Loop Quantum Cosmology: Interplay between Theory and Observations

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Will summarize the work of many researchers; especially:
Agullo, Barrau, Bojowald, Cailleatau, Campiglia, Corichi, Grain, Gupt, Kaminski, Lewandowski, Mielczarek, Mena, Nelson, Olmedo, Pawlowski, Singh, Sloan, Velhinho ...

Beyond the First Century of General Relativity , RIT; May 30th, 2015

1. Setting the stage

- Observational and theoretical advances in cosmology have been spectacular. Starting from the tiny, one part in 10^{-5}, inhomogeneities seen in the CMB, known physics predicts that inhomogeneities will grow and lead to the observed large scale structure of the universe.

A Triumph: analogous to staring with a snapshot of a 1 day old baby and predicting what the person would look like when (s)he is 100 years old!!

- Physics is more ambitious: What is the origin of the tiny inhomogeneities in CMB? Primary motivation is conceptual and led to the inflationary scenario. Issue of origins pushed back from energy density (or curvature) of $\sim 2.5 \times 10^{-114}\,\rho_{\rm Pl}$ to $\sim 7.32 \times 10^{12}\rho_{\rm Pl}$!

- Spectacular as this is, inflationary scenario is conceptually incomplete because it assumes a classical space-time all the way back to the big bang. Even before we reach the big-bang, we face the so-called 'Trans Planckian Problems'.

- Goal of this talk is to argue that loop quantum cosmology has matured sufficiently to extend the inflationary scenario to the Planck regime, creating a useful dialog between theory and observations.

Conceptual summary: AA, Agullo & Nelson PRL 109, 251301 (2012);

Updated Status: AA & Barrau arXiv:1504.07559 ;

Viewpoint article: P. Singh, Physics: Spotlighting Exceptional Research, 5, 142 (2012.)

Inflationary Paradigm

• Major success: Prediction of inhomogeneities in CMB which serve as seeds for structure formation. Observationally relevant wave numbers in the range $\sim (k_o, 3000k_o)$ (radius of the observable CMB surface $\sim \lambda_o$).

• Rather minimal assumptions:

1. Sometime in its early history, the universe underwent a phase of accelerated expansion during which the Hubble parameter H was nearly constant.

2. Starting from this phase till the CMB era, the universe is well-described by a FLRW background with linear perturbations. Only matter: inflaton(s) in suitable potential(s).

3. At the onset of this 'slow roll inflationary phase' Fourier modes of quantum fields describing perturbations were in the Bunch-Davies vacuum (at least for co-moving wave numbers in the range $\sim (k_o,\, 2000k_o)$); and,

4. Soon after a mode exited the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

• Then, QFT on FLRW space-times (and classical GR) implies the existence of tiny inhomogeneities in CMB seen by the 7 year WMAP data.

All large scale structure emerged from vacuum fluctuations!

Limitations

But in its current status, inflation faces two sets of limitations.

Particle Physics Issues:

• Where from the inflaton? A single inflaton or multi-inflatons? Interactions between them? How are particles/fields of the standard model created during 'reheating'? ...

Quantum Gravity Issues: Focus of this talk. (Brandenberger, Martin, Starobinsky, ..)

• Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped since the 1970's? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

• Does the 'slow-roll' used to account for CMB observations arise from natural initial conditions 'at the Beginning' that replaces the big bang in quantum gravity?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

• Is the pre-inflationary dynamics compatible with the standard assumptions underlying inflation? Does it leave an observational imprint? Can observations inform theory?

Answers from Loop Quantum Cosmology: Status

- The Big Bang singularity is naturally resolved in LQG; replaced by a Big Bounce. Goals laid out by Wheeler, Misner and others were achieved, thanks to the quantum geometry effects that lie at the heart of LQG.

  In FLRW Models: $a(t), \phi(t) \quad \rightarrow \quad \Psi_o(a, \phi)$.

- Quantum states $\psi_{\rm pert}(T^{(1)}, T^{(2)}, \mathcal{R})$ of perturbations propagate on the quantum FLRW geometry $\Psi_o(a, \phi)$. Planck scale issues faced squarely.

