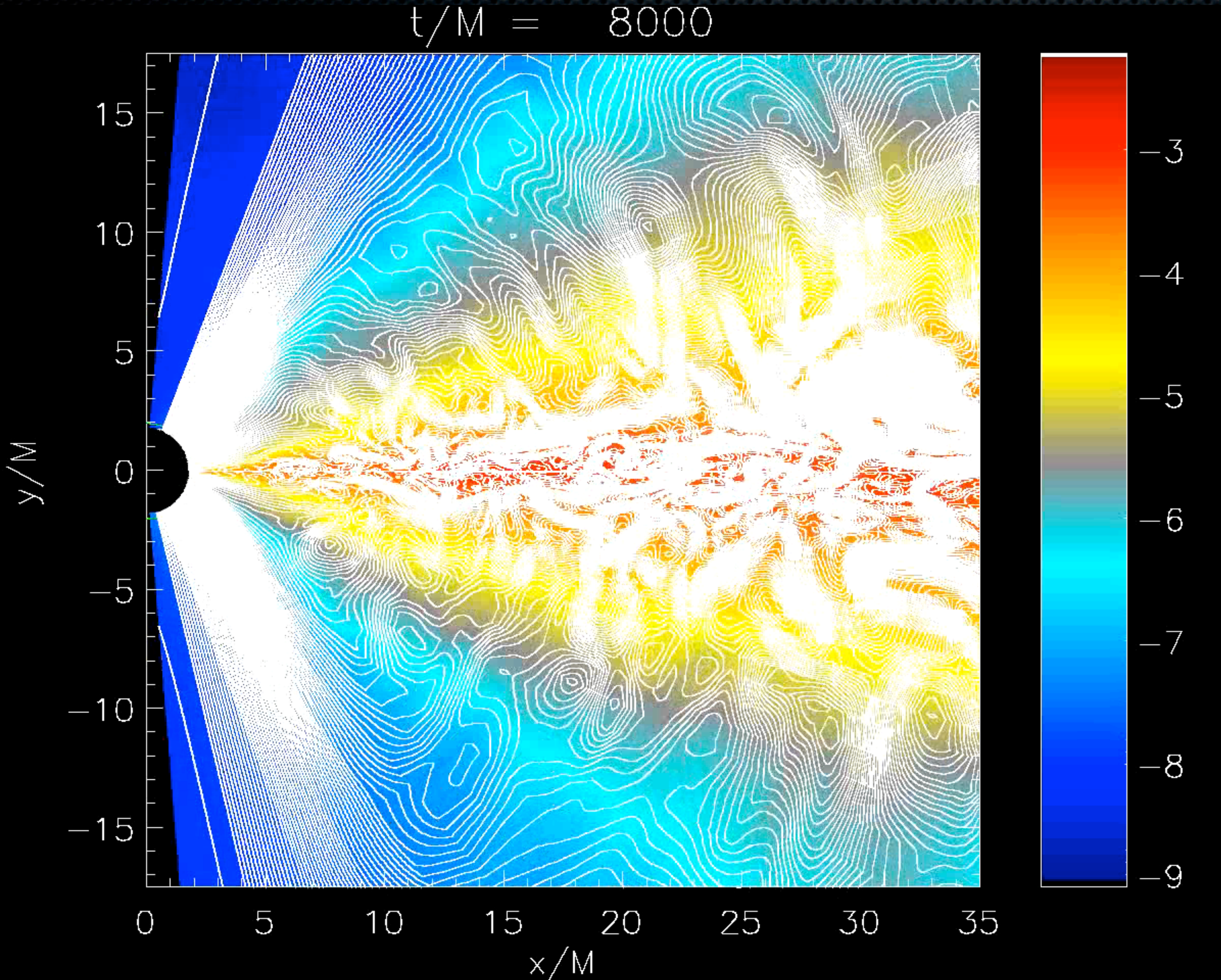
ThinHR: H/R = 0.06 912x160x64 $a = 0M$

ρ

SCN, Krolik, Hawley 2010

ThinHR: H/R = 0.06 912x160x64 $a = 0M$



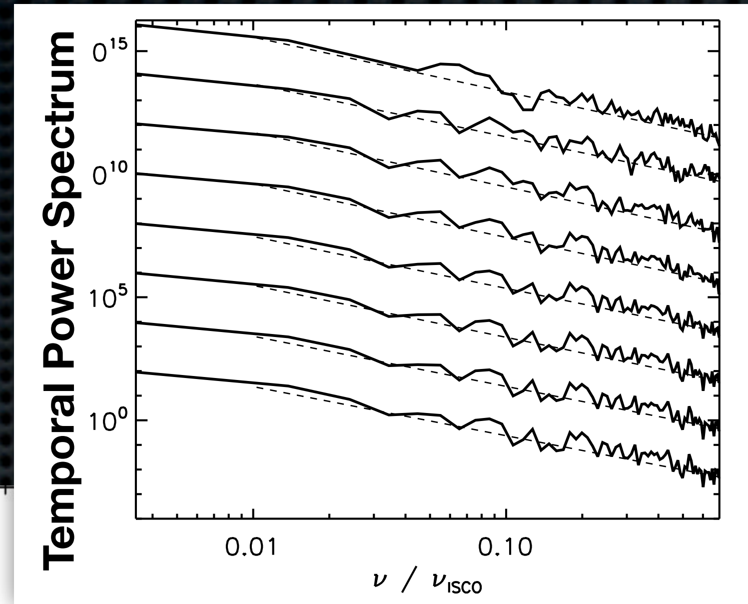

t/M = 8000
y/M
x/M
15
10
5
0
−5
−10
−15
0
5
10
15
20
25
30
35
−3
−4
−5
−6
−7
−8
−9

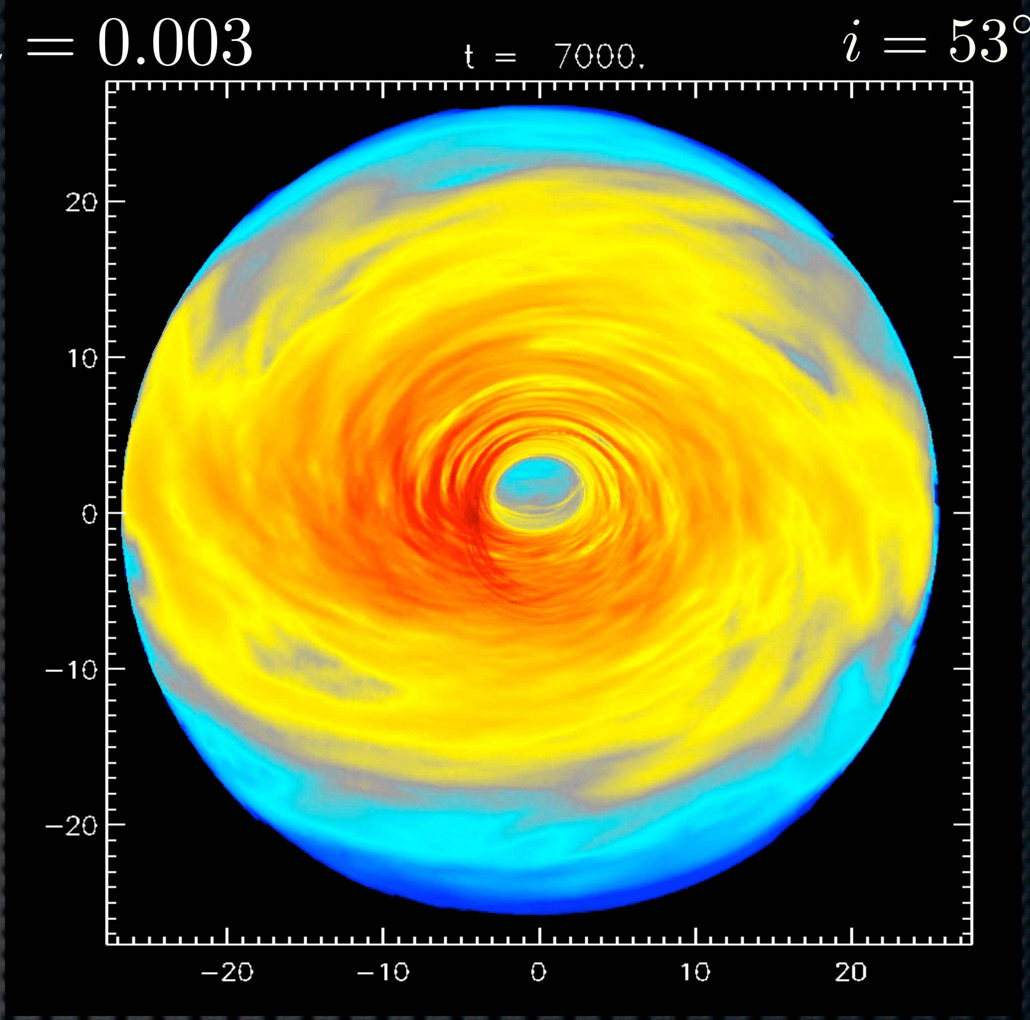
Corona's X-ray Variability:

Noble & Krolik 2009

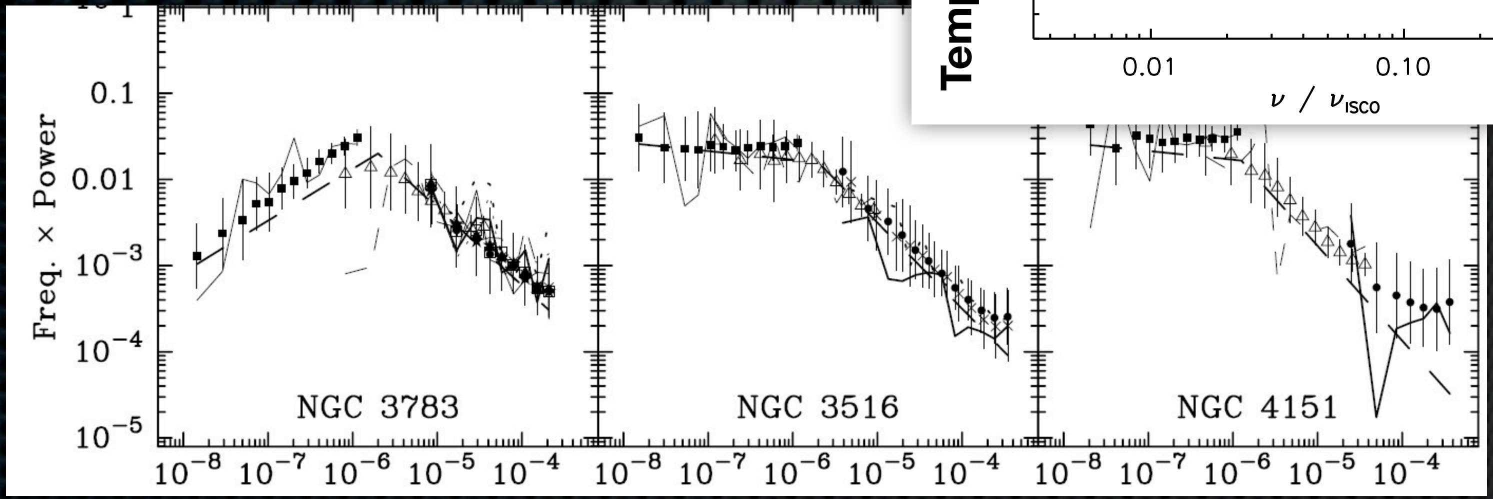
$$P \sim \nu^{\alpha}$$

$$-3 < \alpha < -1$$



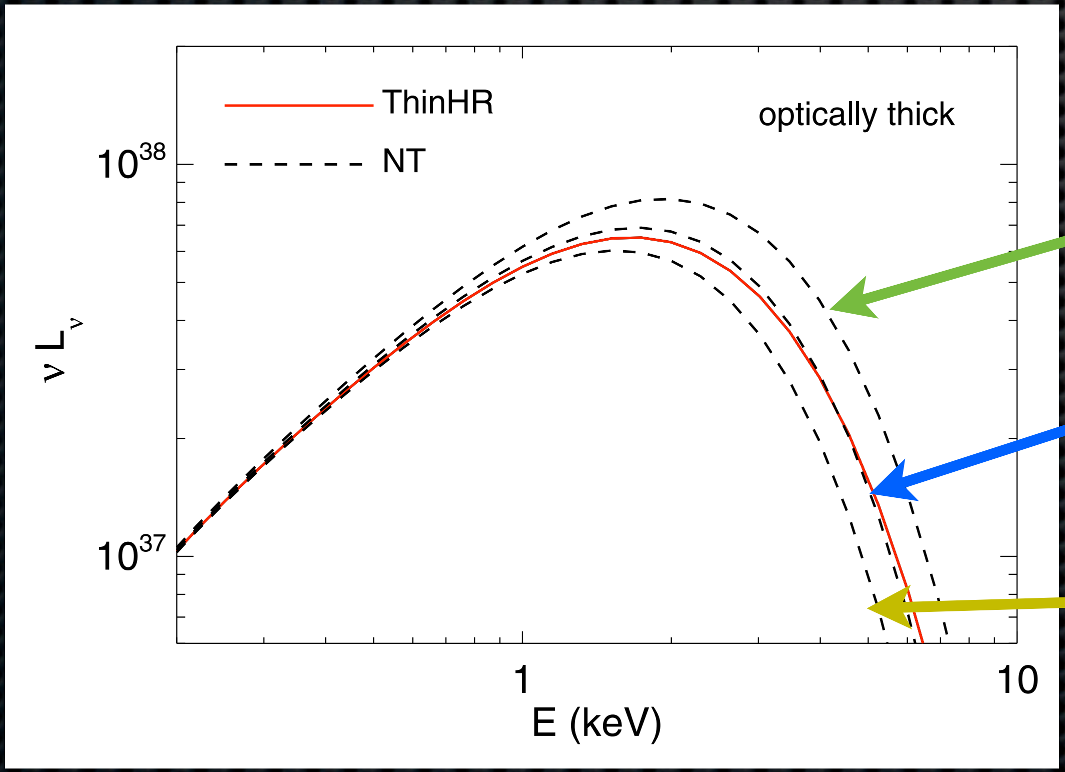


Markowitz et al 2003



Thermal Spectrum of Thin Disks:

