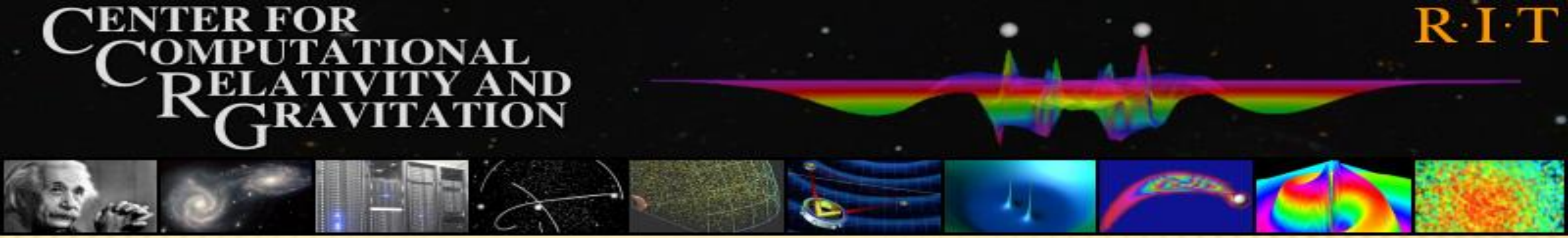

Null Geodesics in the 3+1 Formalism of Numerical Relativity

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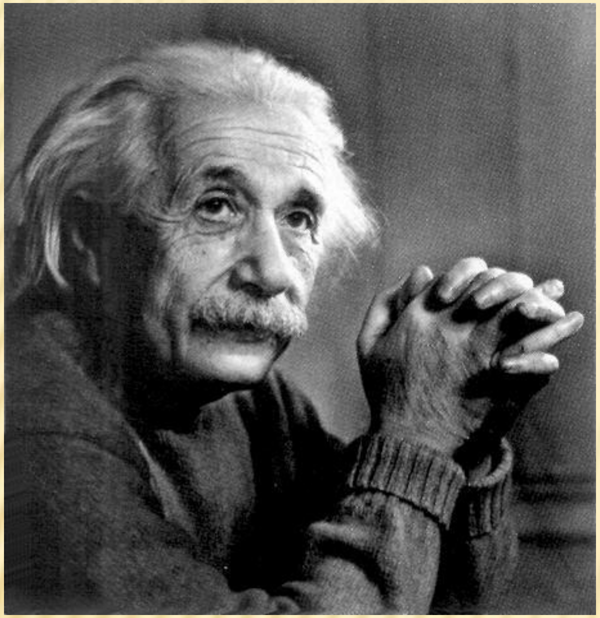
GENERAL RELATIVITY



•In 1905 Albert Einstein revolutionized conventional thinking by introducing his theory of Special Relativity.

•In 1915 Albert Einstein revealed to the world the Einstein's Field Equations, thus changing the Newtonian concept of gravity

•In General Relativity (GR), gravity is geometry …

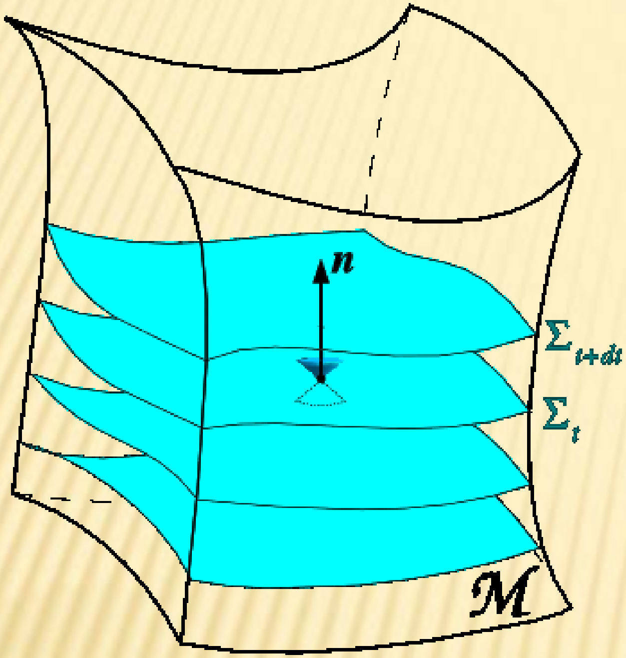
• Einstein's Equations relate the curvature of the space-time with the stress-energy of matter and fields.