- Natural initial conditions for $\Psi_o \otimes \psi_{\rm pert}$ at the bounce such that: i) The desired slow roll is achieved; and ii) back reaction of perturbations in $\psi_{\rm pert}$ on the quantum background Ψ_o remains negligible from the bounce to the onset of slow roll (evolution over 11 orders of magnitude in curvature).

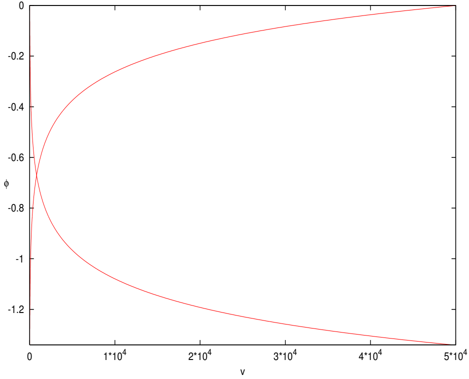
- Agreement with the standard inflation for $\ell \gtrsim 30$. But the pre-inflationary dynamics can leave imprints on large wave length modes. e.g., Power spectrum can be suppressed for $\ell \lesssim 30$. (Seems counter-intuitive at first. Will explain.)

Thus, LQG leads to an interplay between theory and observations in a direction that is complementary to those based in particle physics.

The rest of the talk will be devoted to explaining these points.

2. Singularity Resolution in LQC

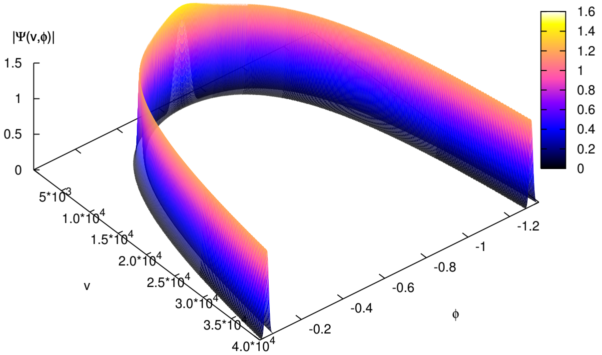
The Simplest Model: The k=0, $\Lambda = 0$ FRW Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

LQC Evolution

k=0 LQC with massless scalar field

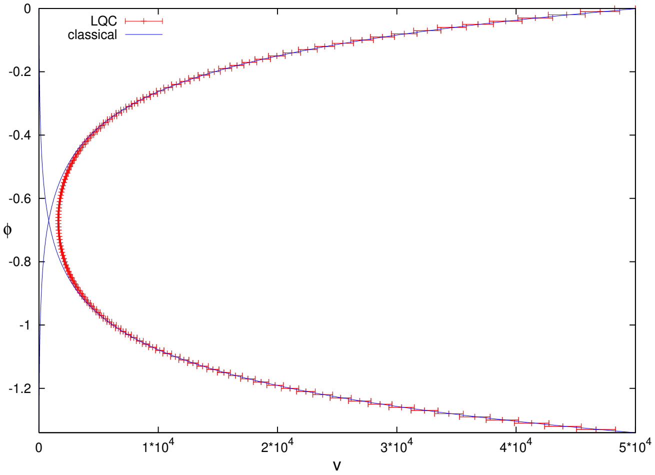


Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

LQC Evolution

k=0 LQC with massless scalar field



Expectations values (and dispersions) of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh)

What is behind this singularity resolution?

• LQG is based on a specific quantum Riemannian geometry. In FLRW models, quantum Einstein's equations dictate the (relational) evolution of $\Psi_o(a, \phi)$.