Noble, Schnittman, Krolik, Hawley 2011



spin = 0.4

spin = 0.2

spin = 0.0

NT = Novikov-Thorne
= Standard time-axi-symmetric cold disk solution

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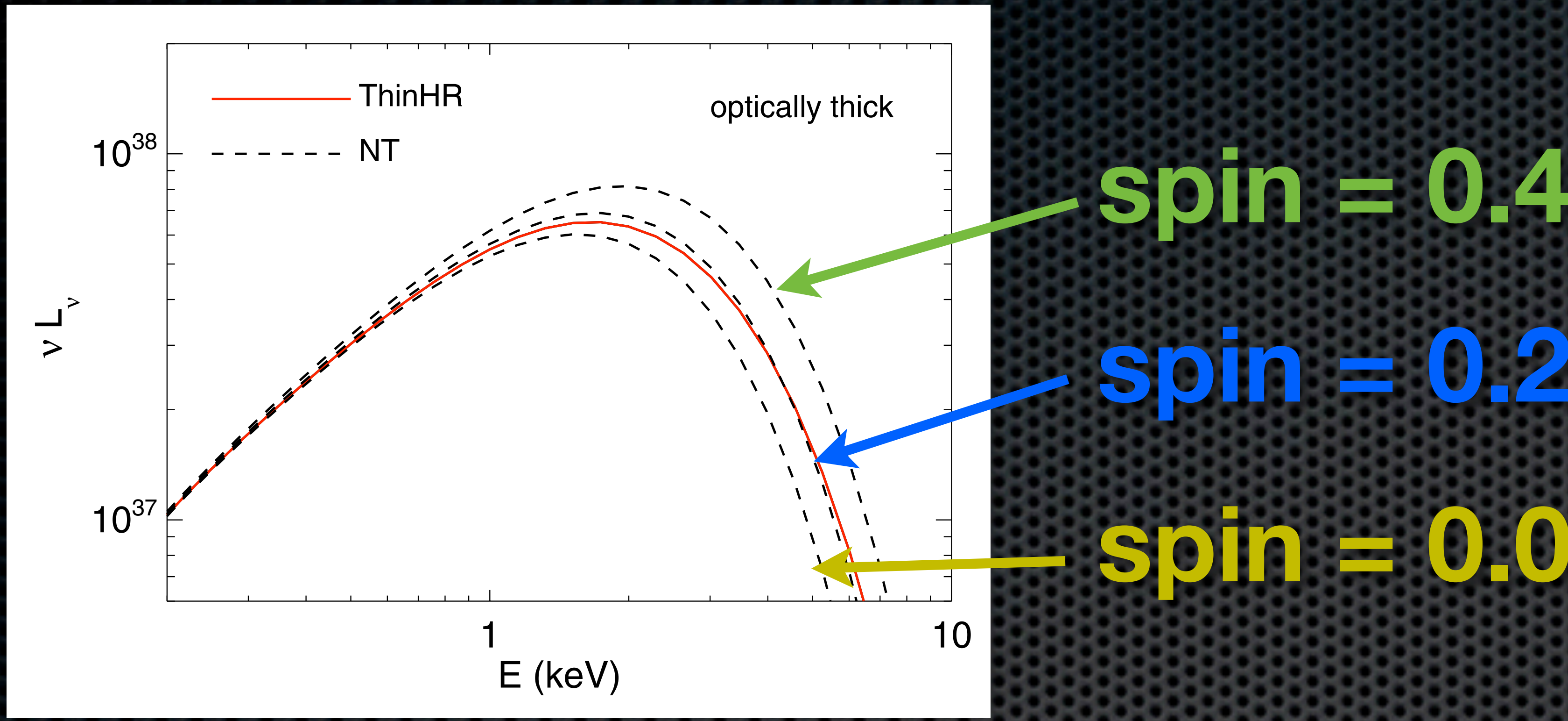

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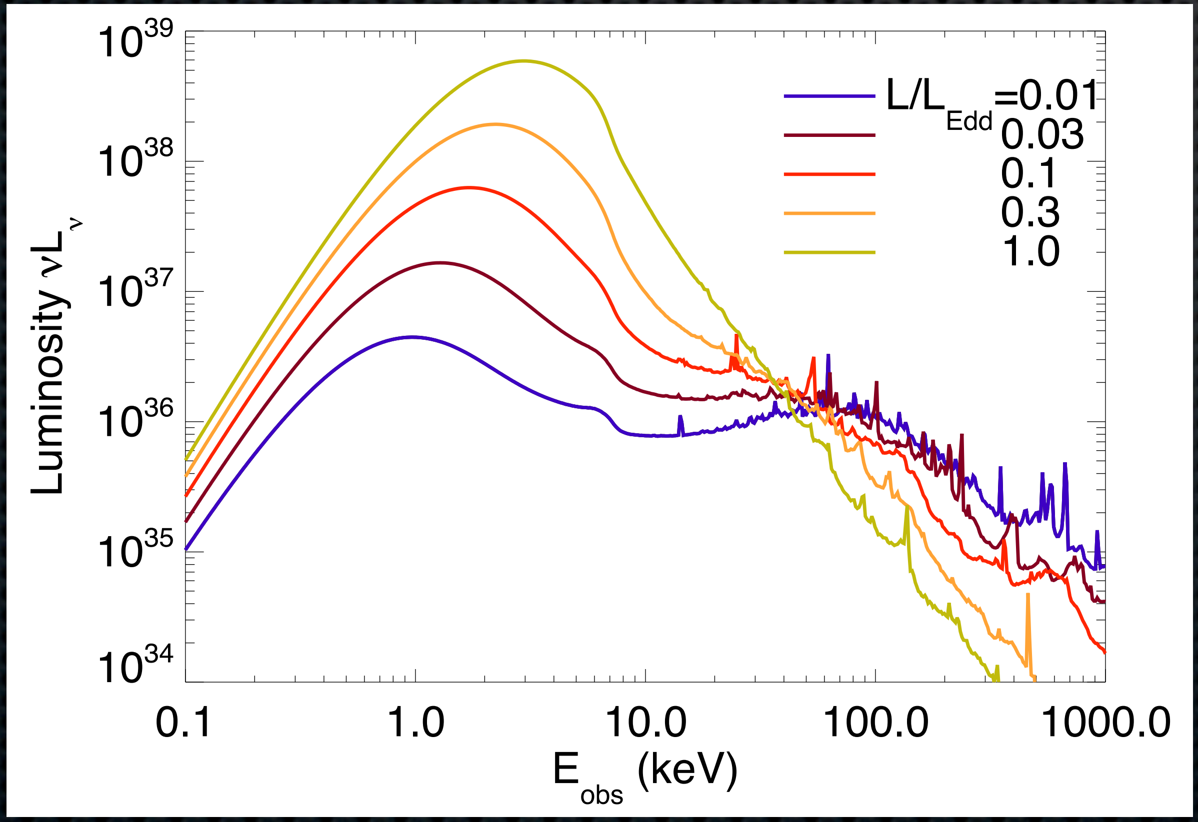
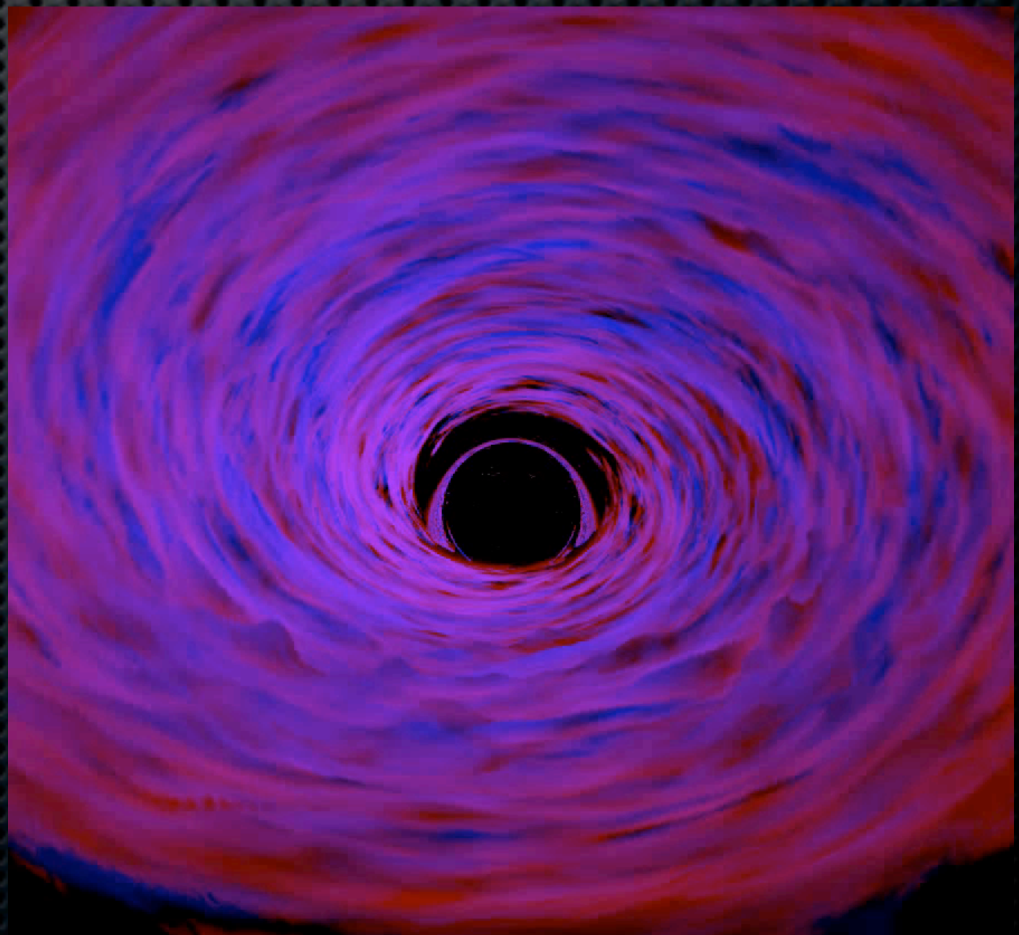
Monte Carlo Inverse Compton Emission

Schnittman, Krolik, Noble 2012



Bremsstrahlung:
Red = Disk, Soft X-rays
Blue = Corona, Hard X-rays



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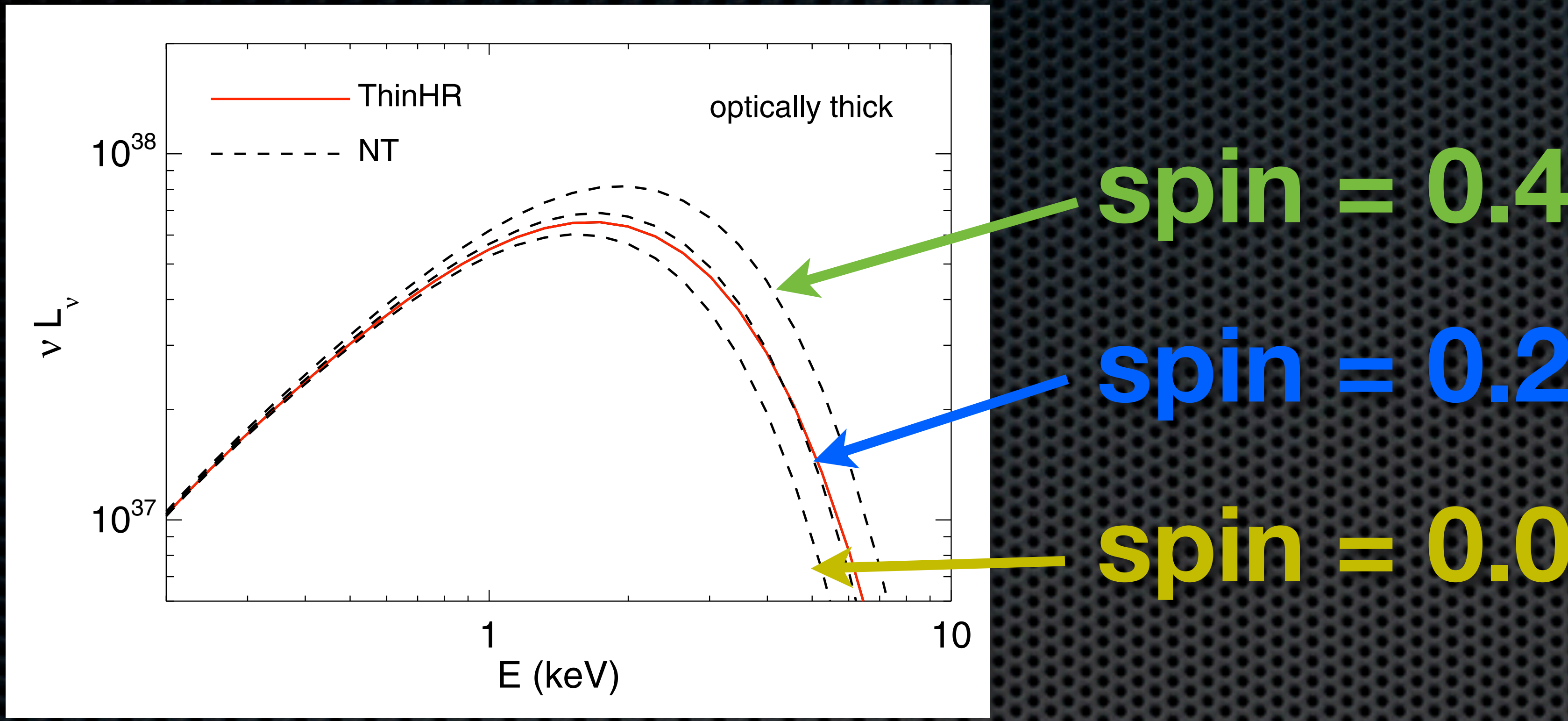