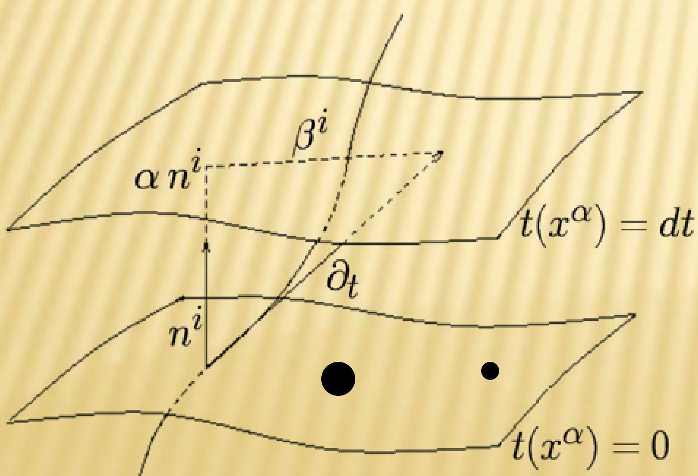
$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

• Gravity is geometry affected by mass-energy. They are intricately coupled in a bi-directional relationship.

•Geodesics are the paths the freely falling particles follow in space-time.

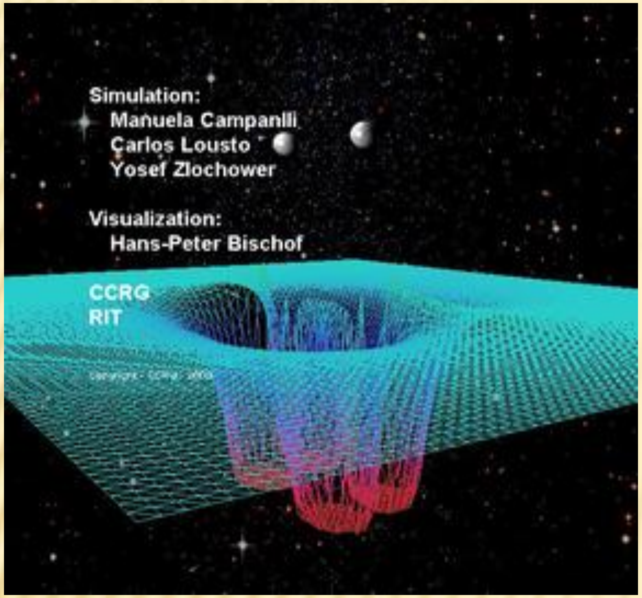
NUMERICAL RELATIVITY IN 3+1

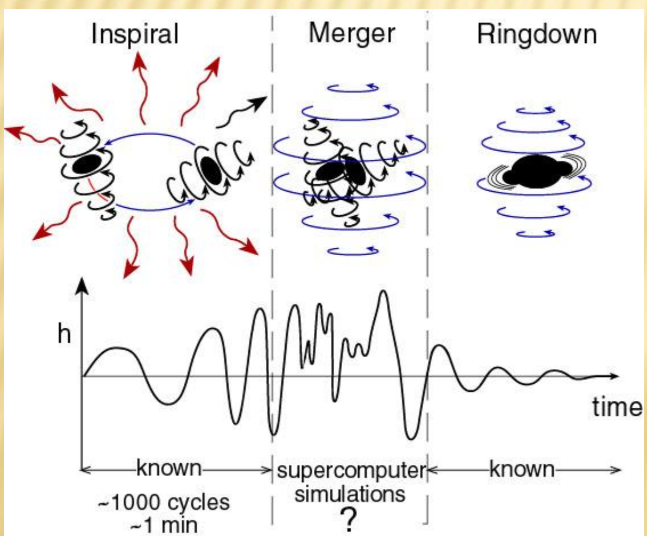




- Einstein Field Equations describe the gravitational field in a covariant way. That is to describe space-time where there is no clear distinction between space and time.
- But what if we are interested in the evolution of the gravitational field in time?
- We split the space-time into 3-D spacelike (t = const.) surface slices.
- We then represent black holes as *punctures* in an initial slice (t=0) (with given masses, spins and orbital parameters) and then solve the constraint equations.
- We choose a *lapse* α function (which tells us how time evolves at each point on the initial slice) and a *shift vector* β^i (which tells us how the spatial coordinates on each slice are related to each other).