• The key modification well-captured in effective equations. For example, the effective Friedmann equation (AA, Pawlowski, Singh) :

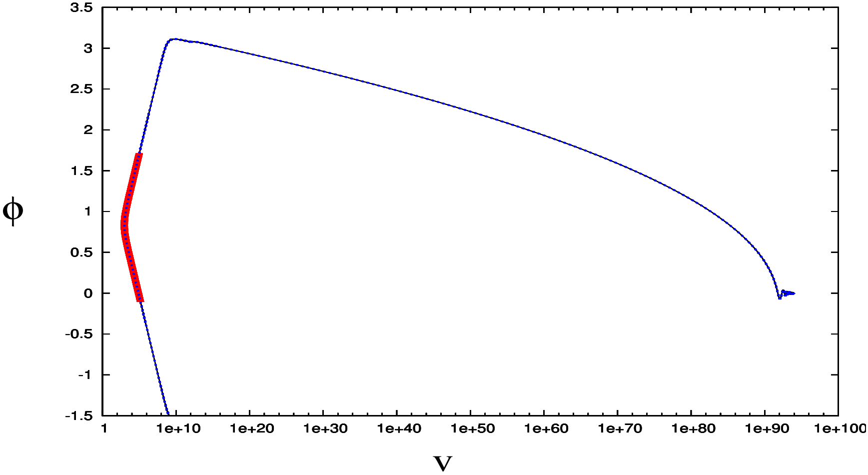
$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where} \quad \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce. Separation of scales: effects become negligible for $\rho << \rho_{\rm Pl}$. Eigenvalues of physical observables, such as matter density and curvature have an absolute upper bound on the physical Hilbert space. (AA, Corichi, Singh)

• Mechanism: No unphysical matter or new boundary conditions. Quantum geometry creates a brand new repulsive force in the Planck regime, overwhelming classical attraction. Understood in the Hamiltonian, Path integral and consistent histories frameworks. (AA, Campiglia, Henderson; Craig & Singh)

• Many generalizations: inclusion of spatial curvature, a cosmological constant Λ, anisotropies, . . . (Bojowald; AA, Pawlowski, Singh, Vandersloot; Lewandowski; Corichi; Wilson-Ewing; Brezuela, Martin-Benito, Mena, . . .). Qualitative summary: Every time a curvature scalar enters the Planck regime, the quantum geometry repulsive force dilutes it, preventing a blow up.

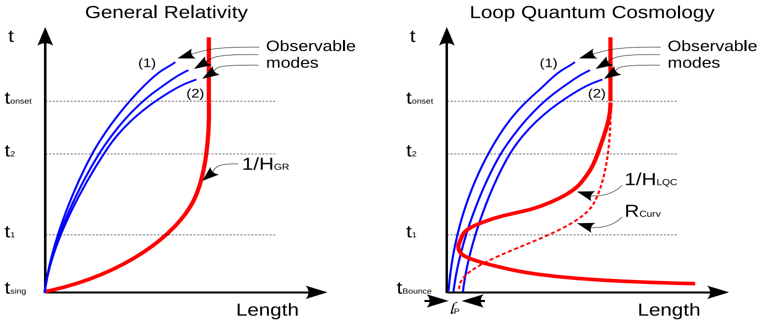
Singularity Resolution: $(1/2)m^2\phi^2$ Potential



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh). The Big Bang is replaced by a Big Bounce. Similar resolution in a wide class of cosmological models.

Why pre-inflationary dynamics matters

Contrary to a wide-spread belief, pre-inflationary dynamics does matter because modes with $\lambda_{\rm phys} > R_{\rm curv}$, the curvature radius, in the pre-inflationary era are excited and populated at the onset of inflation. They can leave imprints on CMB, naturally leading to 'anomalies' at low ℓs .



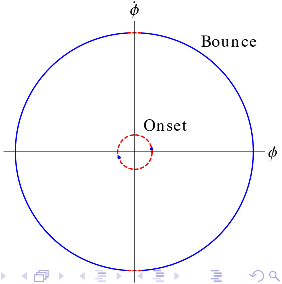
The UV LQG regularization tames the FLRW singularity. The new FLRW dynamics in turn affects the IR behavior of perturbations!