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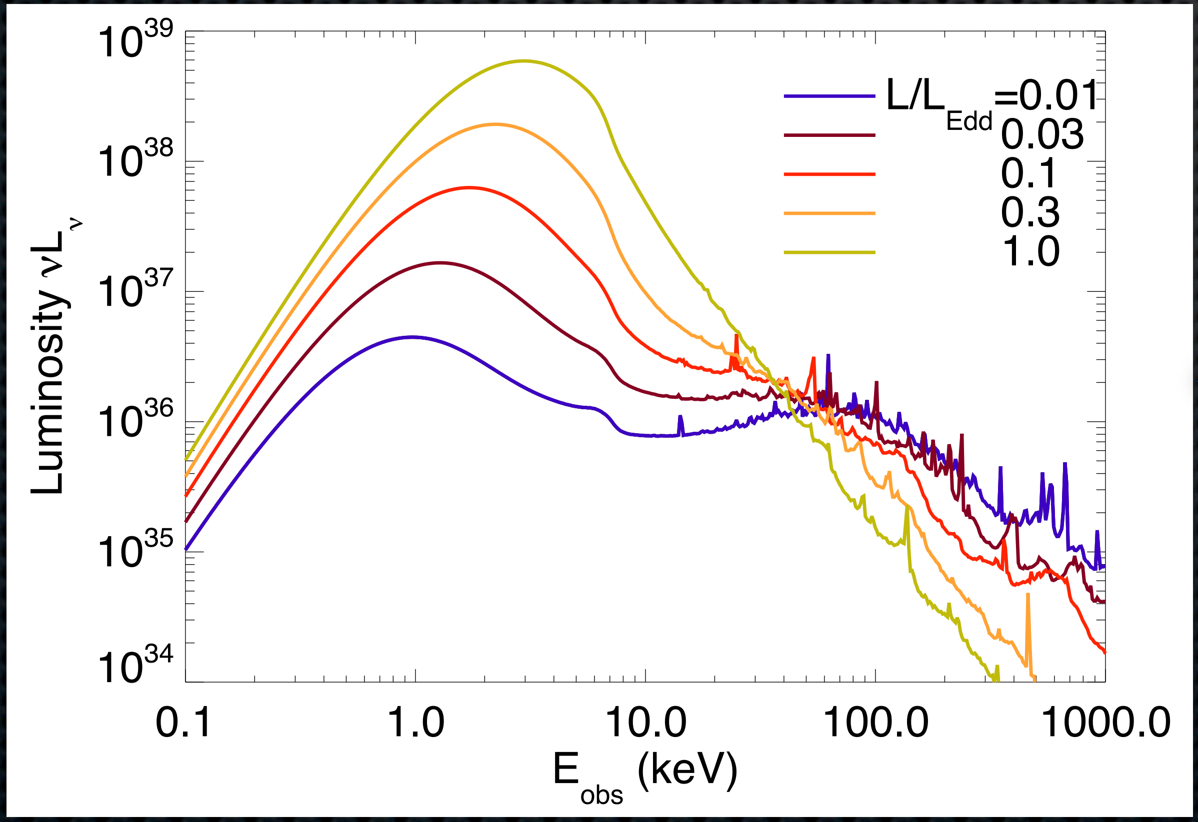
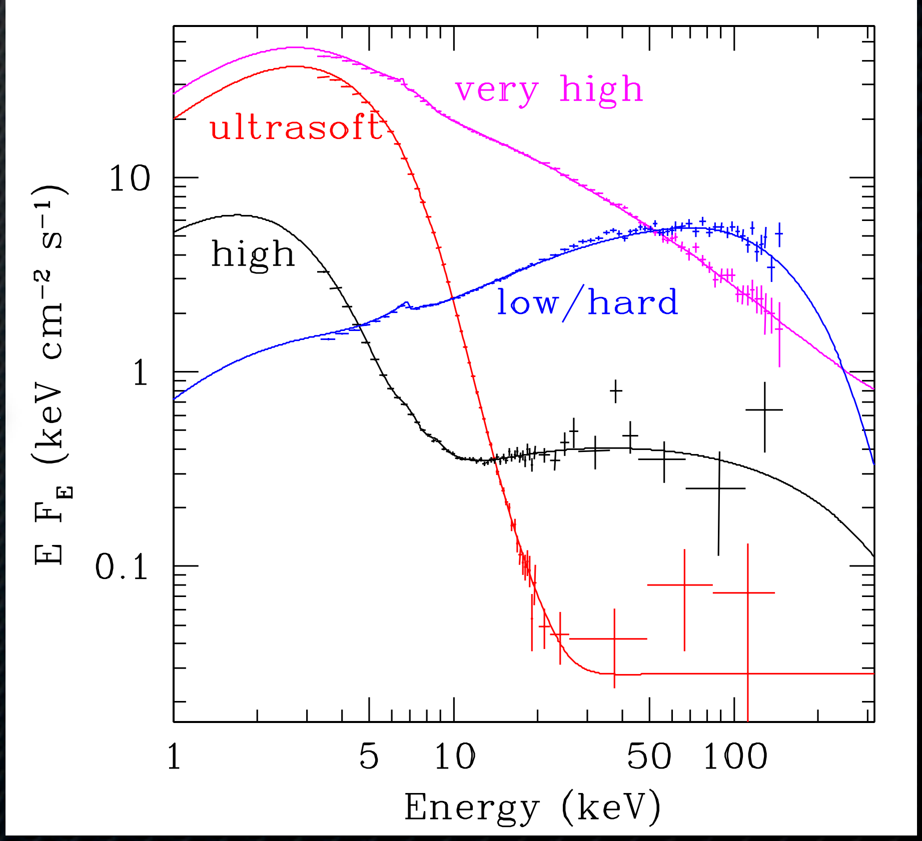
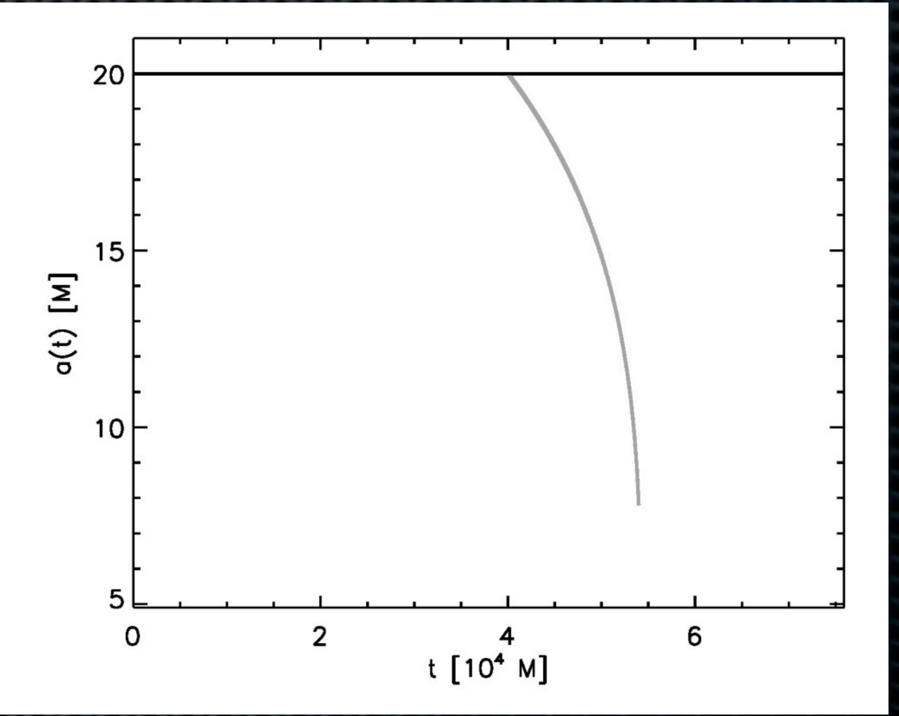




Binary Black Hole Spacetimes

1) Solve Einstein's Equations: Numerical Relativity

- Set of 12 second-order non-linear PDEs with constraints and gauge choices;
- Two-body problem solved only after ~30 years of research;
- Require grid refinement hierarchies that follow BHs --- not amenable for disk evolutions
- Please recall talks by Rezzolla, Loffler, Montero;

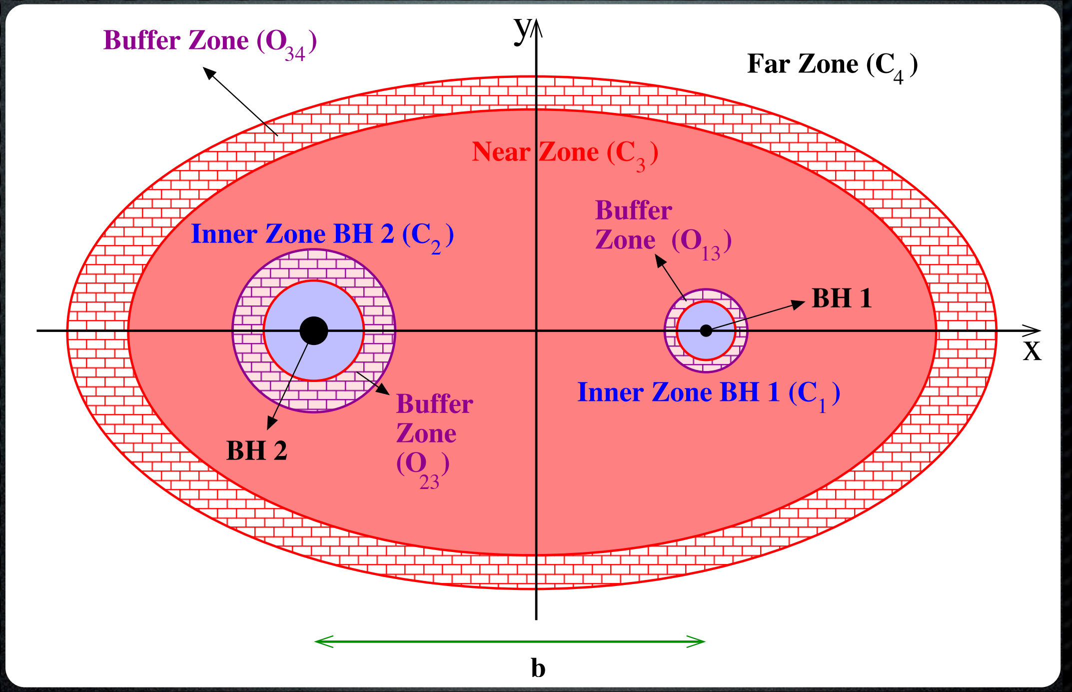


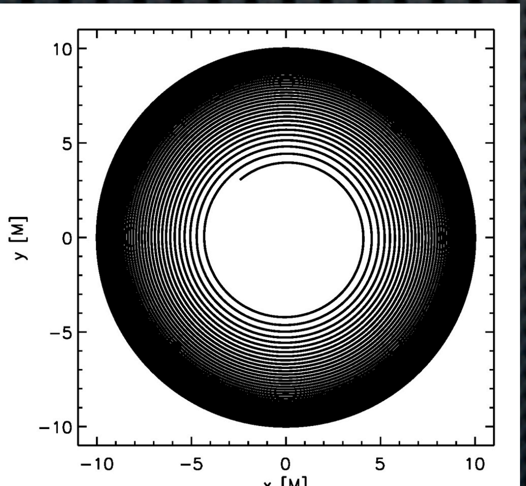
2) Approximate Spacetimes

- Solve Einstein's Equations approximately, perturbatively;
- Expand equations to orders of 2.5 Post-Newtonian order

$$\epsilon_i = m_i / r_i \sim (v_i / c)^2$$

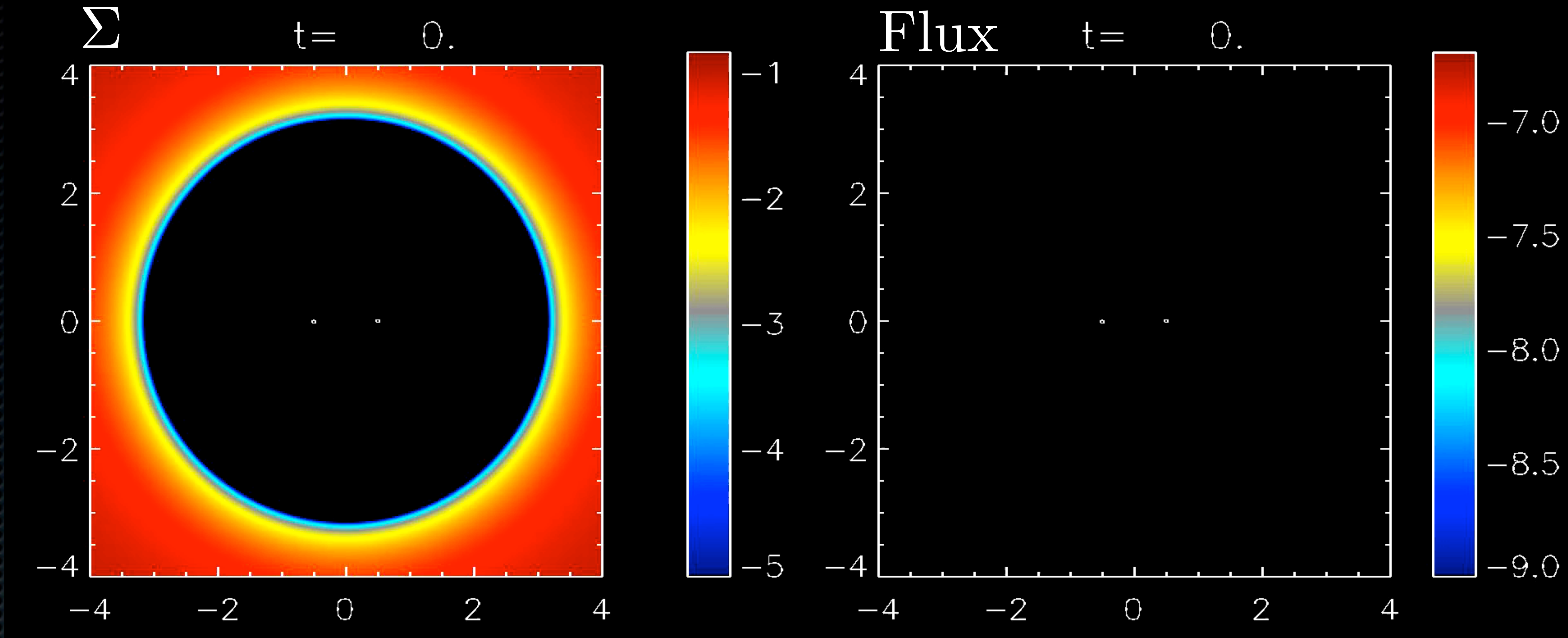
- Used as initial data of Numerical Relativity simulations;
- BHs rigidly rotate at Post-Newtonian Frequency;
- a_0 = 20M;
- Domain: r = [0.75 a_0, 13a_0] = [15M, 260M];
- Keep binary at fixed separation until t = 40,000M;
- For t > 40,000M, let BHB inspiral according to PN;