WHAT NUMERICAL RELATIVITY CAN DO TODAY …





- Accurately and stably evolve arbitrary black hole binaries.

- Neutron star + Neutron star, Neutron star + black hole, other systems of BH and matter

- Produce accurate gravitational waveforms (black hole binaries convert up to 10% of mass to GW)
  - For use in GW data analysis
  - To calculate Recoil kicks (asymmetric radiation)
  - To calculate Remnant BH masses and spins

- Prove the validity of Post-Newtonian methods and BH's perturbation theory.

- Compare NR / PN dynamics and determine PN region of validity.

- Provide testable models for remnant kicks, masses and spins, merged BH retention and BH dynamics

GEODESICS IN THE 3+1 FORMALISM

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_k\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{\alpha^2} & \frac{\beta^i}{\alpha^2} \\ \frac{\beta^i}{\alpha^2} & \gamma^{ij} - \frac{\beta^i\beta^j}{\alpha^2} \end{pmatrix}$$

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda}(\partial_\nu g_{\sigma\lambda} + \partial_\sigma g_{\lambda\nu} - \partial_\lambda g_{\nu\sigma})$$

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^{\mu}_{\nu\sigma}\frac{dx^\nu}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0$$

$$\frac{dT^\mu}{d\lambda} + \Gamma^{\mu}_{\nu\sigma}T^\nu T^\sigma = 0$$

- Geodesics are the paths in space-time that parallel transports the tangent vector of the curve.

- A geodesic in Minkowski (flat space-time) is the notion of a straight line.

- In order to understand geodesics, we need to introduce the metric or line element. This mathematical abstraction encodes the geometry of our space time.

- The metric in 3+1 formalism can be shown as a 2 by 2 matrix, which components only contain terms if the shift vector, the lapse and the 3-metric.

- The geodesic equation reduces to the equation of the line in flat space.

- The Cristoffel symbols are the corrections to the notion of a straight line in curved space.

- The geodesic equation can also be expressed in covariant form, by introducing the tangent vector to the curve.

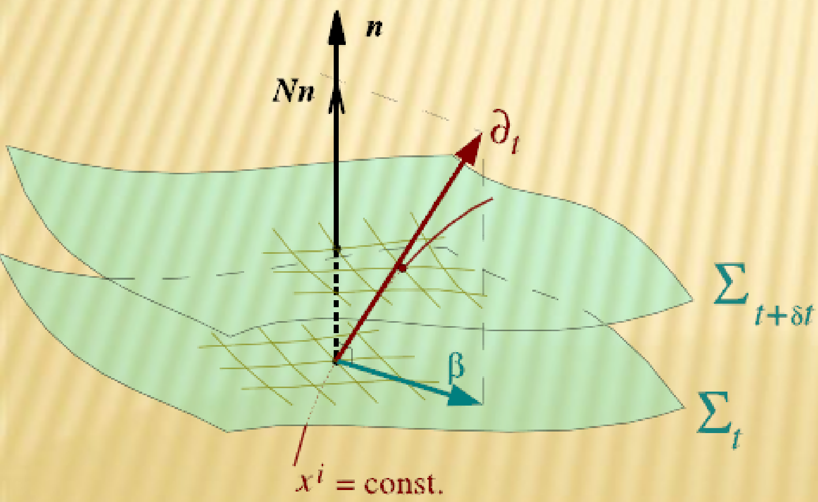
GEODESICS IN THE 3+1 FORMALISM

$$\frac{dT_\mu}{d\lambda} - \frac{1}{2}\partial_\mu g_{\rho\zeta} T^{(\rho}T^{\zeta)} = 0$$