3. Background Quantum Geometry Ψ_o

• Let us begin with the effective theory, consider generic data at the bounce and evolve. Will the solution enter slow roll at curvature scale $\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4$ determined from the CMB data ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

• **Answer: YES**. In LQC, $|\phi_{\rm B}| \in (0, 7.47 \times 10^5)$. If $\phi_{\rm B} \geq 0.93$, the initial data evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future. In this sense, 'almost every' initial data at the bounce evolves to a solution that encounters the desired slow roll sometime in the future. (AA & Sloan; Further results: Corichi & Karami; Barrau & Linsefors)

• For the background quantum geometry, we can choose a 'coherent' state Ψ_o sharply peaked at an effective trajectory with $\phi_{\rm B} > 0.93$ and evolve using LQC. It remains sharply peaked on that effective trajectory. Hence the desired slow roll automatically occurs in this quantum geometry!



• Choice of the background geometry Ψ_o is dictated by ϕ_B; Free parameter in LQC.

4. Perturbations ψ on the Quantum Geometry Ψ_o

- Strategy: Assume perturbations ψ can be regarded as test fields on the quantum geometry Ψ_o, find solutions $\Psi_o \otimes \psi_{\rm pert}$, and finally check self consistency. Then, the Planck regime is dealt with squarely provided $\rho_{\rm Pert} \ll \rho_{\rm BG}$ all the way from the bounce to the onset of slow roll.

- Unforeseen Simplification: dynamics of perturbations $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o is mathematically equivalent to that of $\hat{T}^{(1)}, \hat{T}^{(2)}$, $\hat{\mathcal{R}}$ as quantum fields on a smooth space-time with a 'dressed' effective, c-number metric $\bar{g}_{ab}$ (whose coefficients depend on $\hbar$):

$$\bar{g}_{ab} dx^a dx^b = \bar{a}^2(-d\bar{\eta}^2 + d\vec{x}^2)$$

with

$$d\bar{\eta} = \langle \hat{H}_o^{-1/2} \rangle \, [\langle \hat{H}_o^{-1/2} \hat{a}^4 \hat{H}_o^{-1/2} \rangle]^{1/2} \, d\phi; \qquad \bar{a}^4 = (\langle \hat{H}_o^{-1/2} \, \hat{a}^4 \hat{H}_o^{-1/2} \rangle)/\langle \hat{H}_o^{-1} \rangle$$

where H_o is the Hamiltonian governing dynamics of Ψ_o. For the $\hat{\mathcal{R}}$ there is also a quantum corrected effective potential, $\bar{\mathcal{U}}(\bar{a}, \phi)$. Analogy with light propagating in a medium. (AA, Lewandowski, Kaminski; AA, Agullo, Nelson)

- Because of this, the mathematical machinery of adiabatic states, regularization and renormalization can be lifted to the QFT on cosmological QSTs under consideration. Result: Full mathematical control on dynamics starting from the bounce.

Initial Conditions on Perturbations ψ

(Recall: Ψ_o: Peaked at a generic effective trajectory.)

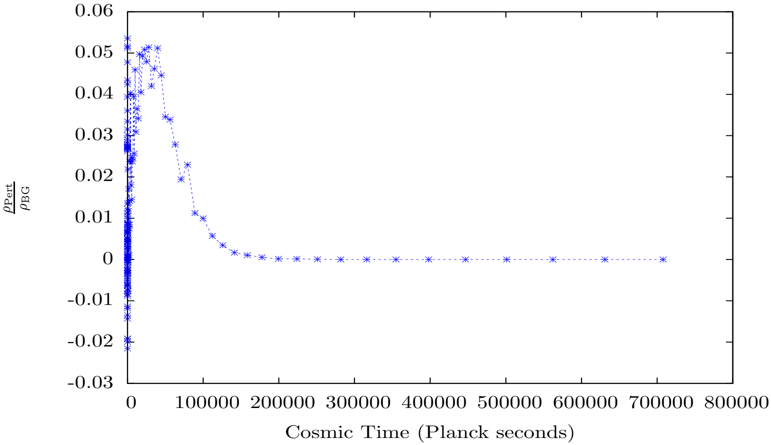
- ψ: Cannot use the BD vacuum because the pre-inflationary phase is far from de Sitter! Demand (i) Symmetry: Since Ψ_o (and hence $\bar{g}_{ab}$) is homogeneous and isotropic, demand that $\psi \in \mathcal{H}$ also invariant under translations and rotations; (ii) Regularity: is regular of 4th adiabatic order (w.r.t. $\bar{g}_{ab}$) and such that the back reaction of the perturbation ψ on the background Ψ_o can be ignored at the bounce; and, (iii) Initial conditions: energy density in ψ is negligible compare to that in the background Ψ_o.