Yunes++2006

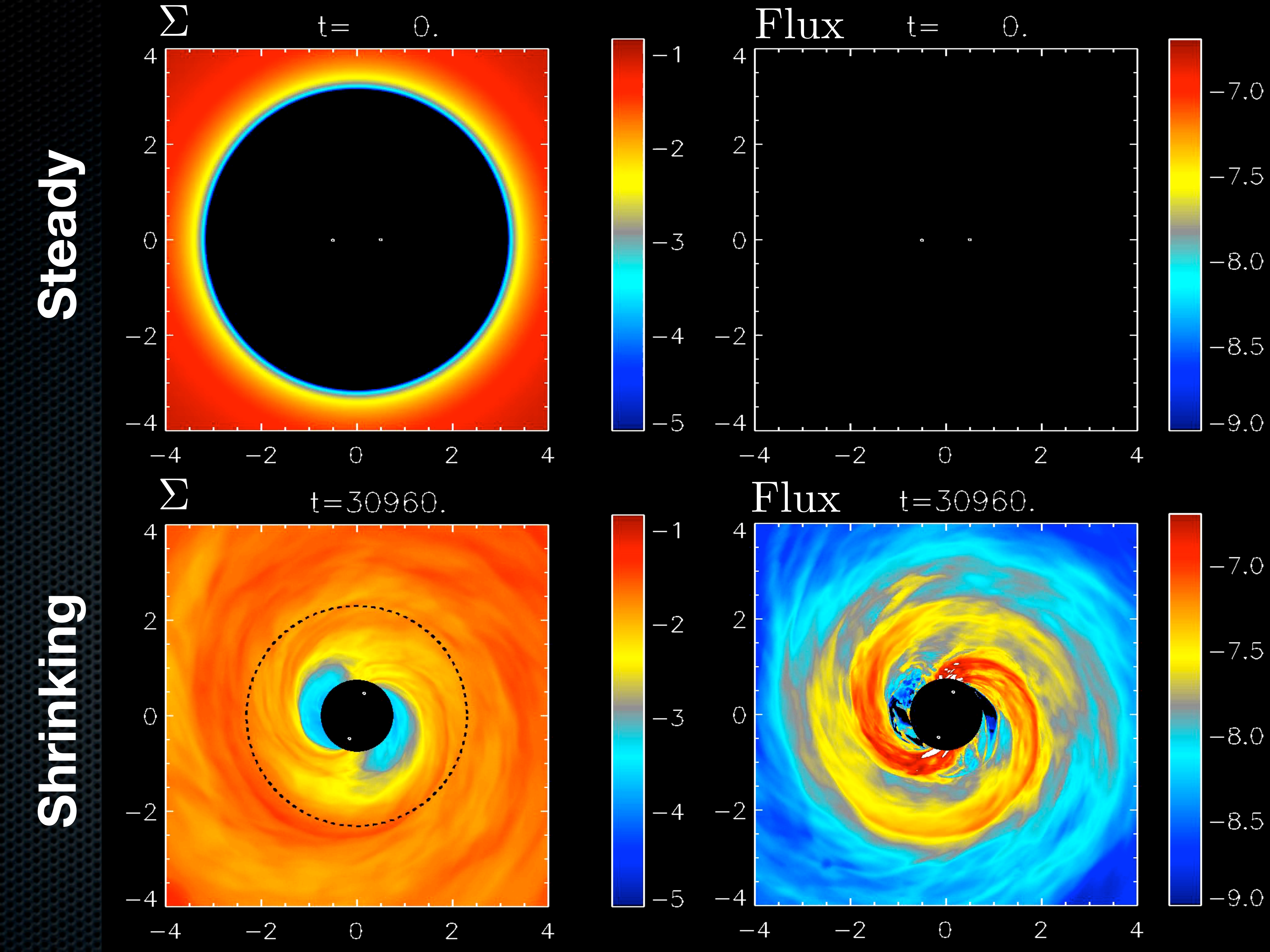
Steady

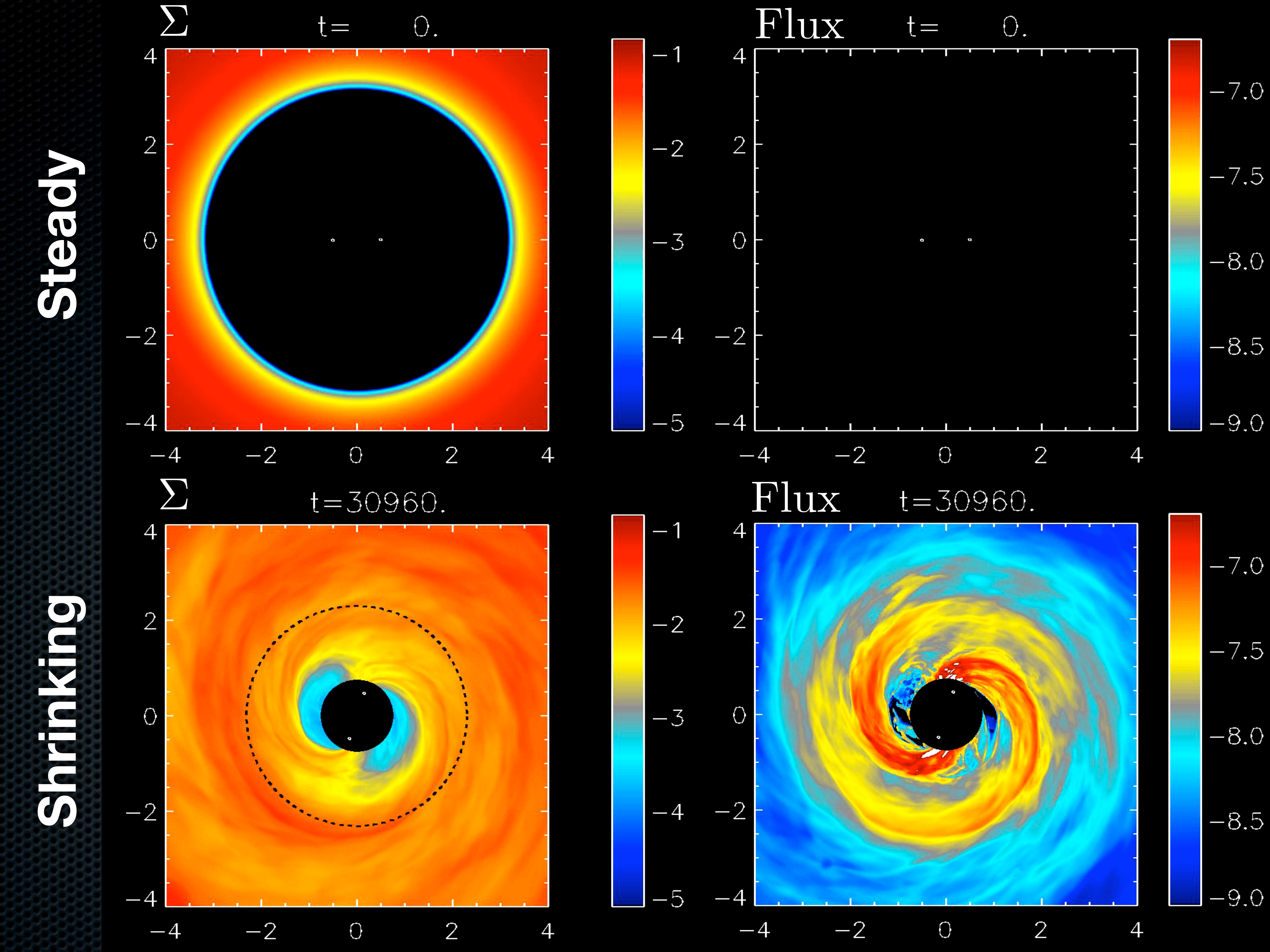


Shrinking

Σ

Flux

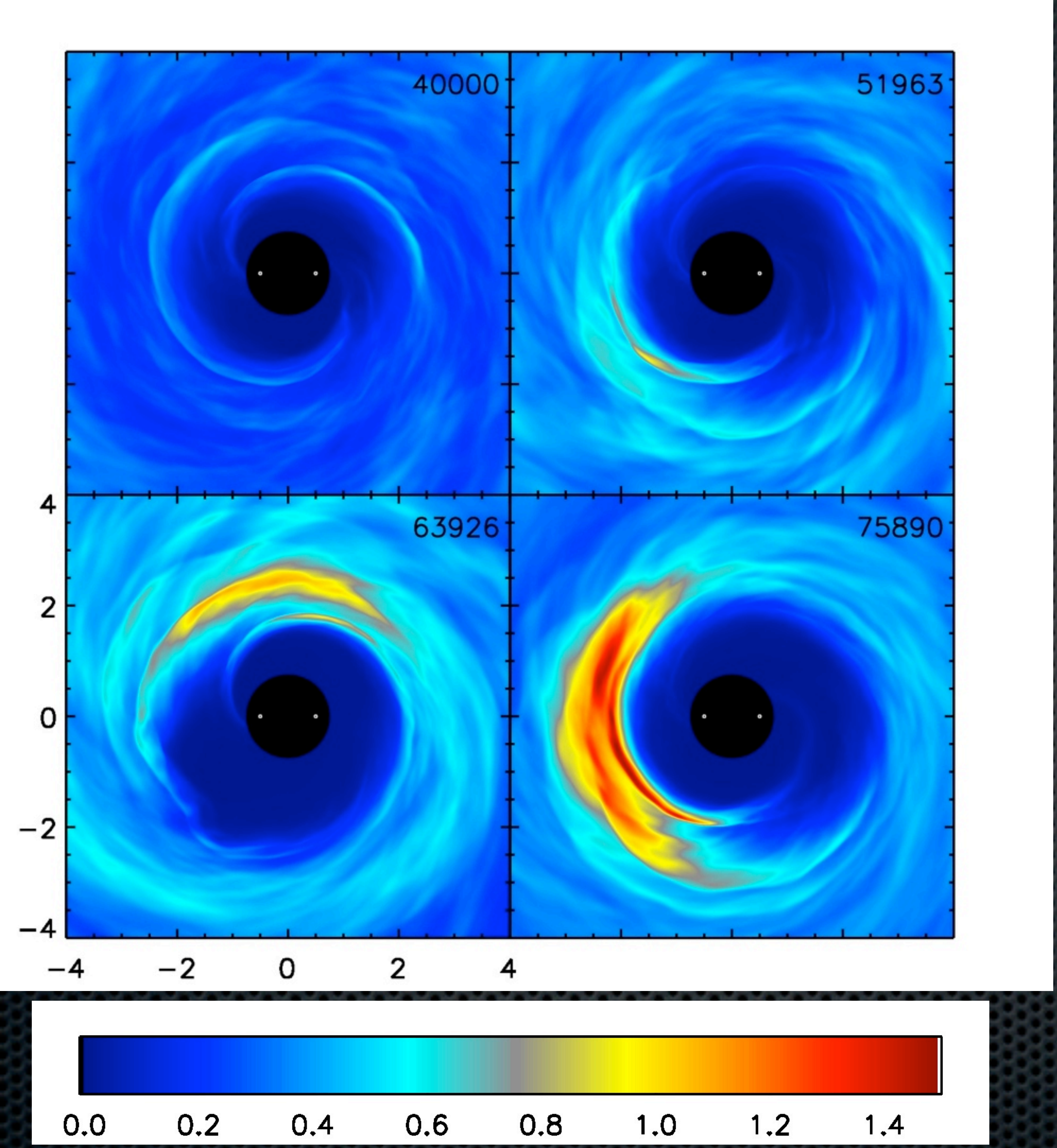

Steady
Shrinking
Σ t= 0.
Flux t= 0.
Σ t=30960.
Flux t=30960.



Steady
Shrinking
Σ
t= 0.
Flux
t= 0.
Σ
t=30960.
Flux
t=30960.

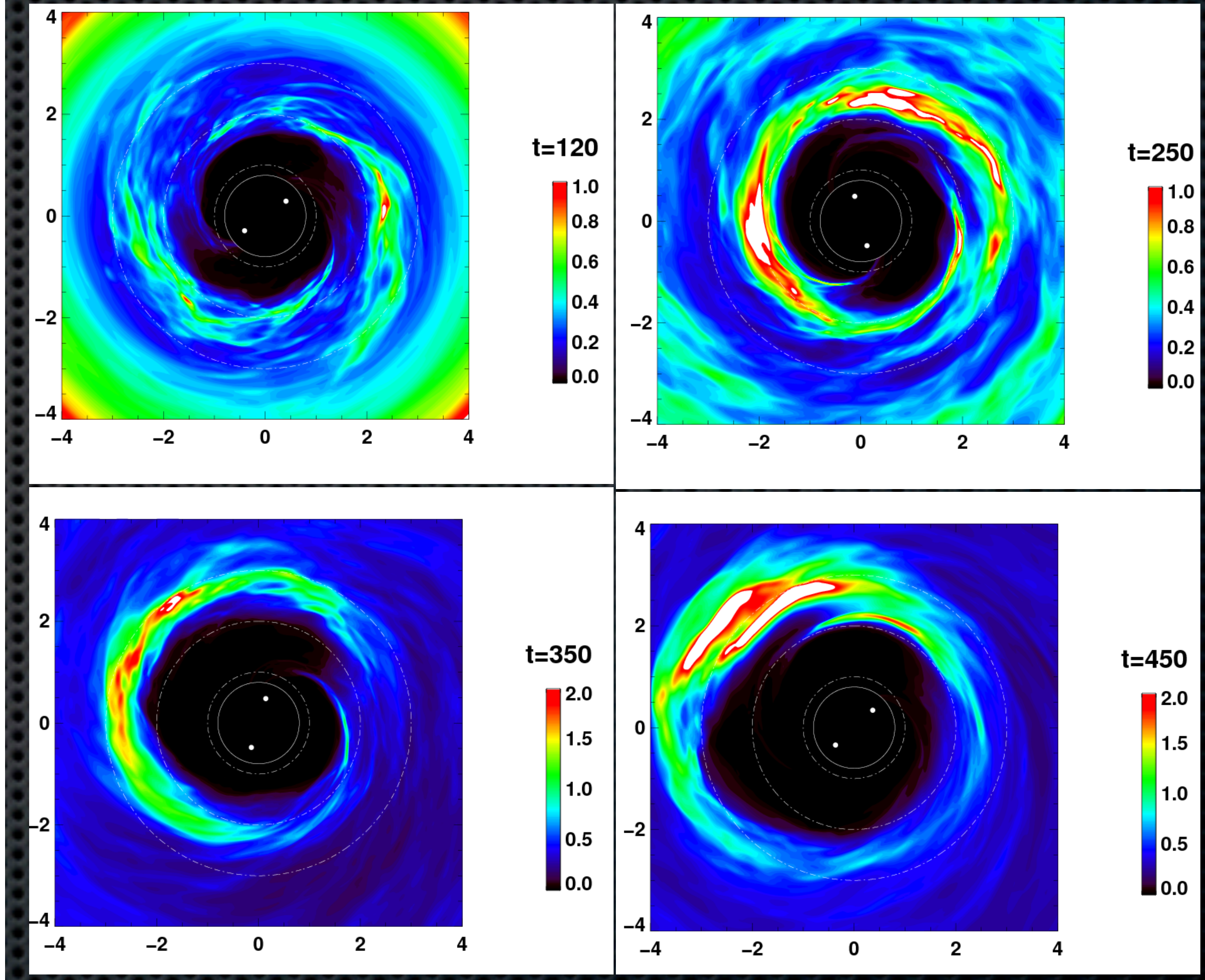
The "Lump"