$$T_\mu = f n_\mu + p_\mu$$

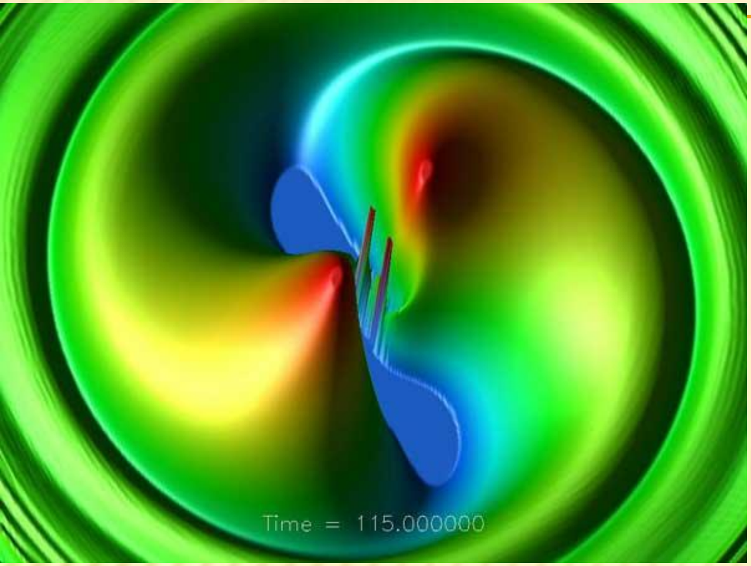
$$\frac{dp_a}{d\lambda} = \frac{1}{2}\partial_a g_{\mu\nu} T^\mu T^\nu$$

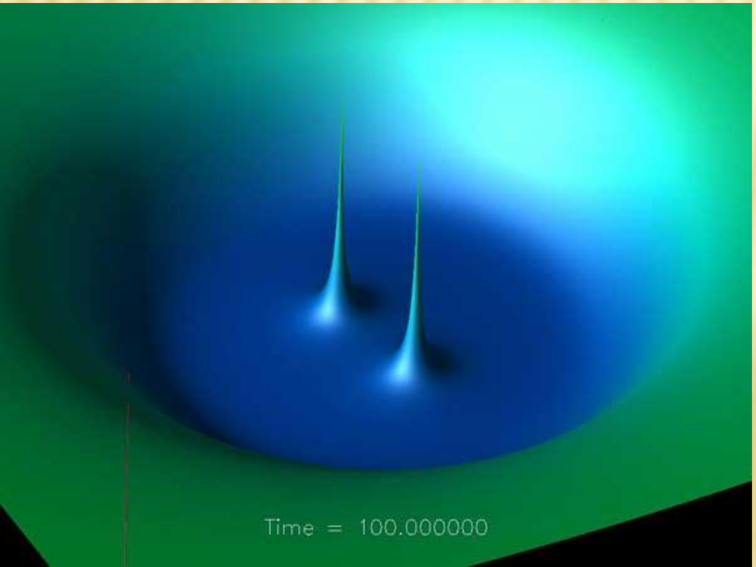
$$\frac{dp_a}{d\lambda} = -\alpha(p^0)^2\partial_a\alpha + p^0 p_k \partial_a \beta^i + \frac{p_i p_j}{2}\partial_a \gamma_{ij}$$



- Geodesics can be timelike, spacelike or null.
- Timelike geodesics are the paths that free falling massive particles follow. Null geodesics are the paths that freely falling massless particles follow.
- It can be shown, that the geodesic equation can be reduced to the derivative of the covariant tangent vector minus a term that only depends on spatial partial derivatives of the metric.
- The tangent vector to the curve can be defined as a linear combination of the normal vector and a tangent vector to the hypersurface.
- It is because of the definition of the tangent vector to the curve in terms of the normal and tangent vectors of the hypersurface that we can re-express the geodesic equation in terms of the tangent vector to the hypersurface.
- This last equation is used by Miguel Alcubierre's TimeGeodesic thorn in the Cactus framework. A thorn that integrates timelike geodesics.

NULL GEODESICS





• Why research null geodesics in 3+1 formalism?

• One answer is that it will allow us to understand what happens with electromagnetic radiation during binary black hole mergers where there is matter in the area of the event.

• Electromagnetic radiation is carried by a massless particle, the photon.

•By studying the paths that the photons will take during one of these events, we could understand what kind of electromagnetic signature these events produce.

• Observational astrophysicist could then look for these signatures in the universe and find direct evidence of BBH mergers.

• **"The important thing is not to stop questioning. Curiosity has its own reason for existing." – Albert Einstein**