- So far the emphasis has been only on establishing that one can extend the inflationary scenario to the Planck regime. For this, these three conditions suffice. Non-trivial feature: State has to be close to the BD vacuum at the onset of inflation for compatibility with observations!

- Work in progress on vastly narrowing down the initial conditions, i.e. looking for principles that may lead to a very small set of states $\Psi_o \otimes \psi$ at the bounce.

5. Dynamics and Results

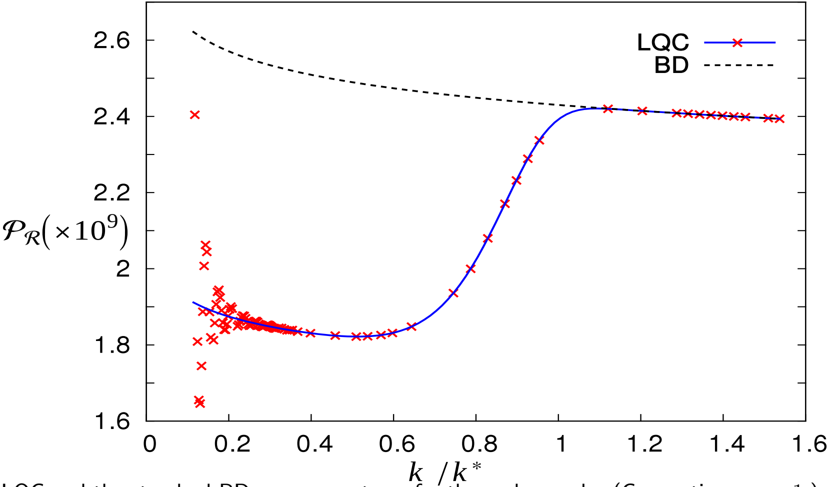
Facing trans-Planckian issues squarely: Is $\rho_{\rm Pert}/\rho_{\rm BG} \ll 1$ all the way from the bounce to the onset of slow roll? If so, self-consistency.



Yes!. Our initial conditions on ψ do ensure self-consistency of the test field approximation as hoped. Illustrative plot. (Agullo, AA, Nelson)

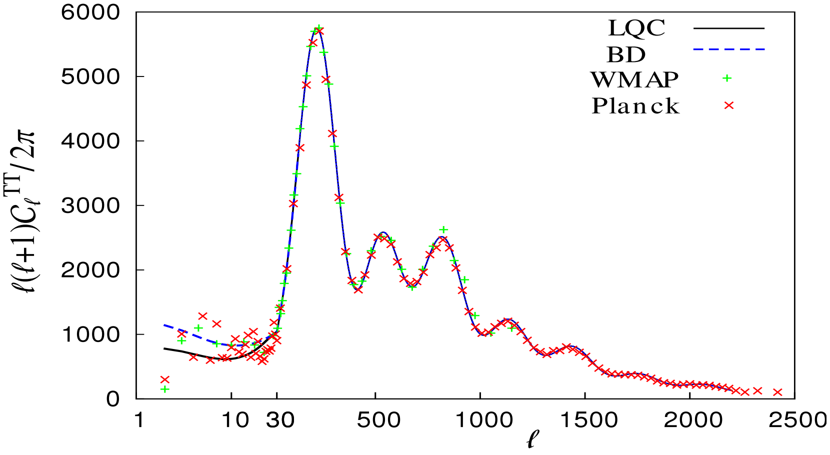
The Scalar Power spectrum

"Top-down approach"