$$\Sigma(r,\phi) \equiv \int d\theta \sqrt{-g}\,\rho / \sqrt{g_{\phi\phi}}$$



GRMHD: Noble++2012

Newtonian MHD:
Shi++2012

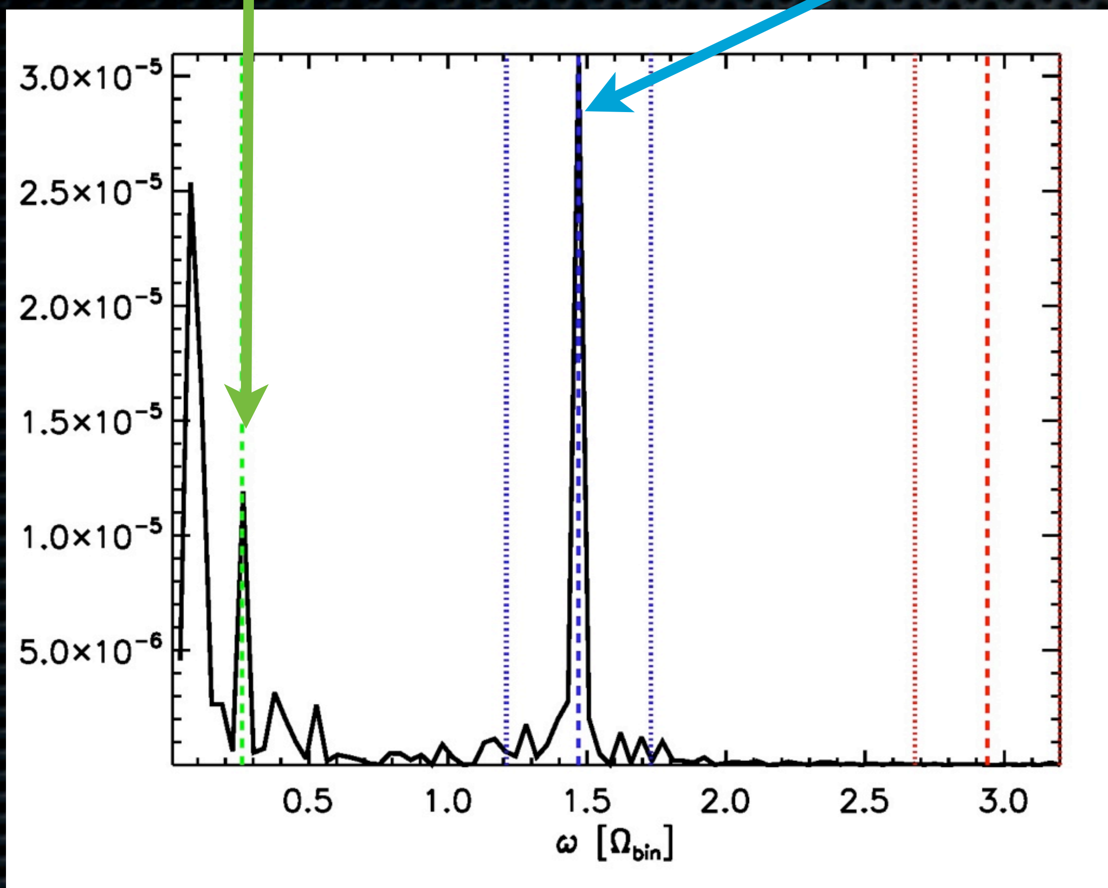


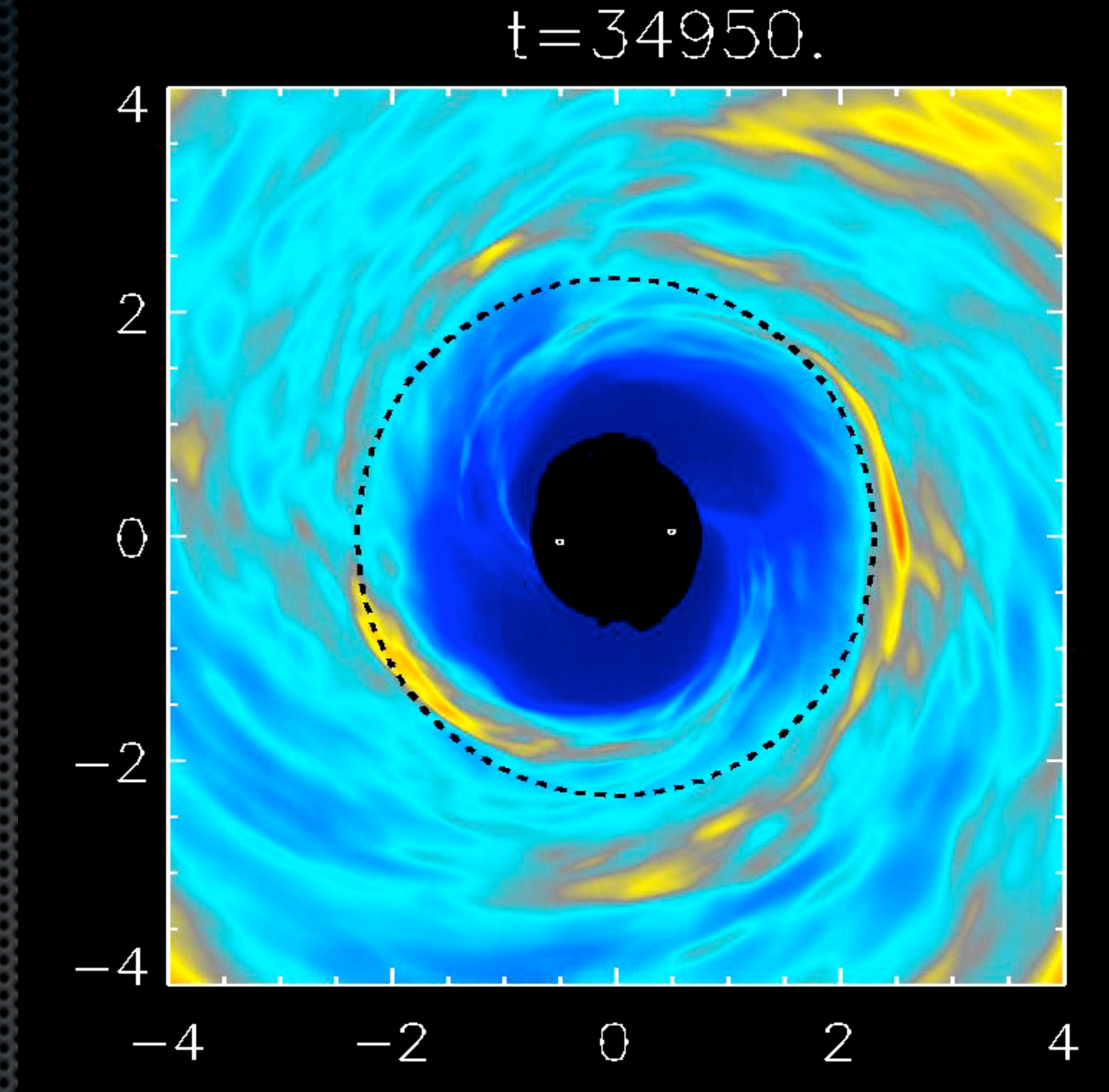
- Also, seen in:
  - Self-gravitating Newtonian hydro:
    - D'Orazio++2012
    - Roedig++2012

Periodic Signal

$r_{\rm lump} \simeq 2.5a$
$\Omega_K(r_{\rm lump})$

$1.47\Omega_{\rm bin}$





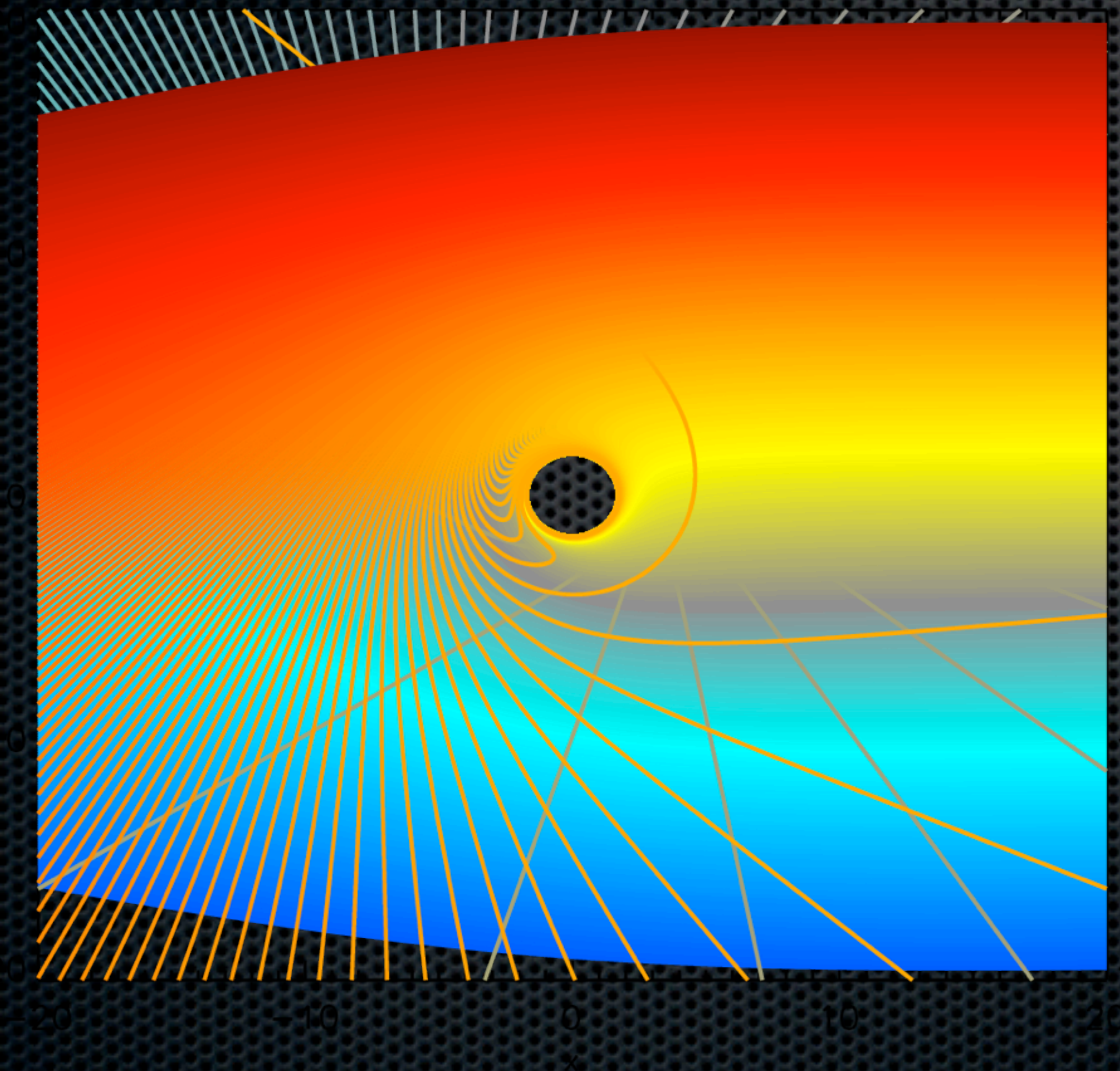
$$\omega_{\rm peak} = 2\left(\Omega_{\rm bin} - \Omega_{\rm lump}\right) \rightarrow 1 < \frac{\omega_{\rm peak}}{\left(\Omega_{\rm bin} - \Omega_{\rm lump}\right)} < 2 \qquad 0 < \frac{M_2}{M_1} < 1$$

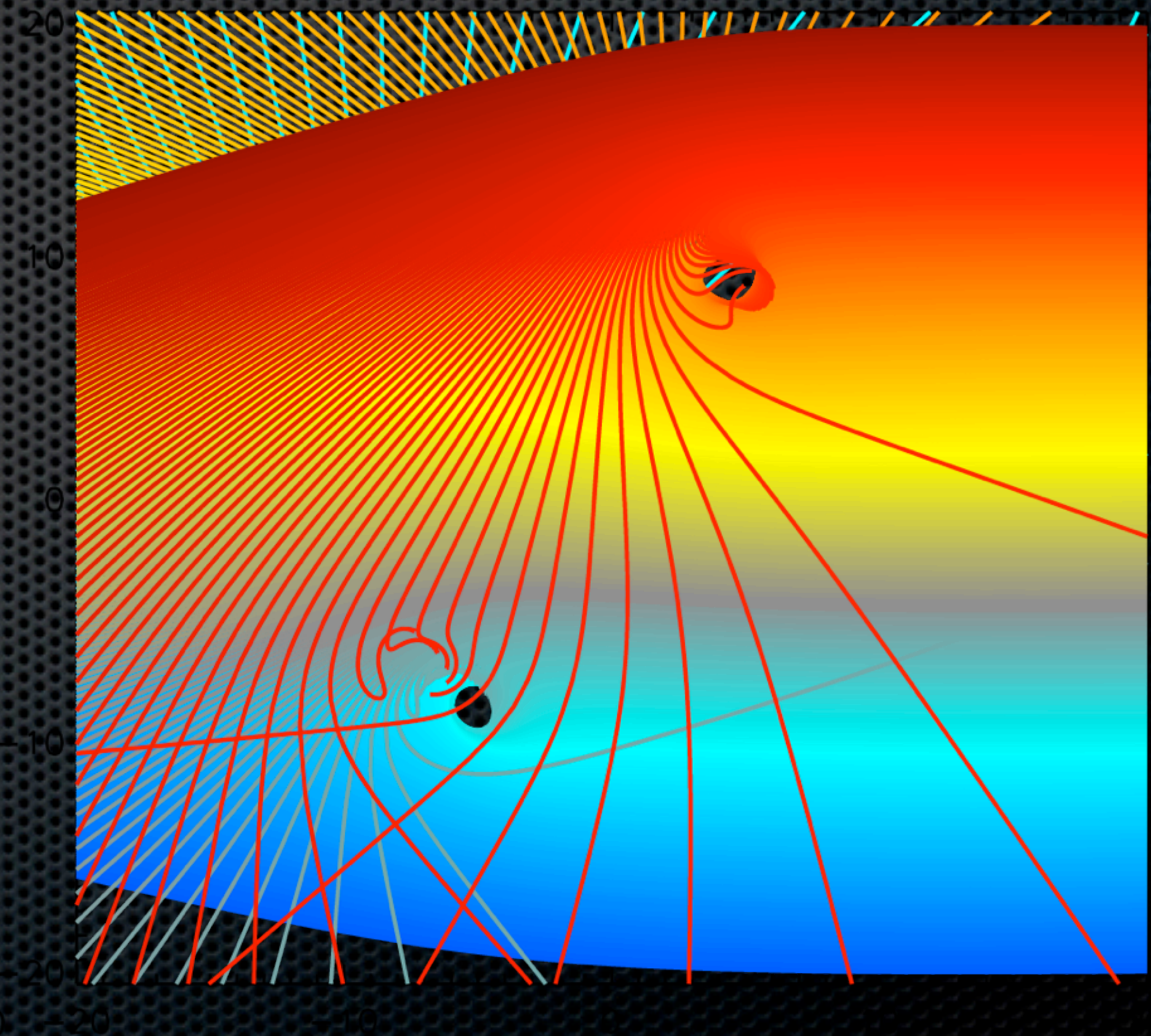
May be obfuscated by “low-pass” filter of disk’s opacity: $0.16\left(\frac{\alpha}{0.3}\right) \lesssim f_{\rm supp} \lesssim 0.32\left(\frac{\alpha}{0.3}\right)$ **--> Ray-tracing may help determine quality of signal**

Current & Future Directions

Binary Black Hole Ray-tracing:

- With Billy Vazquez (grad student);
- Use Superimposed Boosted Dual Kerr-Schild black holes; Bonning++2009
- Binary "orbits" via rigid rotation;

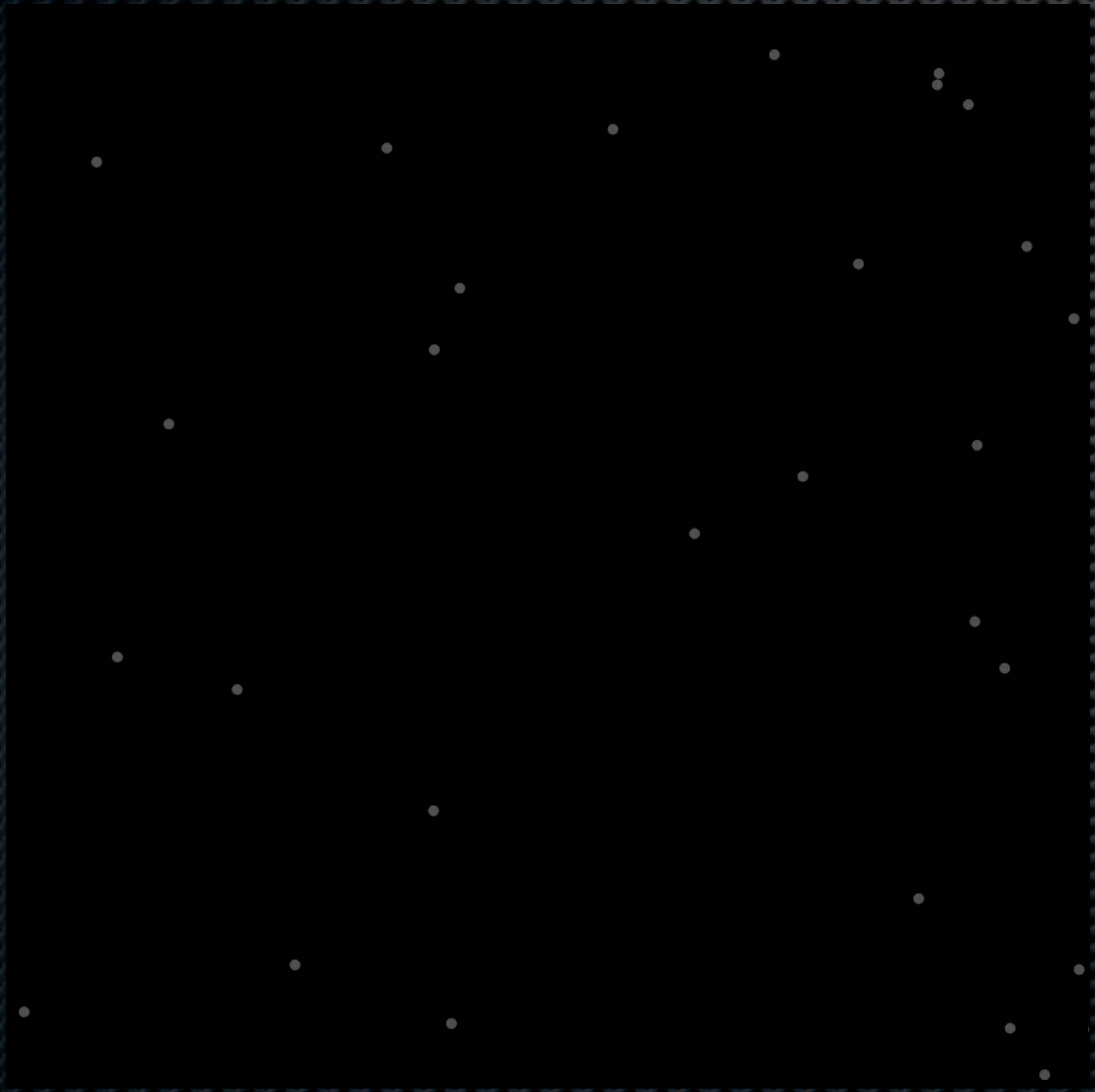




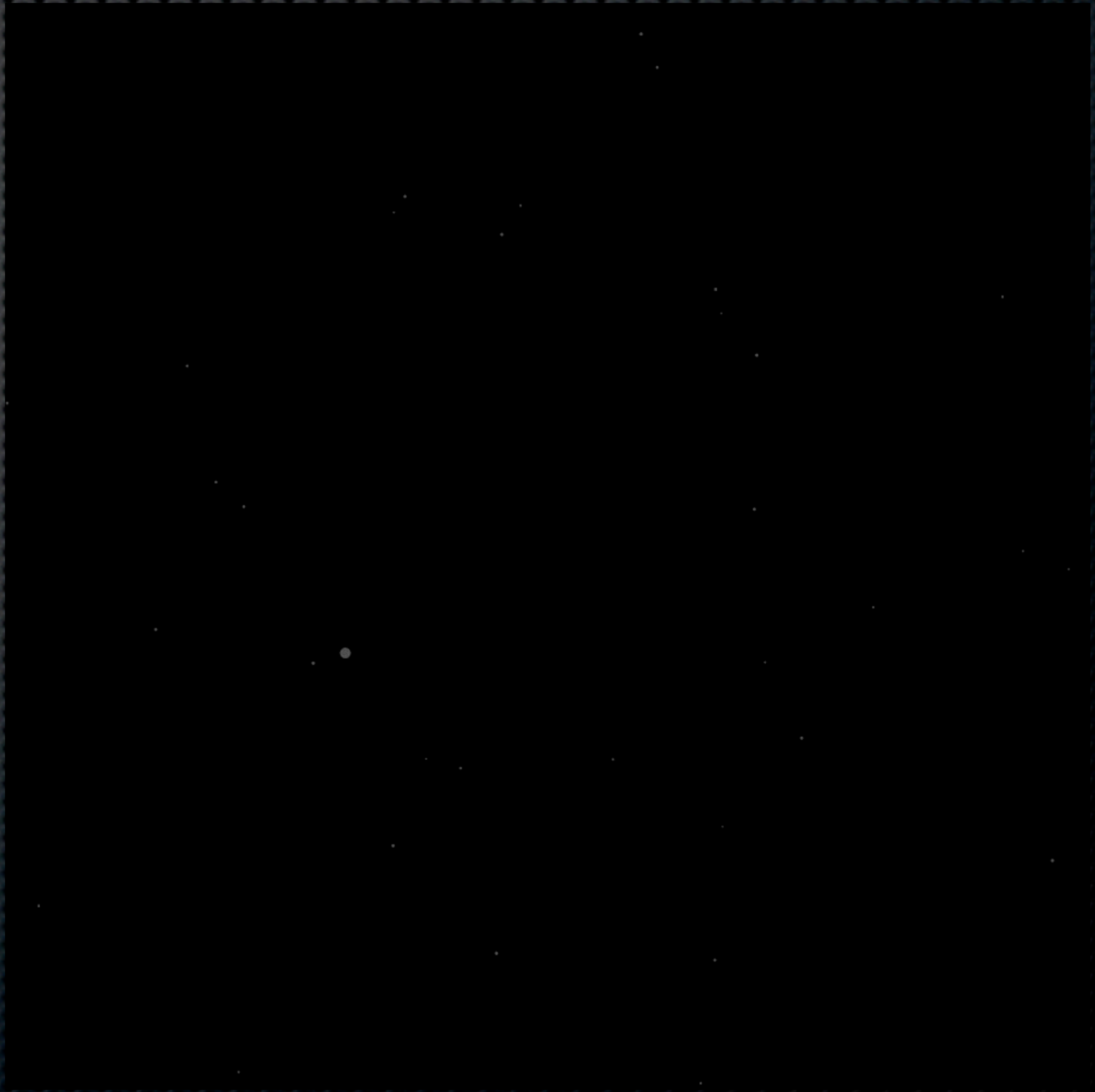
Binary Black Hole Ray-tracing:

- With Billy Vazquez (grad student);
- Use Superimposed Boosted Dual Kerr-Schild black holes; Bonning++2009
- Binary "orbits" via rigid rotation;