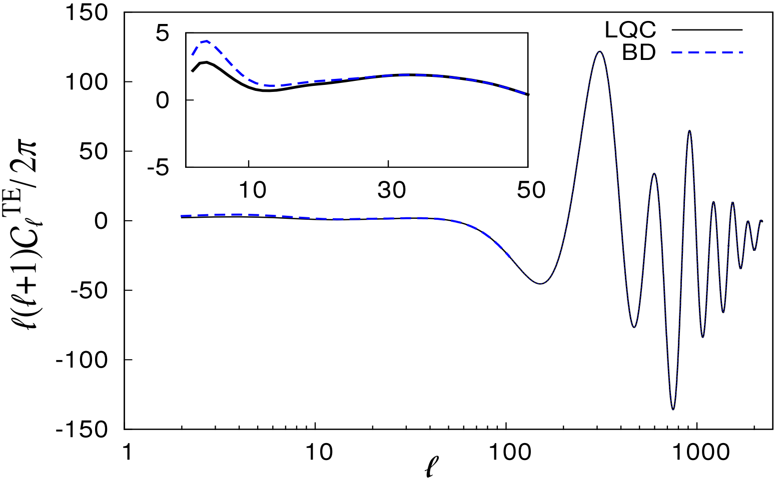
The LQC and the standard BD power spectrum for the scalar mode. (Convention $a_{\rm B} = 1$.) Red: Raw 'data' from LQC. blue: best fit curve. Here, the WMAP reference mode $k^{\star}_{\rm B}/a_{\rm B} = 54 m_{\rm Pl}$ and $k^{\rm min}_{\rm B}/a_{\rm B} = 6.3 m_{\rm Pl}$. (AA, Gupt)

LQC: Predicted TT-Power spectrum



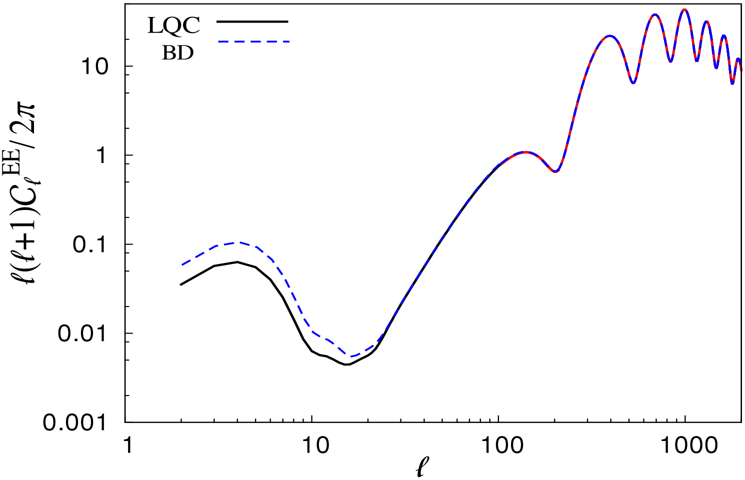
There exist permissible states $\Psi_o \otimes \psi$ such that the LQC power spectrum agrees with the standard BD power spectrum for $\ell \gtrsim 30$, but in LQC power is suppressed for $\ell \lesssim 30$. (AA, Gupt)

LQC: Predicted TE Correlations



The LQC prediction for the TE spectrum, for the initial state that gave the TT-spectrum in the last slide. Small suppression of power at small ℓ is a signature that the TT power suppression is of primordial origin. (AA, Gupt)

LQC: Predicted EE Correlations



The LQC prediction for the TE spectrum, for the initial state that gave the TT-spectrum in the last but one slide. The small suppression of power at small ℓ is a signature that the TT power suppression is of primordial origin. (AA, Gupt)

6. Summary

• The early universe provides an ideal setting to test quantum gravity ideas. Key questions: Can one obtain an extension of successful cosmological scenarios to include the Planck regime? Can the pre-inflationary, Planck scale dynamics leave observable imprints?