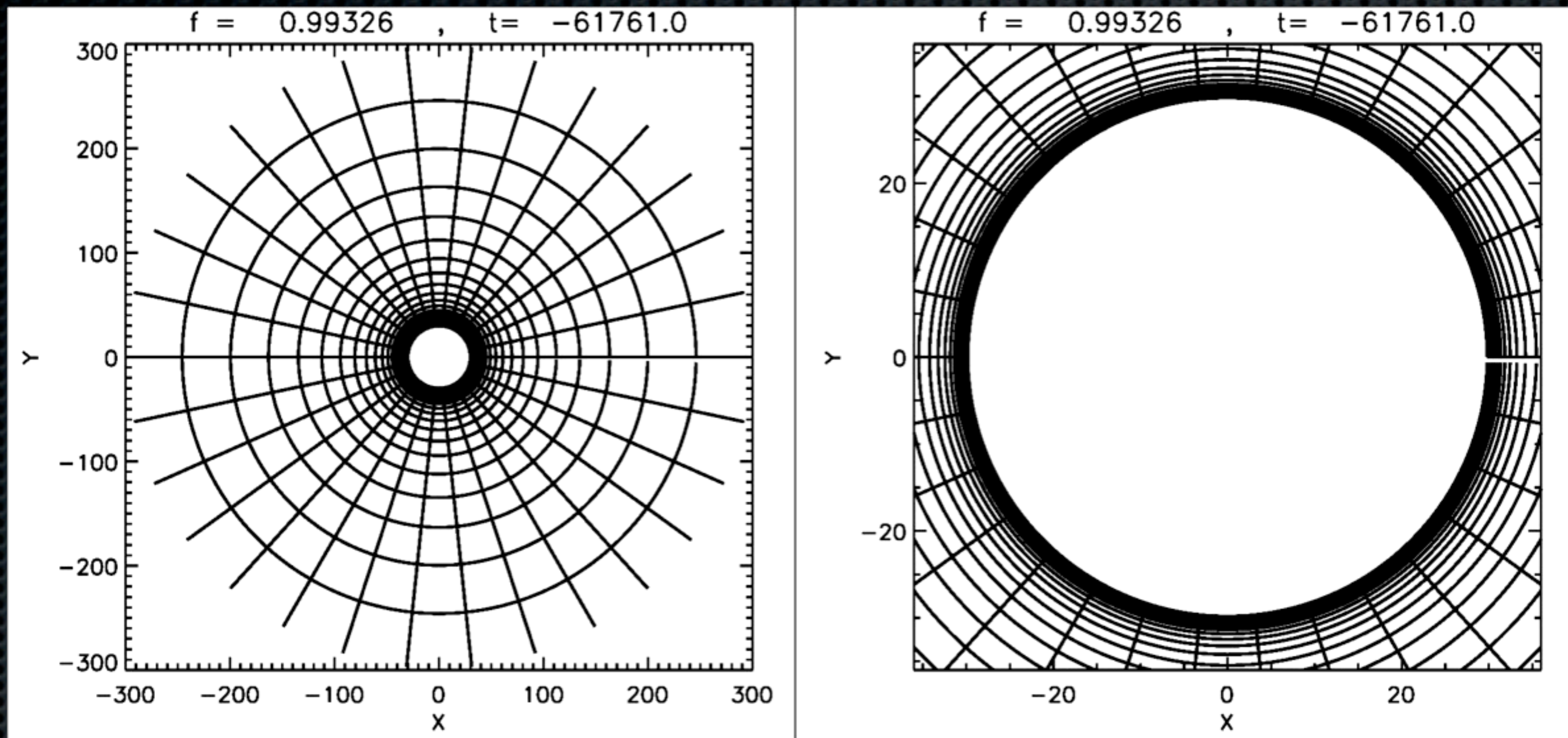
Constrained to BBH's Plane

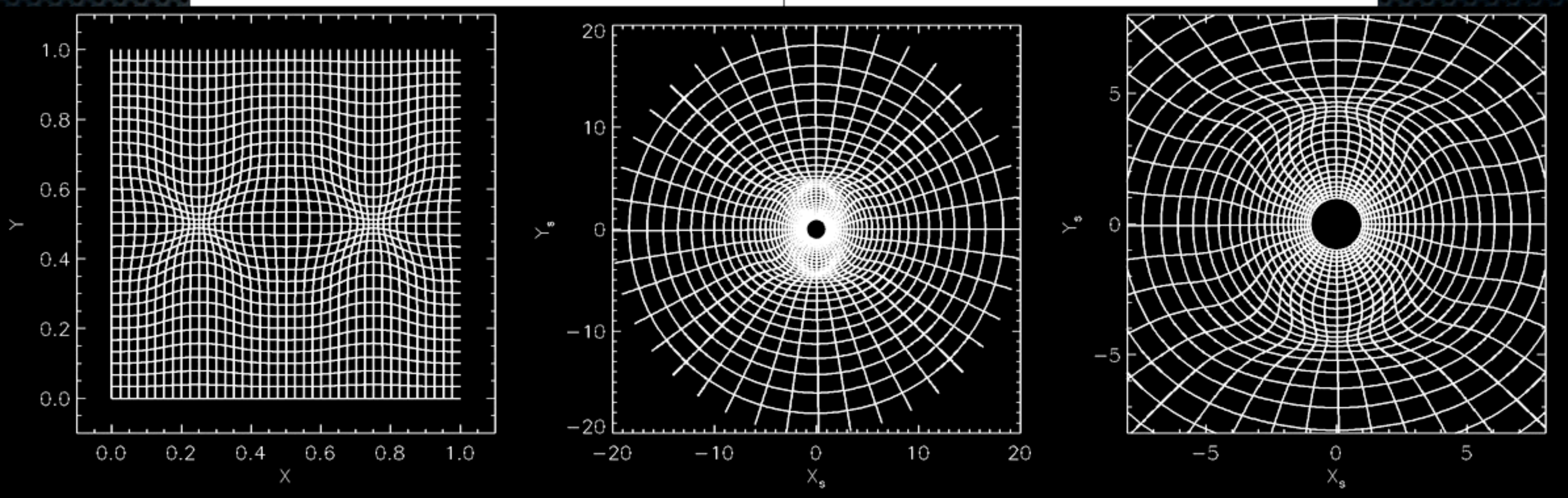


Isotropic

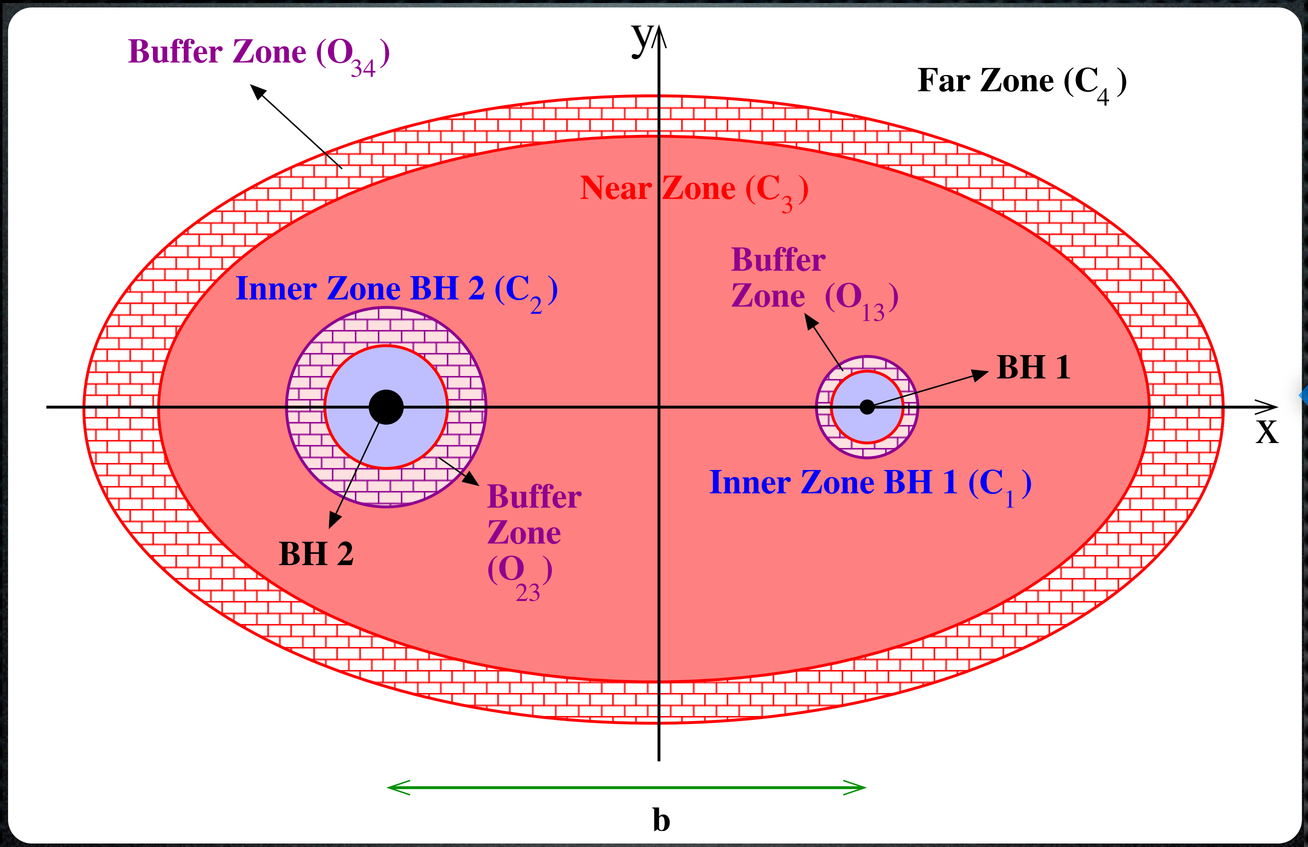


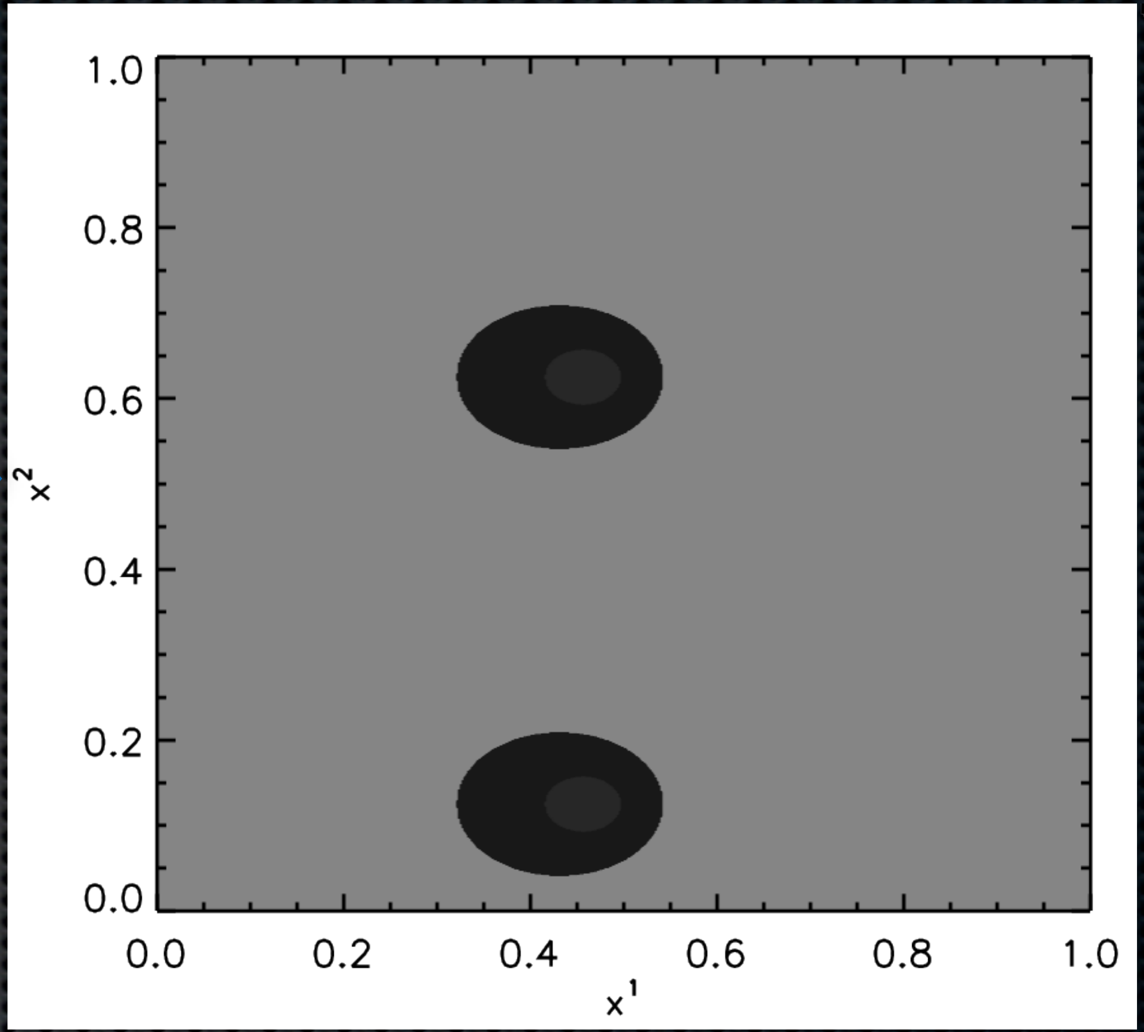
Dynamic Coordinates to Resolve Binary Black Holes





Load Balancing Domain Decomposition





- Different zones of the spacetime vary in computational cost of evaluating metric;
- Black Holes (or zones) move through the grid --> "dynamic" load balancer;

Zone	Relative Cost Per Cell
Inner	3
Inner/Near Buffer	4
Near	1
Near/Far Buffer	2
Far	~1

Conclusions

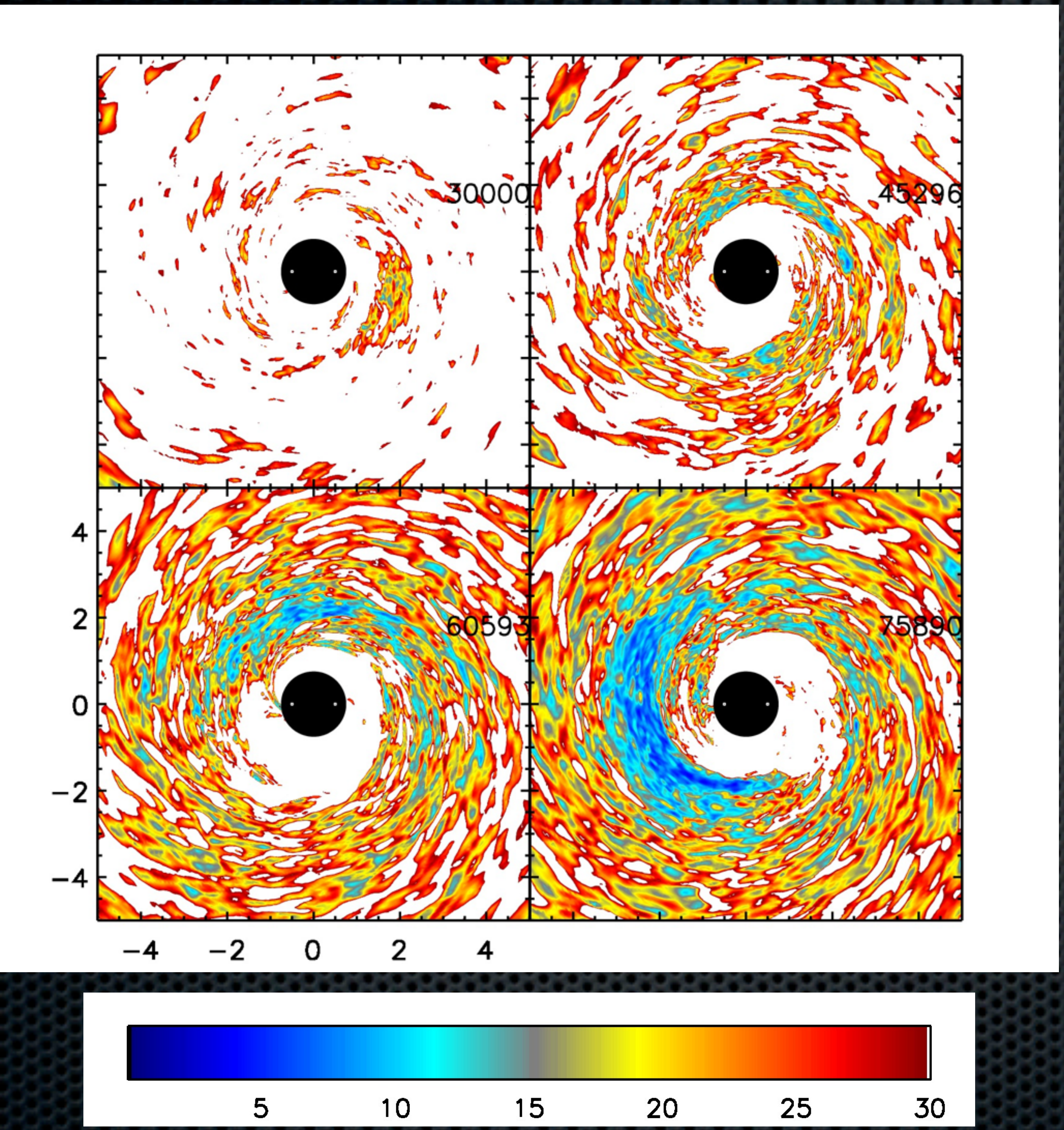
- We have the tools to model single black hole accretion disks in 3D;
- We have the tools to make observational predictions from these simulations;
- We are in the process of applying these tools to the binary case;
  - Predicted a periodic EM signal that could be used for identifying close binaries by all-sky high-cadence campaigns (e.g., LSST, Pan-STARRS);
  - Additional computational techniques are required for the sake of runtime efficiency, load balancing and scaling to O (10^4 - 10^5) cores;

Extra Slides

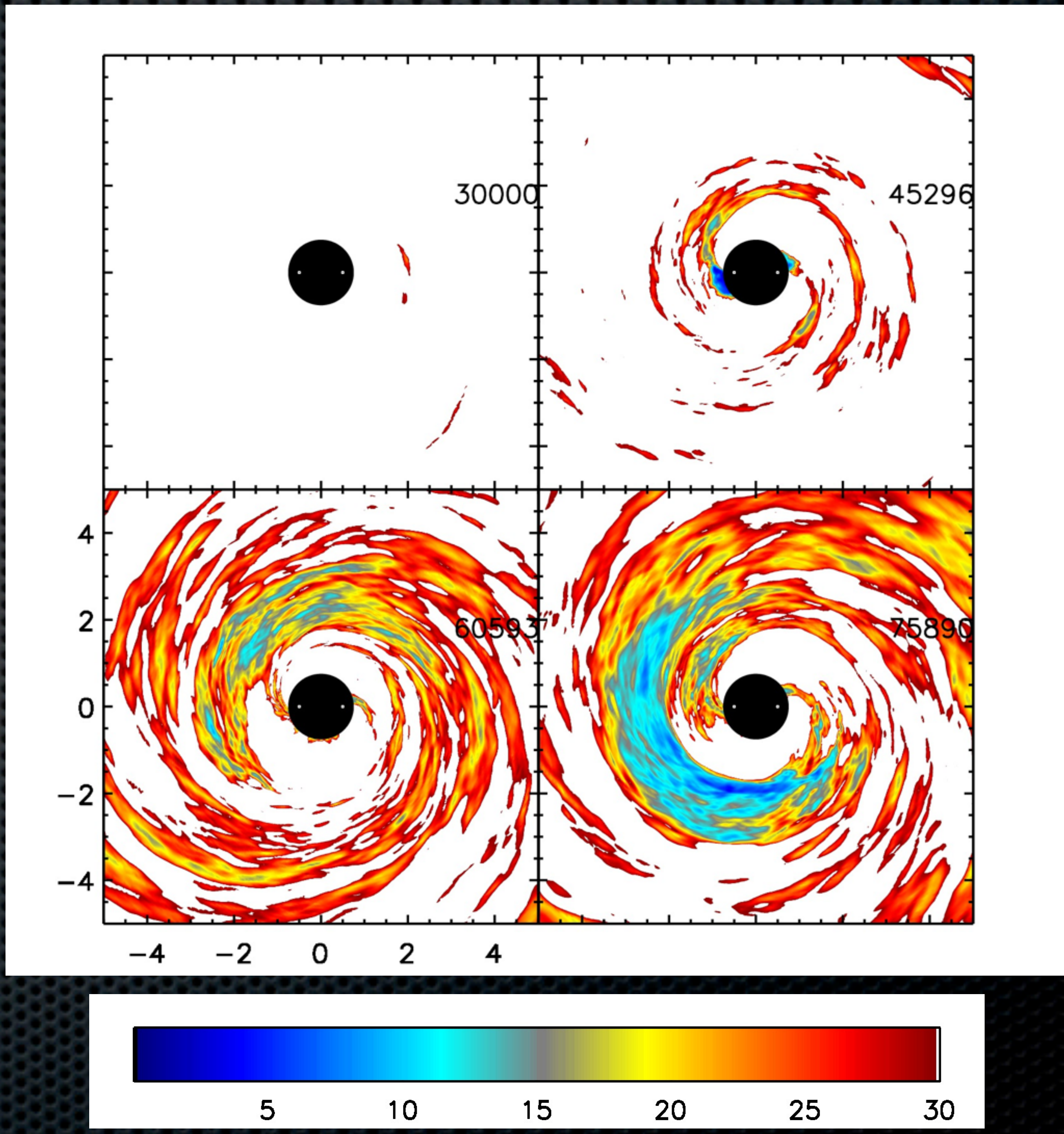
MRI Resolution

$$Q^i = \frac{2\pi \left|b^i\right|}{\Delta x^i \Omega(r) \sqrt{\rho h + 2p_m}}$$