• No approach to quantum gravity is complete. Still in LQG progress could be made by truncating the classical theory to the physical problem under consideration and then passing to the quantum theory using LQG techniques. For inflation, the relevant sector: FLRW background with an inflation ϕ in a suitable potential as matter, together with first order perturbations.

Result: LQC provides a self-consistent extension of this sector.

• *Background geometry:* Singularity Resolution and precise quantum geometry for the Planck regime. Interesting to note that Einstein would not have been surprised:

> *"One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense."*
>
> *A. Einstein, 1945*

Perturbations

• Since they propagate on quantum geometry, using QFT on cosmological quantum geometries, (AA, Lewandowski, Kaminski), trans-Planckian issues can be handled systematically provided the test field approximation holds. There exist natural states $\Psi_o \otimes \psi_{\rm pert}$ in which it does. (Agullo, AA, Nelson).

In this scenario, the observable universe was a ball of radius $\sim 10\ell_{\rm Pl}$ at the Big Bounce. Qualitatively, the quantum geometry repulsive force of LQG provides a mechanism to 'explain' the extraordinary initial homogeneity and isotropy in this ball, making the pre-big-bounce history largely irrelevant for foreseeable observations.

• There are natural restrictions on initial conditions on $\Psi_{o\otimes}\psi$ at the bounce. In this allowed class, there is agreement with standard (BD-based) inflation for $\ell > 30$ or so. In this sense, LQC provides a natural extension of the inflationary paradigm over 12 orders of magnitude in curvature from the bounce to the onset of inflation.

Theory and Observations

• But for low values of ℓ, there can be deviations (in a small window for the parameter $\phi_{\rm B}$. For these states, pre-inflationary dynamics leaves an imprint. A new mechanism for primordial power suppression. For these states, LQC differs from the standard, BD-based inflation also for E-E and E-T correlations for $\ell < 30$. Other 'standard' predictions, such as the consistency relation $r = -8n_t$, is also modified for a single inflaton. These results open an avenue to see fundamental Planck scale physics in the cosmological observations.

• The issue of initial conditions. General physical considerations already constraint the state $\Psi_o \otimes \psi$ at the bounce. But it is not unique. Work in progress on uniqueness. Observations can potentially inform the theory! Possibility being pursued: A new physical principle (such as the quantum version of Penrose's Weyl curvature hypothesis) could lead to a preferred 'initial' state. Thus Loop quantum gravity has now sufficiently matured to create a 2-way bridge between the the Planck scale geometry and observations of the very early universe.

• But note that, so far, LQC does not take into account any of the particle physics issues. The analysis simply assumes an inflaton and a suitable potential. Therefore, it cannot imply that inflation must have occurred. On the other hand, the LQC framework can be, and is being, used to address quantum gravity issues also in non-inflationary scenarios.

Main References for this talk

- For a summary, see:

AA, Agullo & Nelson PRL 109, 251301 (2012);
Viewpoint article, P. Singh, Physics 5, 142 (2012);
AA, Barrau, **arXiv: 1504.07559**

- More complete references:

AA, Agullo & Nelson, PRD 87, 043507 (2013); CQG 30, 085014 (2013)
AA & Sloan, GRG (2011), PLB (2009); Corichi & Karami, PRD

AA, Corichi & Singh, PRD (2008); AA, Pawlowski, Singh, PRL & PRD (2006).

Other Results Referred to in the Talk:

- Future Observations:

Agullo & Parker PRD & GRG (2011); Agullo & Shandera JCAP (2012); Ganc & Koamtzu PRD (2012).

- A recent detailed Review of Loop Quantum Cosmology

AA & Singh, CQG (2011).

Supplementary Material

The slide that follows represents supplementary material, which was not included in the main talk because of the time limitation. It addresses a general questions.

Merits and Limitations of QC

One's first reaction to Quantum Cosmology is often: Symmetry reduction gives only toy models! Full theory much richer and much more complicated.

But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do not imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases.
(e.g. the work of the Madrid group on Gowdy models which have infinite degrees of freedom).
At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.