Sano++ 2004 Noble++ 2010 Guan, Gammie 2010 Sorathia++ 2010, 2011 Hawley++ 2011

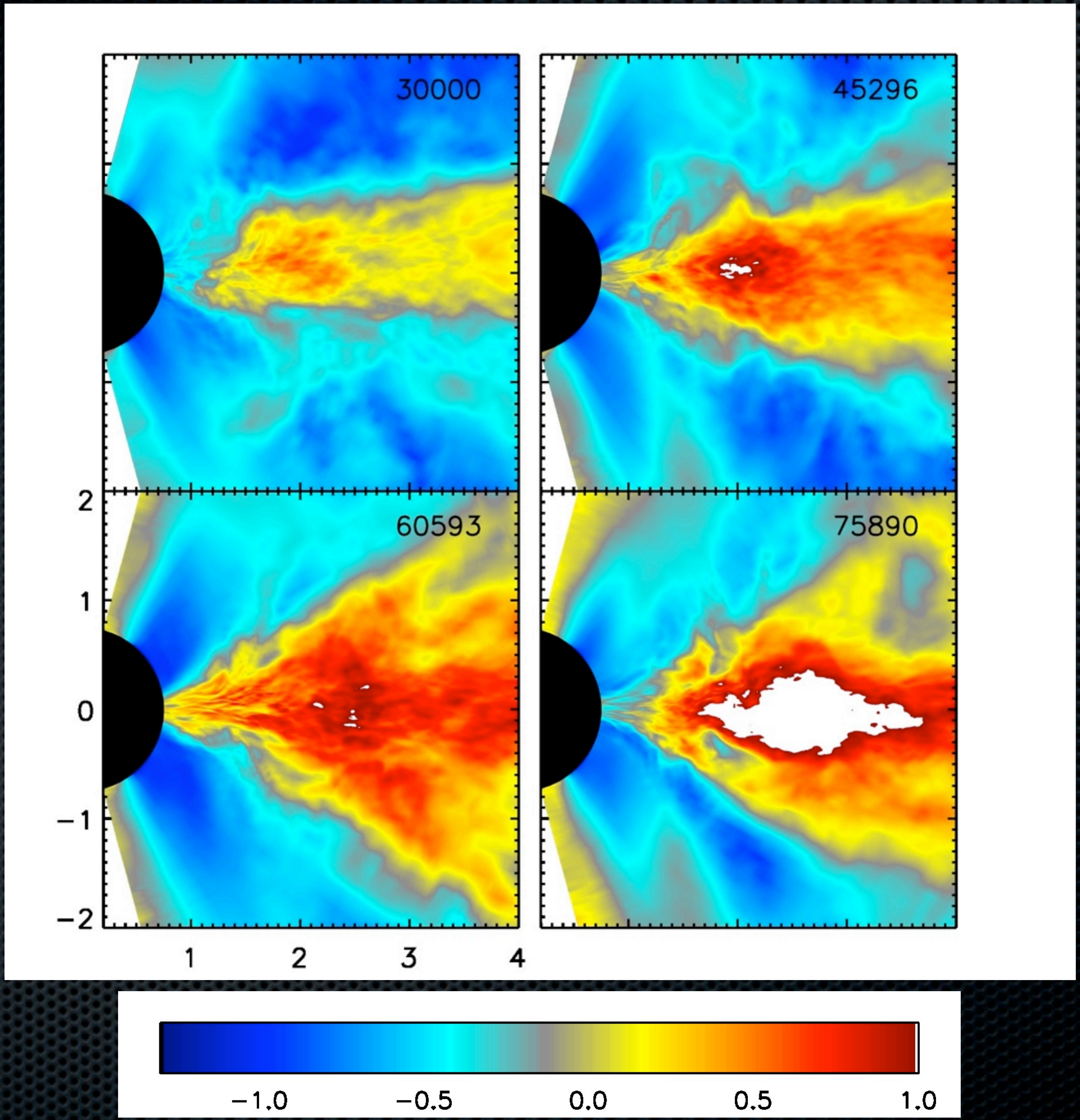


$Q^\theta > 10$



$Q^\phi > 25$

Plasma Beta parameter = pgas / pmag

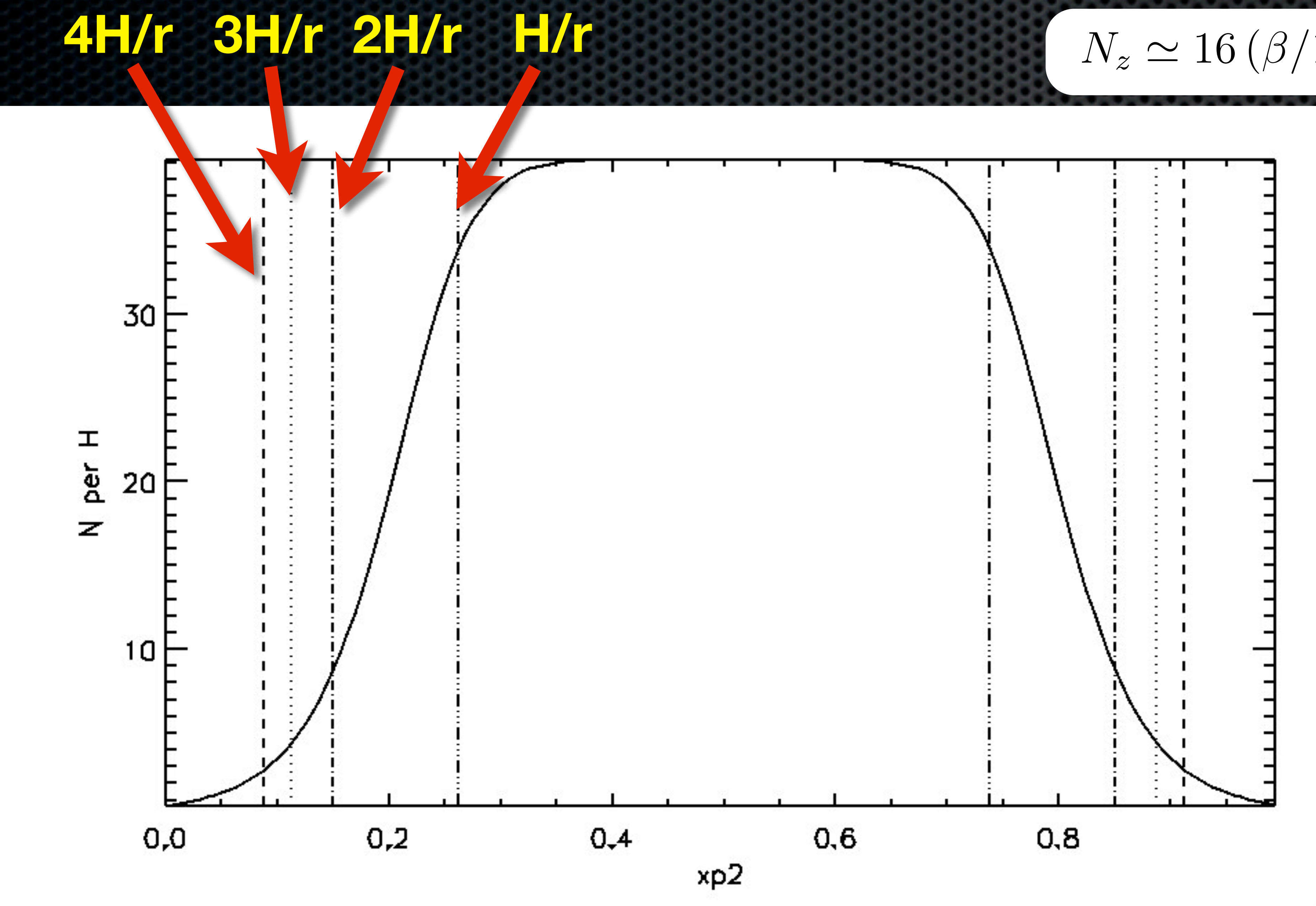


Resolution Constraints: MRI

Sano++ 2004 Noble++ 2010 Guan, Gammie 2010 Sorathia++ 2010, 2011
Hawley++ 2011

$$N_\phi \simeq 1000\ (0.1\ R/H)\ (\beta/100)^{1/2}\ (Q_\phi/10)$$

$$N_z \simeq 16\,(\beta/100)^{1/2}\left(\langle v_A^2\rangle/\langle v_{Az}^2\rangle\right)^{1/2}(Q_z/10)$$



$$Q_z = \lambda_{MRI}/\Delta z = \frac{2\pi|v_{az}|}{\Omega\Delta z},$$

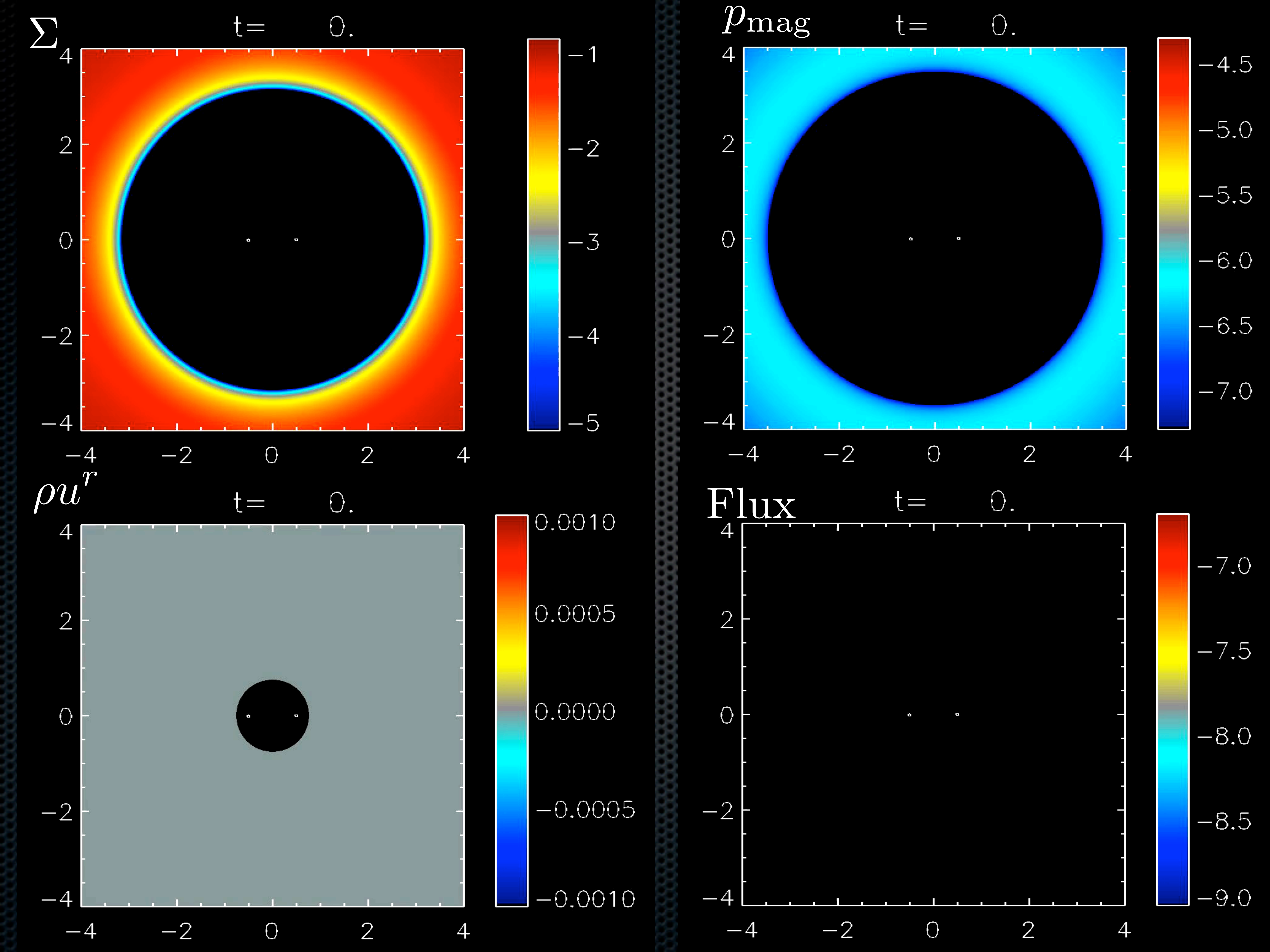
$$\beta_z/\beta \simeq 50$$

$$\beta \simeq 10$$

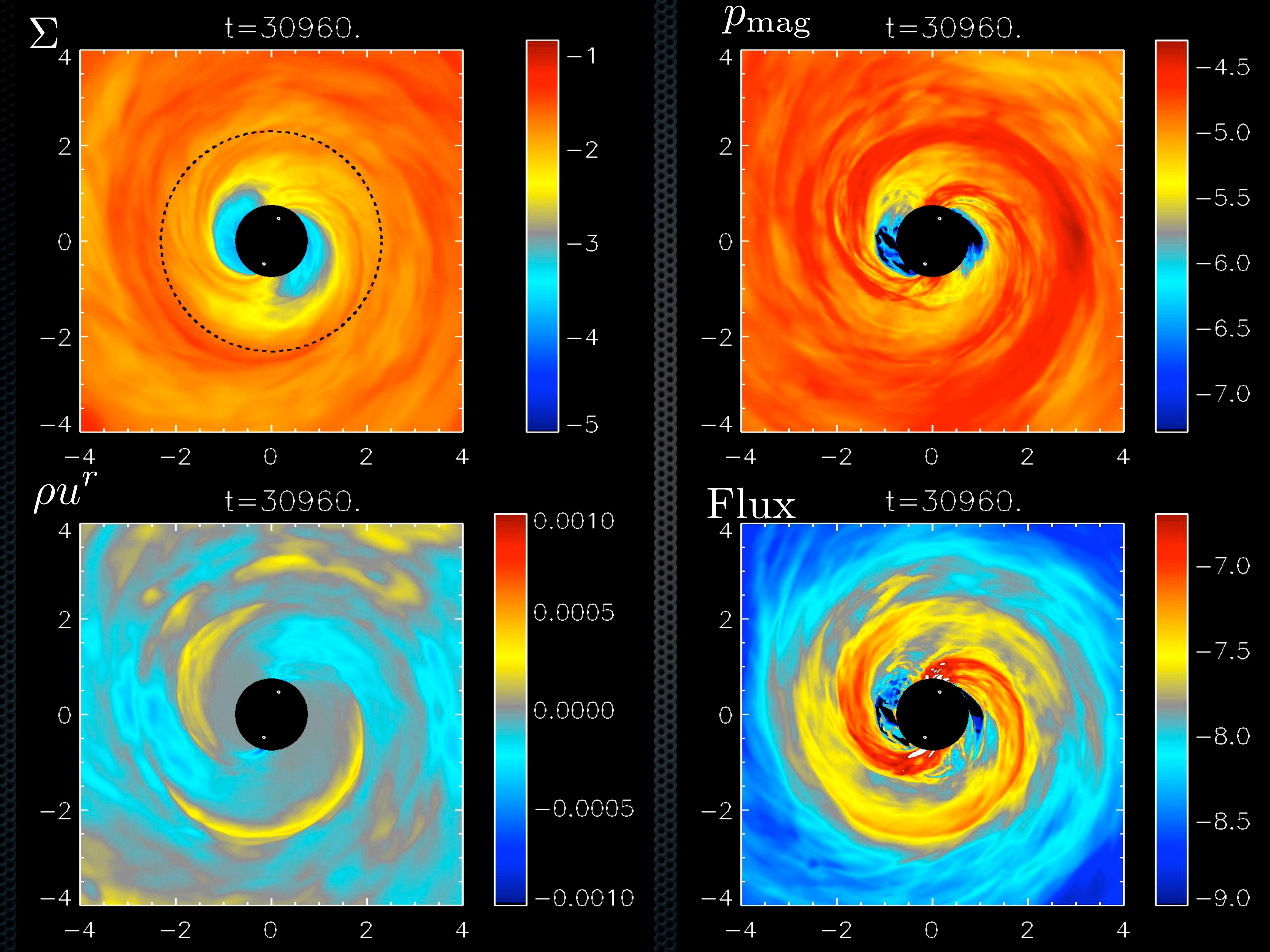
$$N_\theta > 36 \text{ per } H/r$$

$$\theta = \frac{\pi}{2}\left[1 + h_s\left(2x^2-1\right) + \left(1 - h_s - 2\theta_c/\pi\right)\left(2x^2-1\right)^n\right]$$

$$N_\theta = 160,\ h_s = 0.13,\ n = 9,\ \theta_c = 10^{-15}$$



Σ
t= 0.
p_{mag}
t= 0.
ρu^r
t= 0.
Flux
t= 0.
−1
−2
−3
−4
−5
−4.5
−5.0
−5.5
−6.0
−6.5
−7.0
0.0010
0.0005
0.0000
−0.0005
−0.0010
−7.0
−7.5
−8.0
−8.5
−9.0



Σ
t=30960.
p_{mag}
t=30960.
ρu^r
t=30960.
Flux
t=30960.